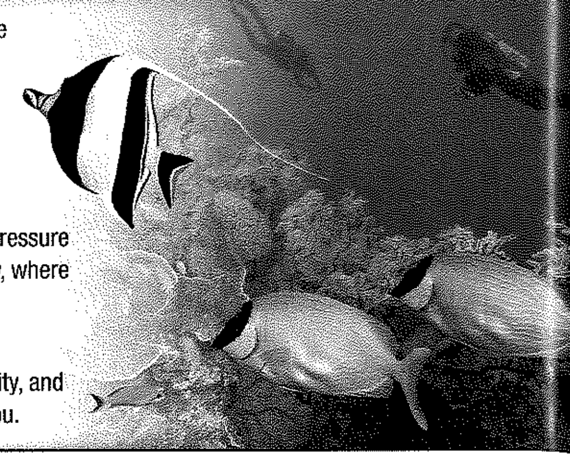
LESSON

1-6 Relations

:: Then	:: Now	:: Why?
You solved equations with one or two variables.	1 Represent relations. 2 Interpret graphs of relations.	The deeper in the ocean you are, the greater pressure is on your body. This is because there is more water over you. The force of gravity pulls the water weight down, creating a greater pressure.

The equation that relates the total pressure of the water to the depth is $P = rgh$, where

P = the pressure,
r = the density of water,
g = the acceleration due to gravity, and
h = the height of water above you.



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NewVocabulary

coordinate system
coordinate plane
x- and y-axes
origin
ordered pair
x- and y-coordinates
relation
mapping
domain
range
independent variable
dependent variable

CCSS Common Core State Standards

Content Standards

A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

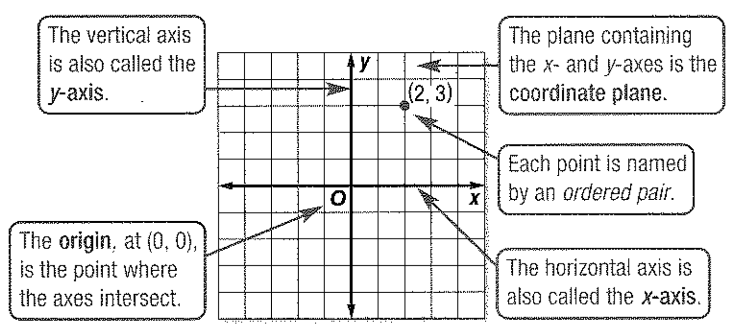
Mathematical Practices

1 Make sense of problems and persevere in solving them.

1 Represent a Relation

This relationship between the depth and the pressure exerted can be represented by a line on a coordinate grid.

A **coordinate system** is formed by the intersection of two number lines, the *horizontal axis* and the *vertical axis*.

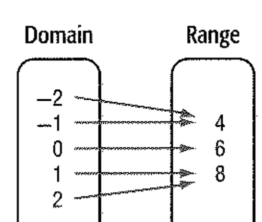


A point is represented on a graph using ordered pairs.

- An **ordered pair** is a set of numbers, or *coordinates*, written in the form (x, y).
- The x-value, called the **x-coordinate**, represents the horizontal placement of the point.
- The y-value, or **y-coordinate**, represents the vertical placement of the point.

A set of ordered pairs is called a **relation**. A relation can be represented in several different ways: as an equation, in a graph, with a table, or with a mapping.

A **mapping** illustrates how each element of the *domain* is paired with an element in the *range*. The set of the first numbers of the ordered pairs is the **domain**. The set of second numbers of the ordered pairs is the **range** of the relation. This mapping represents the ordered pairs $(-2, 4)$, $(-1, 4)$, $(0, 6)$ $(1, 8)$, and $(2, 8)$.



StudyTip

CCSS Sense-Making Each representation of the same relation serves a different purpose. Graphing the points can show the pattern between the points. A mapping shows you at a glance if elements are paired with the same element.

Study the different representations of the same relation below.

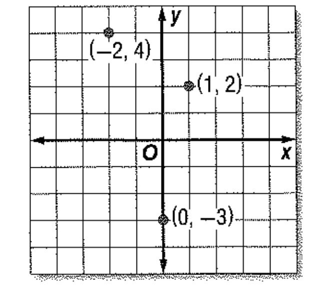
Ordered Pairs

(1, 2)
(−2, 4)
(0, −3)

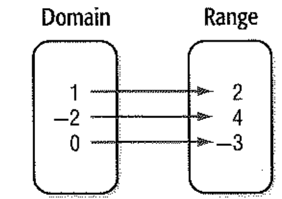
Table

x	y
1	2
−2	4
0	−3

Graph



Mapping



The x-values of a relation are members of the domain and the y-values of a relation are members of the range. In the relation above, the domain is $\{-2, 1, 0\}$ and the range is $\{-3, 2, 4\}$.

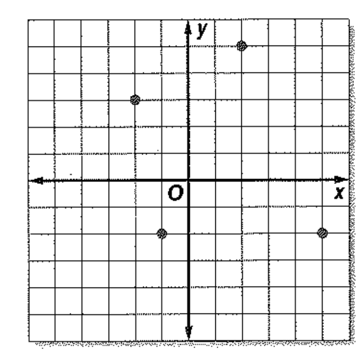
Example 1 Representations of a Relation

a. Express $\{(2, 5), (-2, 3), (5, -2), (-1, -2)\}$ as a table, a graph, and a mapping.

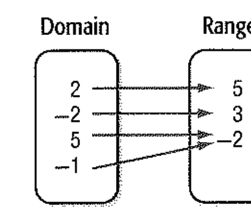
Table
Place the x-coordinates into the first column of the table. Place the corresponding y-coordinates in the second column of the table.

x	y
2	5
−2	3
5	−2
−1	−2

Graph
Graph each ordered pair on a coordinate plane.



Mapping
List the x-values in the domain and the y-values in the range. Draw arrows from the x-values in the domain to the corresponding y-values in the range.



b. Determine the domain and the range of the relation.

The domain of the relation is $\{2, -2, 5, -1\}$. The range of the relation is $\{5, 3, -2\}$.

GuidedPractice

1A. Express $\{(4, -3), (3, 2), (-4, 1), (0, -3)\}$ as a table, graph, and mapping.

1B. Determine the domain and range.

In a relation, the value of the variable that determines the output is called the **independent variable**. The variable with a value that is dependent on the value of the independent variable is called the **dependent variable**. The domain contains values of the independent variable. The range contains the values of the dependent variable.

Real-World Example 2 Independent and Dependent Variables

Identify the independent and dependent variables for each relation.

a. DANCE The dance committee is selling tickets to the Fall Ball. The more tickets that they sell, the greater the amount of money they can spend for decorations.

The number of tickets sold is the independent variable because it is unaffected by the money spent on decorations. The money spent on decorations is the dependent variable because it depends on the number of tickets sold.

b. MOVIES Generally, the average price of going to the movies has steadily increased over time.

Time is the independent variable because it is unaffected by the cost of attending the movies. The price of going to the movies is the dependent variable because it is affected by time.

Real-WorldLink

In 1948, a movie ticket cost \$0.36. In 2008, the average ticket price in the United States was \$7.18.

Source: National Association of Theatre Owners

GuidedPractice

Identify the independent and dependent variables for each relation.

2A. The air pressure inside a tire increases with the temperature.

2B. As the amount of rain decreases, so does the water level of the river.

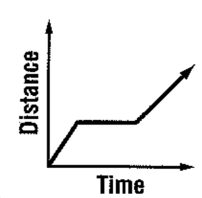
2 Graphs of a Relation

A relation can be graphed without a scale on either axis. These graphs can be interpreted by analyzing their shape.

Example 3 Analyze Graphs

The graph represents the distance Francesca has ridden on her bike. Describe what happens in the graph.

Bike Ride



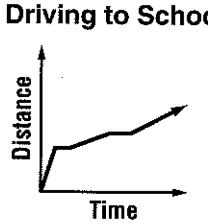
As time increases, the distance increases until the graph becomes a horizontal line.

So, time is increasing but the distance remains constant. At this section Francesca stopped. Then she continued to ride her bike.

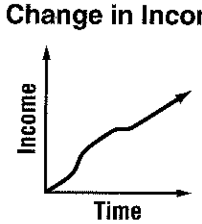
GuidedPractice

Describe what is happening in each graph.

3A. Driving to School



3B. Change in Income



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Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Example 1 **Express each relation as a table, a graph, and a mapping. Then determine the domain and range.**

1. {(4, 3), (−2, 2), (5, −6)}

2. {(5, −7), (−1, 4), (0, −5), (−2, 3)}

Example 2 **Identify the independent and dependent variables for each relation.**

3. Increasing the temperature of a compound inside a sealed container increases the pressure inside a sealed container.

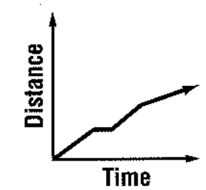
4. Mike's cell phone is part of a family plan. If he uses more minutes than his share, then there are fewer minutes available for the rest of his family.

5. Julian is buying concert tickets for himself and his friends. The more concert tickets he buys the greater the cost.

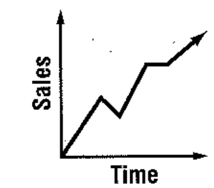
6. A store is having a sale over Labor Day weekend. The more purchases, the greater the profits.

Example 3 **CCSS MODELING Describe what is happening in each graph.**

7. The graph represents the distance the track team runs during a practice.



8. The graph represents revenues generated through an online store.



Practice and Problem Solving

Extra Practice is on page R1.

Example 1 **Express each relation as a table, a graph, and a mapping. Then determine the domain and range.**

9. {(0, 0), (−3, 2), (6, 4), (−1, 1)}

10. {(5, 2), (5, 6), (3, −2), (0, −2)}

11. {(6, 1), (4, −3), (3, 2), (−1, −3)}

12. {(−1, 3), (3, −6), (−1, −8), (−3, −7)}

13. {(6, 7), (3, −2), (8, 8), (−6, 2), (2, −6)}

14. {(4, −3), (1, 3), (7, −2), (2, −2), (1, 5)}

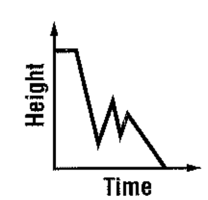
Example 2 **Identify the independent and dependent variables for each relation.**

15. The Spanish classes are having a fiesta lunch. Each student that attends is to bring a Spanish side dish or dessert. The more students that attend, the more food there will be.

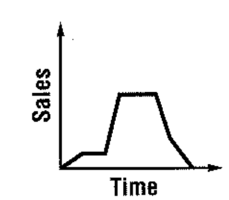
16. The faster you drive your car, the longer it will take to come to a complete stop.

Example 3 **CCSS MODELING Describe what is happening in each graph.**

17. The graph represents the height of a bungee jumper.

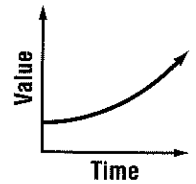


18. The graph represents the sales of lawn mowers.

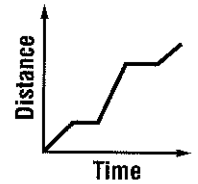


CCSS MODELING Describe what is happening in each graph.

19. The graph represents the value of a rare baseball card.

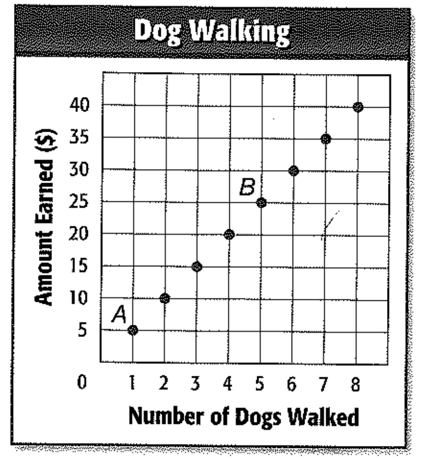


20. The graph represents the distance covered on an extended car ride.



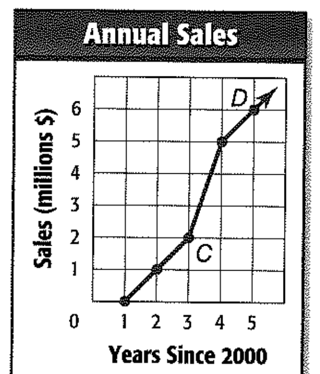
For Exercises 21–23, use the graph at the right.

21. Name the ordered pair at point *A* and explain what it represents.
22. Name the ordered pair at point *B* and explain what it represents.
23. Identify the independent and dependent variables for the relation.



For Exercises 24–26, use the graph at the right.

24. Name the ordered pair at point *C* and explain what it represents.
25. Name the ordered pair at point *D* and explain what it represents.
26. Identify the independent and dependent variables.

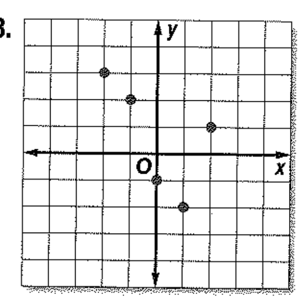


Express each relation as a set of ordered pairs. Describe the domain and range.

27.

Buying Aquarium Fish	
Number of Fish	Total Cost
1	$2.50
2	$4.50
5	$10.50
8	$16.50

28.

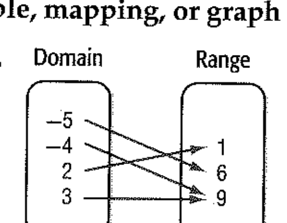


Express the relation in each table, mapping, or graph as a set of ordered pairs.

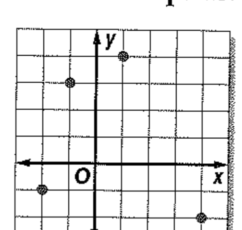
29.

x	y
4	−1
8	9
−2	−6
7	−3

30.

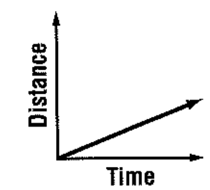


31.

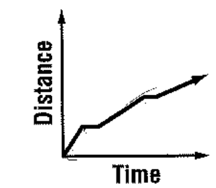


32. **SPORTS** In a triathlon, athletes swim 2.4 miles, bicycle 112 miles, and run 26.2 miles. Their total time includes transition time from one activity to the next. Which graph best represents a participant in a triathlon? Explain.

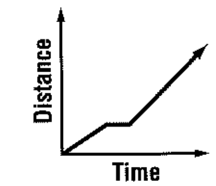
Graph A



Graph B



Graph C



Draw a graph to represent each situation.

33. **ANTIQUES** A grandfather clock that is over 100 years old has increased in value rapidly from when it was first purchased.
34. **CAR** A car depreciates in value. The value decreases quickly in the first few years.
35. **REAL ESTATE** A house typically increases in value over time.
36. **EXERCISE** An athlete alternates between running and walking during a workout.

37. **PHYSIOLOGY** A typical adult has about 2 pounds of water for every 3 pounds of body weight. This can be represented by the equation $w = 2\left(\frac{b}{3}\right)$, where w is the weight of water in pounds and b is the body weight in pounds.
   a. Make a table to show the relation between body and water weight for people weighing 100, 105, 110, 115, 120, 125, and 130 pounds. Round to the nearest tenth if necessary.
   b. What are the independent and dependent variables?
   c. State the domain and range, and then graph the relation.
   d. Reverse the independent and dependent variables. Graph this relation. Explain what the graph indicates in this circumstance.

H.O.T. Problems Use Higher-Order Thinking Skills

38. **OPEN ENDED** Describe a real-life situation that can be represented using a relation and discuss how one of the quantities in the relation depends on the other. Then represent the relation in three different ways.
39. **CHALLENGE** Describe a real-world situation where it is reasonable to have a negative number included in the domain or range.
40. **CCSS PRECISION** Compare and contrast dependent and independent variables.
41. **CHALLENGE** The table presents a relation. Graph the ordered pairs. Then reverse the y-coordinate and the x-coordinate in each ordered pair. Graph these ordered pairs on the same coordinate plane. Graph the line $y = x$. Describe the relationship between the two sets of ordered pairs.

x	y
0	1
1	3
2	5
3	7

42. **WRITING IN MATH** Use the data about the pressure of water on page 40 to explain the difference between dependent and independent variables.

Standardized Test Practice

43. A school's cafeteria employees surveyed 250 students asking what beverage they drank with lunch. They used the data to create the table below.

Beverage	Number of Students
milk	38
chocolate milk	112
juice	75
water	25

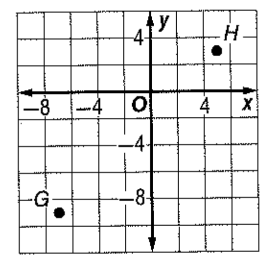
What percent of the students surveyed preferred drinking juice with lunch?

A 25% **C** 35%
B 30% **D** 40%

44. Which of the following is equivalent to $6(3 - g) + 2(11 - g)$?

F $2(20 - g)$ **H** $8(5 - g)$
G $8(14 - g)$ **J** $40 - g$

45. SHORT RESPONSE Grant and Hector want to build a clubhouse at the midpoint between their houses. If Grant's house is at point G and Hector's house is at point H, what will be the coordinates of the clubhouse?



46. If $3b = 2b$, which of the following is true?

A $b = 0$
B $b = \frac{2}{3}$
C $b = 1$
D $b = \frac{3}{2}$

Spiral Review

Solve each equation. (Lesson 1-5)

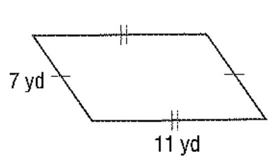
47. $6(a + 5) = 42$ **48.** $92 = k + 11$ **49.** $17 = \frac{45}{w} + 2$

50. HOT-AIR BALLOON A hot-air balloon owner charges $150 for a one-hour ride. If he gave 6 rides on Saturday and 5 rides on Sunday, write and evaluate an expression to describe his total income for the weekend. (Lesson 1-4)

51. LOLLIPOPS A bag of lollipops contains 19 cherry, 13 grape, 8 sour apple, 15 strawberry, and 9 orange flavored lollipops. What is the probability of drawing a sour apple flavored lollipop? (Lesson 0-11)

Find the perimeter of each figure. (Lesson 0-7)

52.



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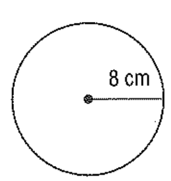

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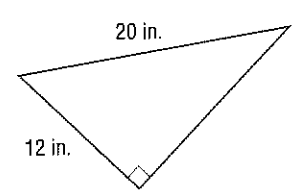


Skills Review

Evaluate each expression.

55. 8^2 **56.** $(-6)^2$ **57.** $(2.5)^2$

58. $(-1.8)^2$ **59.** $(3 + 4)^2$ **60.** $(1 - 4)^2$

LESSON 1-7 Functions

::Then

- You solved equations with elements from a replacement set.

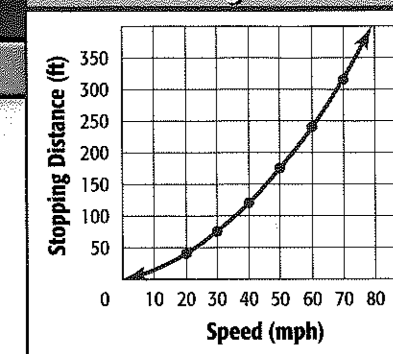
::Now

- **1** Determine whether a relation is a function.
- **2** Find function values.

::Why?

- The distance a car travels from when the brakes are applied to the car's complete stop is the stopping distance. This includes time for the driver to react. The faster a car is traveling, the longer the stopping distance. The stopping distance is a function of the speed of the car.

Stopping Distance of a Passenger Car



NewVocabulary

function
discrete function
continuous function
vertical line test
function notation
nonlinear function

Common Core State Standards

Content Standards

F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Mathematical Practices

3 Construct viable arguments and critique the reasoning of others.

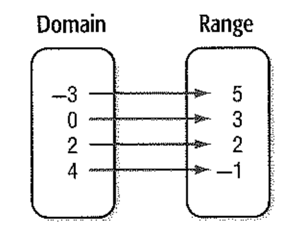
1 Identify Functions

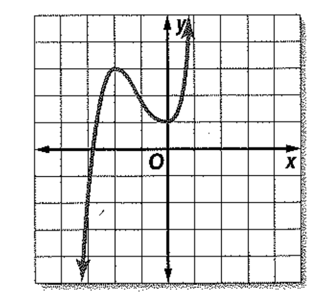
A **function** is a relationship between input and output. In a function, there is exactly one output for each input.

KeyConcept Function

Words A function is a relation in which each element of the domain is paired with *exactly* one element of the range.

Examples

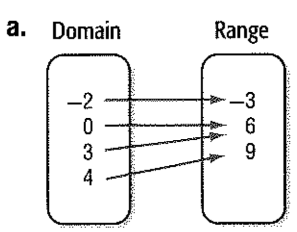




Example 1 Identify Functions

Determine whether each relation is a function. Explain.

a.



For each member of the domain, there is only one member of the range. So this mapping represents a function. It does not matter if more than one element of the domain is paired with one element of the range.

b.

Domain	1	3	5	1
Range	4	2	4	−4

The element 1 in the domain is paired with both 4 and −4 in the range. So, when x equals 1 there is more than one possible value for y. This relation is not a function.

GuidedPractice

1. $\{(2, 1), (3, -2), (3, 1), (2, -2)\}$