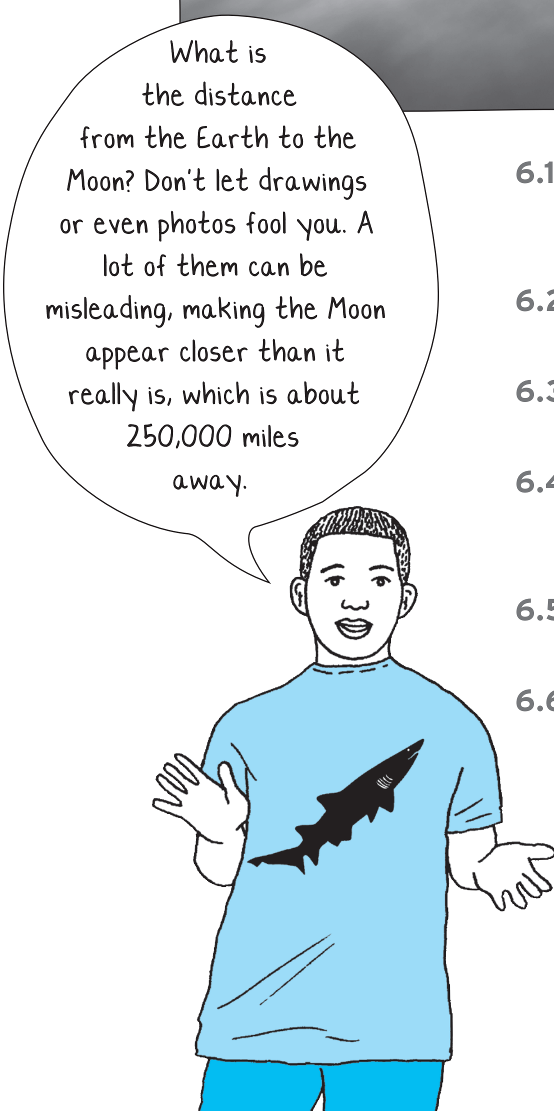
6 Pythagorean Theorem



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6.1 Soon You Will Determine the Right Triangle Connection

The Pythagorean Theorem

Learning Goals

In this lesson, you will:

- Use mathematical properties to discover the Pythagorean Theorem.
- Solve problems involving right triangles.

Key Terms

- right triangle
- right angle
- leg
- hypotenuse
- diagonal of a square
- Pythagorean Theorem
- theorem
- postulate
- proof

What do firefighters and roofers have in common? If you said they both use ladders, you would be correct! Many people who use ladders as part of their job must also take a class in ladder safety. What type of safety tips would you recommend? Do you think the angle of the ladder is important to safety?

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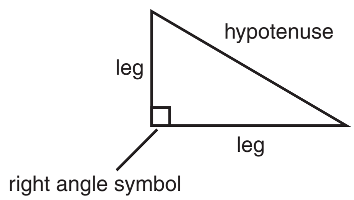
Problem 1 Identifying the Sides of Right Triangles

A **right triangle** is a triangle with a right angle. A **right angle** has a measure of 90° and is indicated by a square drawn at the corner formed by the angle. A **leg** of a right triangle is either of the two shorter sides. Together, the two legs form the right angle of a right triangle. The **hypotenuse** of a right triangle is the longest side. The hypotenuse is opposite the right angle.



1. The side lengths of right triangles are given. Determine which length represents the hypotenuse.

   a. 5, 12, 13

   b. 1, 1, $\sqrt{2}$

   c. 2.4, 5.1, 4.5

   d. 75, 21, 72

   e. 15, 39, 36

   f. 7, 24, 25

2. How did you decide which length represented the hypotenuse?

Problem 2 Exploring Right Triangles

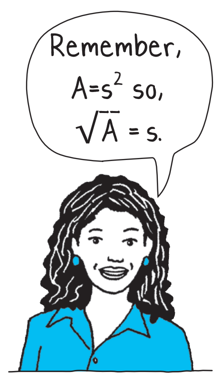
In this problem, you will explore three different right triangles. You will draw squares on each side of the triangles and then answer questions about the completed figures.

A **diagonal of a square** is a line segment connecting opposite vertices of the square. Let's explore the side lengths of more right triangles.

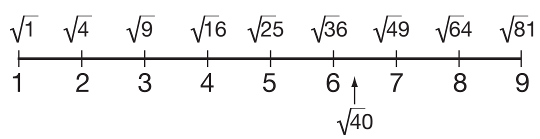
1. An isosceles right triangle is drawn on the grid shown on page 317.
   - **a.** A square on the hypotenuse has been drawn for you. Use a straightedge to draw squares on the other two sides of the triangle. Then use different colored pencils to shade each small square.
   - **b.** Draw two diagonals in each of the two smaller squares.
   - **c.** Cut out the two smaller squares along the legs. Then, cut those squares into fourths along the diagonals you drew.
   - **d.** Redraw the squares on the figure in the graphic organizer on page 327. Shade the smaller squares again.
   - **e.** Arrange the pieces you cut out to fit inside the larger square on the graphic organizer. Then, tape the triangles on top of the larger square.

Answer these questions in the graphic organizer.

   - **f.** What do you notice?
   - **g.** Write a sentence that describes the relationship among the areas of the squares.
   - **h.** Determine the length of the hypotenuse of the right triangle. Justify your solution.



Remember that you can estimate the value of a square root by using the square roots of perfect squares.



The square root of 40 is between $\sqrt{36}$ and $\sqrt{49}$, or between 6 and 7. $\sqrt{40} \approx 6.3$.

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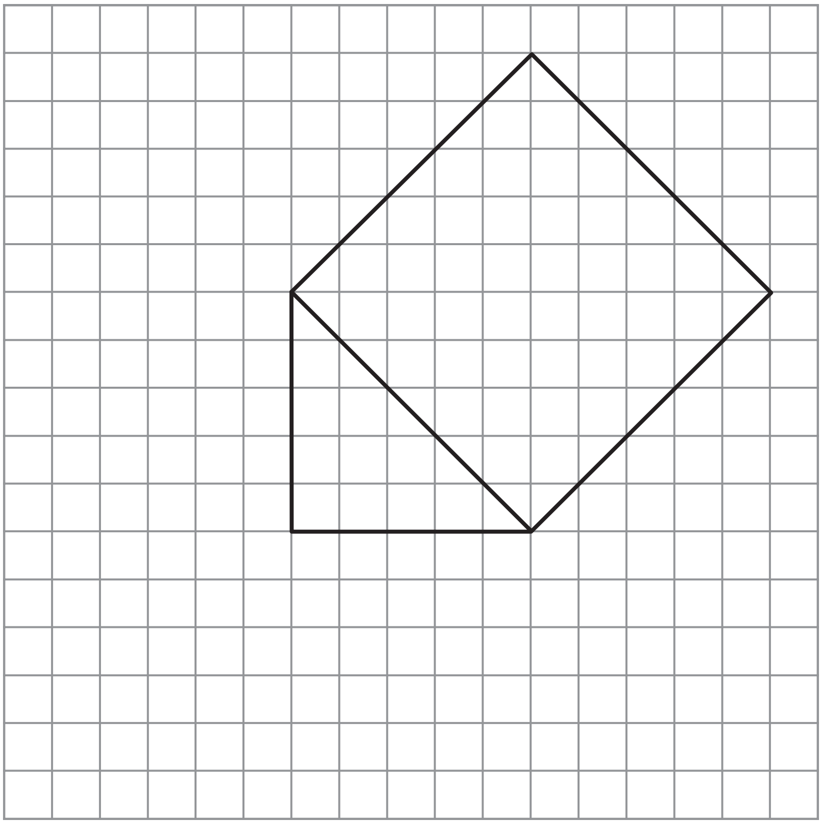

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2. A right triangle is shown on page 321 with one leg 4 units in length and the other leg 3 units in length.

   a. Use a straightedge to draw squares on each side of the triangle. Use different colored pencils to shade each square along the legs.

   b. Cut out the two smaller squares along the legs.

   c. Cut the two squares into strips that are either 4 units by 1 unit or 3 units by 1 unit.

   d. Redraw the squares on the figure in the graphic organizer on page 328. Shade the smaller squares again.

   e. Arrange the strips and squares you cut out on top of the square along the hypotenuse on the graphic organizer. You may need to make additional cuts to the strips to create individual squares that are 1 unit by 1 unit. Then, tape the strips on top of the square you drew on the hypotenuse.

Answer these questions in the graphic organizer.

   f. What do you notice?

   g. Write a sentence that describes the relationship among the areas of the squares.

   h. Determine the length of the hypotenuse. Justify your solution.

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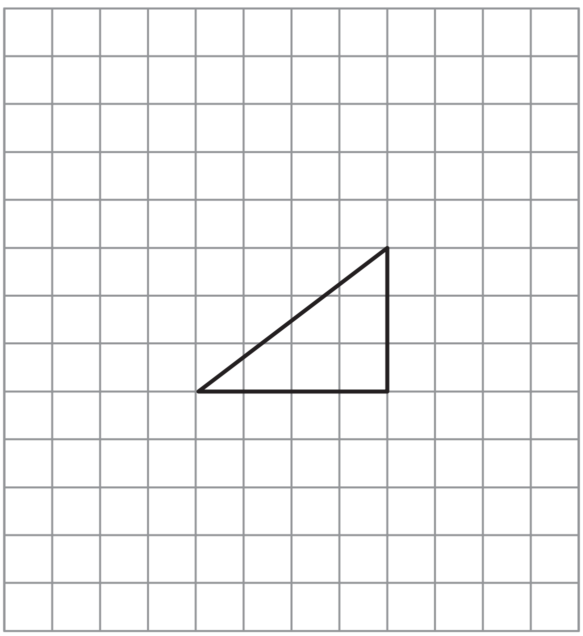

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3. A right triangle is shown on page 325 with one leg 2 units in length and the other leg 4 units in length.

   a. Use a straightedge to draw squares on each side of the triangle. Use different colored pencils to shade each square along the legs.

   b. Cut out the two smaller squares.

   c. Draw four congruent right triangles on the square with side lengths of 4 units. Then, cut out the four congruent right triangles you drew.

   d. Redraw the squares on the figure in the graphic organizer on page 329. Shade the smaller squares again.

   e. Arrange and tape the small square and the 4 congruent triangles you cut out over the square that has one of its sides as the hypotenuse.

Answer these questions in the graphic organizer.

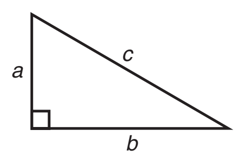
   f. What do you notice?

   g. Write a sentence that describes the relationship among the areas of the squares.

   h. Determine the length of the hypotenuse. Justify your solution.

4. Compare the sentences you wrote for part (f) in Questions 1, 2, and 3. What do you notice?

5. Write an equation that represents the relationship among the areas of the squares. Assume that the length of one leg of the right triangle is "*a*," the length of the other leg of the right triangle is "*b*," and the length of the hypotenuse is "*c*."



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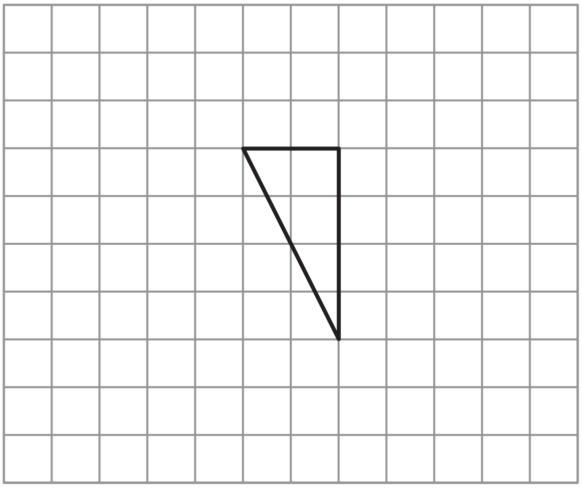

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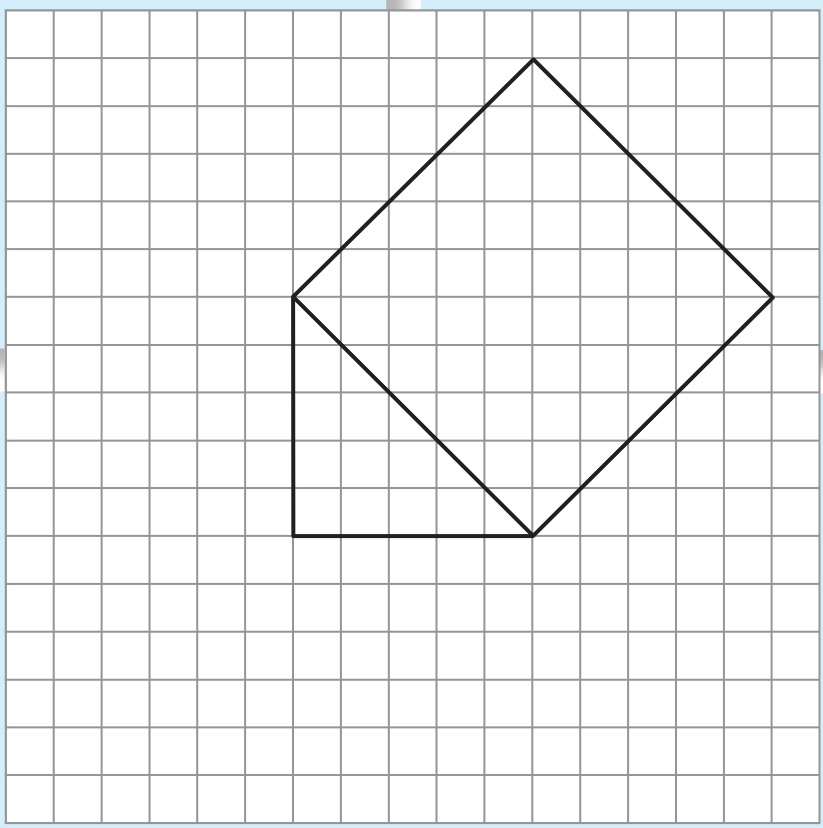
RIGHT TRIANGLE: BOTH LEGS WITH LENGTH OF 5 UNITS

WHAT DO YOU NOTICE?

DESCRIBE THE RELATIONSHIP AMONG THE AREAS OF THE SQUARES.

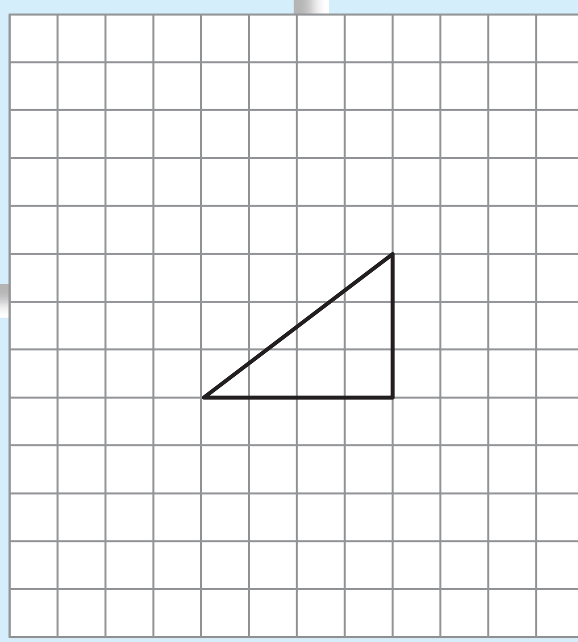


DETERMINE THE LENGTH OF THE HYPOTENUSE.

RIGHT TRIANGLE: ONE LEG WITH LENGTH OF 4 UNITS
AND THE OTHER LEG WITH LENGTH OF 3 UNITS

WHAT DO YOU NOTICE?

DESCRIBE THE RELATIONSHIP AMONG THE AREAS OF THE SQUARES.



DETERMINE THE LENGTH OF THE HYPOTENUSE

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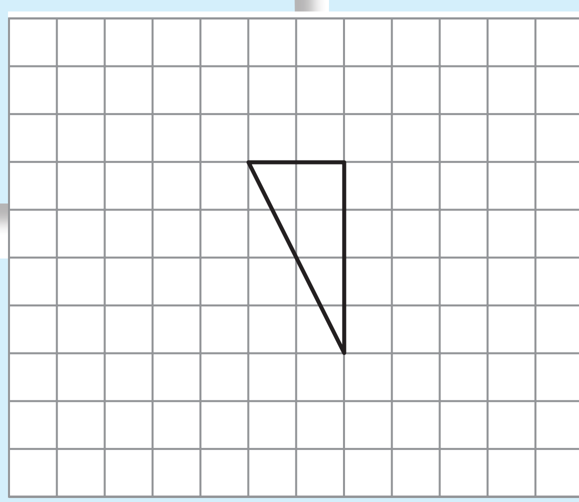
RIGHT TRIANGLE: ONE LEG WITH LENGTH OF 2 UNITS AND THE OTHER LEG WITH LENGTH OF 4 UNITS

WHAT DO YOU NOTICE?

DESCRIBE THE RELATIONSHIP AMONG THE AREAS OF THE SQUARES.



DETERMINE THE LENGTH OF THE HYPOTENUSE.

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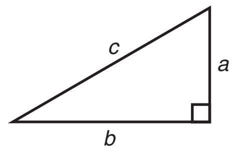
Problem 3 Special Relationships

The special relationship that exists between the squares of the lengths of the sides of a right triangle is known as the *Pythagorean Theorem*. The sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse.

The **Pythagorean Theorem** states that if a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$.



A **theorem** is a mathematical statement that can be proven using definitions, *postulates*, and other theorems. A **postulate** is a mathematical statement that cannot be proved but is considered true. The Pythagorean Theorem is one of the earliest known to ancient civilization and one of the most famous. This theorem was named after Pythagoras (580 to 496 B.C.), a Greek mathematician and philosopher who was the first to *prove* the theorem. A **proof** is a series of steps used to prove the validity of a theorem. While it is called the Pythagorean Theorem, the mathematical knowledge was used by the Babylonians 1000 years before Pythagoras. Many proofs followed that of Pythagoras, including ones proved by Euclid, Socrates, and even the twentieth President of the United States, President James A. Garfield.

1. Use the Pythagorean Theorem to determine the length of the hypotenuse:

   a. in Problem 2, Question 1.

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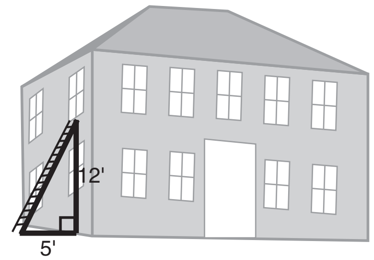
   b. in Problem 2, Question 3.

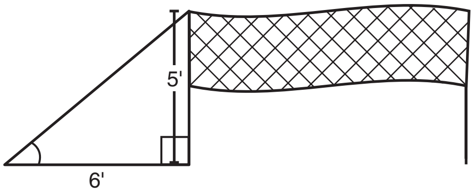
Problem 4 Maintaining School Grounds

Mitch maintains the Magnolia Middle School campus. Use the Pythagorean Theorem to help Mitch with some of his jobs.

1. Mitch needs to wash the windows on the second floor of a building. He knows the windows are 12 feet above the ground. Because of dense shrubbery, he has to put the base of the ladder 5 feet from the building. What ladder length does he need?



2. The gym teacher, Ms. Fisher, asked Mitch to put up the badminton net. Ms. Fisher said that the top of the net must be 5 feet above the ground. She knows that Mitch will need to put stakes in the ground for rope supports. She asked that the stakes be placed 6 feet from the base of the poles. Mitch has two pieces of rope, one that is 7 feet long and a second that is 8 feet long. Will these two pieces of rope be enough to secure the badminton poles? Explain your reasoning.



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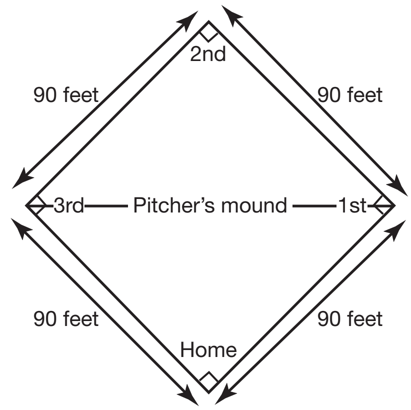
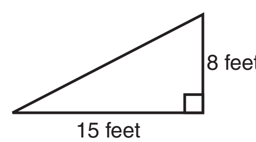
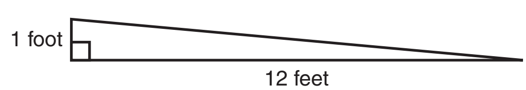
3. Mitch stopped by the baseball field to watch the team practice. The first baseman caught a line drive right on the base. He touched first base for one out and quickly threw the ball to third base to get another out. How far did he throw the ball?



4. The skate ramp on the playground of a neighboring park is going to be replaced. Mitch needs to determine how long the ramp is to get estimates on the cost of a new skate ramp. He knows the measurements shown in the figure. How long is the existing skate ramp?



5. A wheelchair ramp that is constructed to rise 1 foot off the ground must extend 12 feet along the ground. How long will the wheelchair ramp be?



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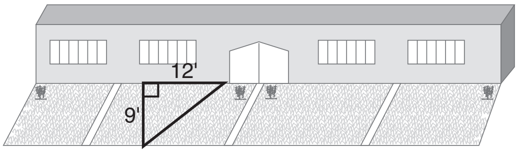
6. The eighth-grade math class keeps a flower garden in the front of the building. The garden is in the shape of a right triangle, and its dimensions are shown. The class wants to install a 3-foot-high picket fence around the garden to keep students from stepping onto the flowers. The picket fence they need costs \$5 a linear foot. How much will the fence cost? Do not calculate sales tax. Show your work and justify your solution.



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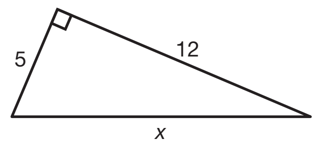
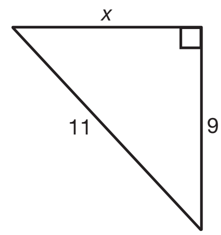
Problem 5 Solving for the Unknown Side

1. Write an equation to determine each unknown length. Then, solve the equation. Make sure your answer is simplified.

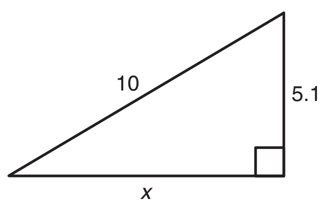
a.



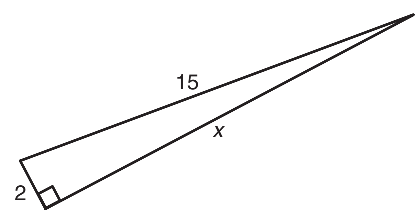
b.



c.



d.



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Be prepared to share your solutions and methods.

6.2 Can That Be Right?

The Converse of the Pythagorean Theorem

Learning Goal

In this lesson, you will:

- Use the Pythagorean Theorem and the Converse of the Pythagorean Theorem to determine unknown side lengths in right triangles.

Key Terms

- converse
- Converse of the Pythagorean Theorem
- Pythagorean triple

"Mind your *p*'s and *q*'s!" This statement usually refers to reminding a person to watch their manners. While the definition is easy to understand, the origin of this saying is not clear. Some people think that it comes from a similar reminder for people to remember their "please and thank-yous" where the "*q*'s" rhymes with "yous." Others believe that it was a reminder to young children not to mix up *p*'s and *q*'s when writing because both letters look very similar.

However, maybe the origin of this saying comes from math. When working with theorems (as you did in the last lesson), mathematicians encounter if-then statements. Often, if-then statements are defined as "if *p*, then *q*," with the *p* representing an assumption and the *q* representing the outcome of the assumption. So, just maybe math played a role in this saying.

Problem 1 The Converse

The Pythagorean Theorem can be used to solve many problems involving right triangles, squares, and rectangles. The Pythagorean Theorem states that in a right triangle, the square of the hypotenuse length equals the sum of the squares of the leg lengths. In other words, if you have a right triangle with a hypotenuse of length c and legs of lengths a and b, then $a^2 + b^2 = c^2$.

The **converse** of a theorem is created when the if-then parts of that theorem are exchanged.

The **Converse of the Pythagorean Theorem** states that if $a^2 + b^2 = c^2$, then the triangle is a right triangle.

If the lengths of the sides of a triangle satisfy the equation $a^2 + b^2 = c^2$, then the triangle is a right triangle.

1. Determine whether the triangle with the given side lengths is a right triangle.

a. 9, 12, 15

b. 24, 45, 51

c. 25, 16, 9

d. 8, 8, 11

You may have noticed that each of the right triangles in Question 1 had side lengths that were integers. Any set of three positive integers a, b, and c that satisfies the equation $a^2 + b^2 = c^2$ is a **Pythagorean triple.** For example, the integers 3, 4, and 5 form a Pythagorean triple because $3^2 + 4^2 = 5^2$.

2. Complete the table to identify more Pythagorean triples.

	a	b	c	Check: $a^2 + b^2 = c^2$
Pythagorean triple	3	4	5	$9 + 16 = 25$
Multiply by 2				
Multiply by 3				
Multiply by 5				

3. Determine a new Pythagorean triple not used in Question 2, and complete the table.

	a	b	c	Check: $a^2 + b^2 = c^2$
Pythagorean triple				
Multiply by 2				
Multiply by 3				
Multiply by 5				

4. Record other Pythagorean triples that your classmates determined.

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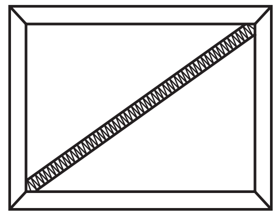
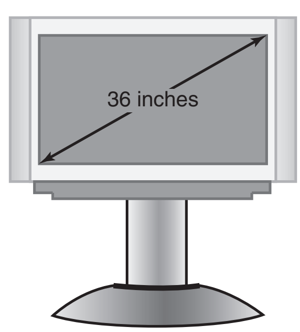
Problem 2 Solving Problems

1. A carpenter attaches a brace to a rectangular-shaped picture frame. If the dimensions of the picture frame are 30 inches by 40 inches, what is the length of the brace?



2. Bill is building a rectangular deck that will be 8 feet wide and 15 feet long. Tyrone is helping Bill with the deck. Tyrone has two boards, one that is 8 feet long and one that is 7 feet long. He puts the two boards together, end to end, and lays them on the diagonal of the deck area, where they just fit. What should he tell Bill?

3. A television is identified by the diagonal measurement of the screen. A television has a 36-inch screen whose height is 22 inches. What is the length of the television screen? Round your answer to the nearest inch.



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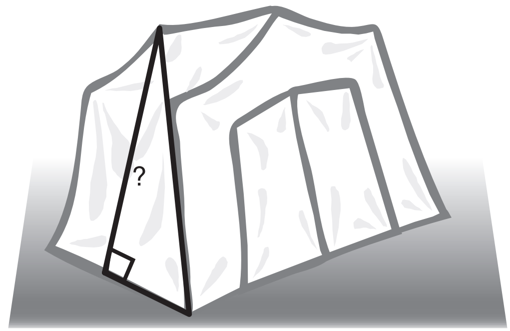
4. Orville and Jerri want to put a custom-made, round table in their dining room. The table top is made of glass with a diameter of 85 inches. The front door is 36 inches wide and 80 inches tall. Orville thinks the table top will fit through the door, but Jerri does not. Who is correct and why?

5. Sherie makes a canvas frame for a painting using stretcher bars. The rectangular painting will be 12 inches long and 9 inches wide. How can she use a ruler to make sure that the corners of the frame will be right angles?

6. A 10-foot ladder is placed 4 feet from the edge of a building. How far up the building does the ladder reach? Round your answer to the nearest tenth of a foot.

7. Chris has a tent that is 64 inches wide with a slant length of 68 inches on each side. What is the height of the center pole needed to prop up the tent?



8. A ship left shore and sailed 240 kilometers east, turned due north, then sailed another 70 kilometers. How many kilometers is the ship from shore by the most direct path?

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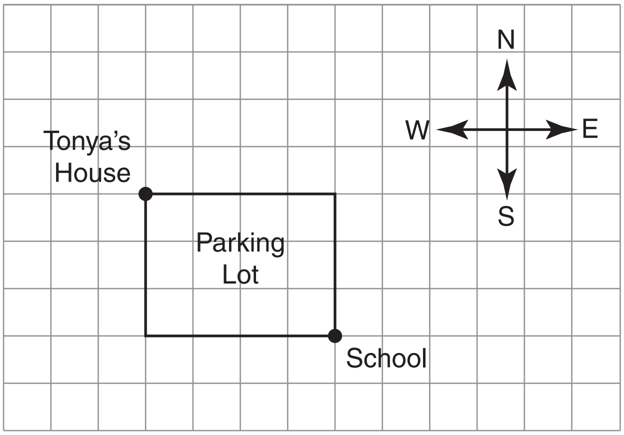
9. Tonya walks to school every day. She must travel 4 blocks east and 3 blocks south around a parking lot. Upon arriving at school, she realizes that she forgot her math homework. In a panic, she decides to run back home to get her homework by taking a shortcut through the parking lot.



a. Describe how many blocks long Tonya's shortcut is.

b. How many fewer blocks did Tonya walk by taking the shortcut?

10. Danielle walks 88 feet due east to the library from her house. From the library, she walks 187 feet northwest to the corner store. Finally, she walks 57 feet from the corner store back home. Does she live directly south of the corner store? Justify your answer.

11. What is the diagonal length of a square that has a side length of 10 cm?

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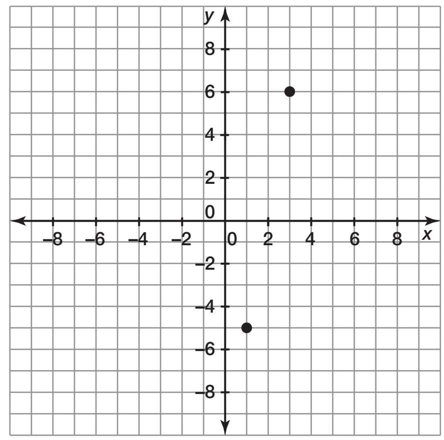
12. Calculate the length of the segment that connects the points (1, −5) and (3, 6).



a. Write your answer as a radical.

b. Write your answer as a decimal rounded to the nearest hundredth.

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Be prepared to share your solutions and methods.

6.3 Pythagoras to the Rescue

Solving for Unknown Lengths

Learning Goal

In this lesson, you will:

- Use the Pythagorean Theorem and the Converse of the Pythagorean Theorem to determine the unknown side lengths in right triangles.

There's a very famous mathematical scene in the movie *The Wizard of Oz*. At the end, when the wizard helps the scarecrow realize that he has had a brain all along, the scarecrow says this:

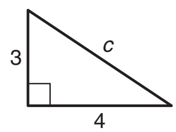
"The sum of the square roots of any two sides of an isosceles triangle is equal to the square root of the remaining side. Oh joy! Rapture! I've got a brain! How can I ever thank you enough?"

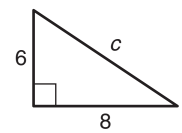
What did the scarecrow get wrong?

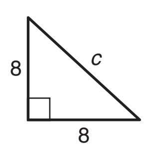
Problem 1 Determining the Length of the Hypotenuse

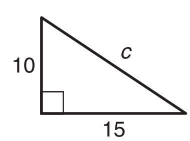
In this lesson, you will investigate solving for different side lengths of right triangles and using the converse of the Pythagorean Theorem.

Determine the length of the hypotenuse of each triangle. Round your answer to the nearest tenth, if necessary.

1. 

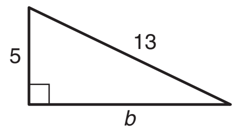
2. 

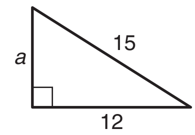
3. 

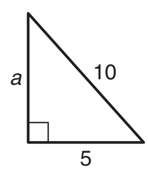
4. 

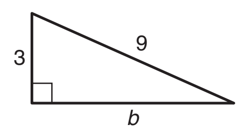
Problem 2 Determining the Length of a Leg

Determine the unknown leg length. Round your answer to the nearest tenth, if necessary.

1. 

2. 

3. 

4. 

Problem 3 Determining the Right Triangle

Use the converse of the Pythagorean Theorem to determine whether each triangle is a right triangle. Explain your answer.

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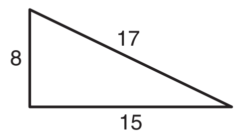

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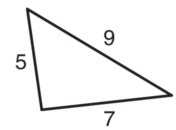

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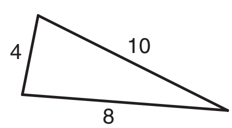

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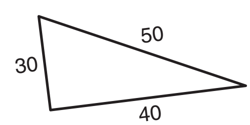


Problem 4 Determining the Unknown Length

Use the Pythagorean Theorem to calculate each unknown length. Round your answer to the nearest tenth, if necessary.

1. Chandra has a ladder that is 20 feet long. If the top of the ladder reaches 16 feet up the side of a building, how far from the building is the base of the ladder?

2. A scaffold has a diagonal support beam to strengthen it. If the scaffold is 12 feet high and 5 feet wide, how long must the support beam be?

3. The length of the hypotenuse of a right triangle is 40 centimeters. The legs of the triangle are the same length. How long is each leg of the triangle?

4. A carpenter props a ladder against the wall of a building. The base of the ladder is 10 feet from the wall. The top of the ladder is 24 feet from the ground. How long is the ladder?

6

Be prepared to share your solutions and methods.

6.4 Meeting Friends

The Distance Between Two Points in a Coordinate System

Learning Goal

In this lesson, you will:

- Use the Pythagorean Theorem to determine the distance between two points in a coordinate system.

Try this in your class. All you need is a regulation size basketball (29.5 inches in diameter), a tennis ball, and a tape measure. Have your teacher hold the basketball, and give the tennis ball to a student. The basketball represents the Earth, and the tennis ball represents the Moon.

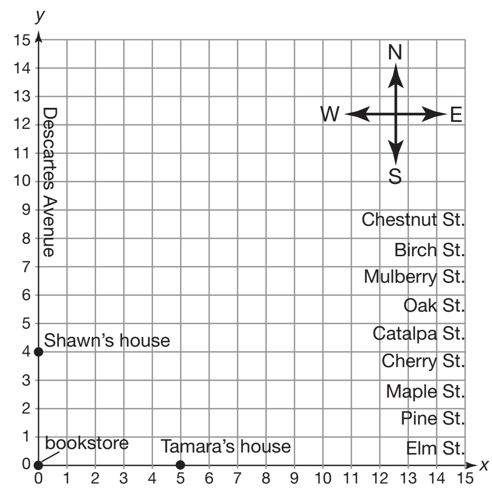
Here's the question each student should guess at: How far away from the basketball should you hold the tennis ball so that the distance between the two represents the actual distance between the Earth and the Moon to scale?

Use the tape measure to record each student's guess. Have your teacher show you the answer when you're done. See who can get the closest.

Problem 1 Meeting at the Bookstore

Two friends, Shawn and Tamara, live in a city in which the streets are laid out in a grid system.

Shawn lives on Descartes Avenue and Tamara lives on Elm Street as shown. The two friends often meet at the bookstore. Each grid square represents one city block.



1. How many blocks does Shawn walk to get to the bookstore?

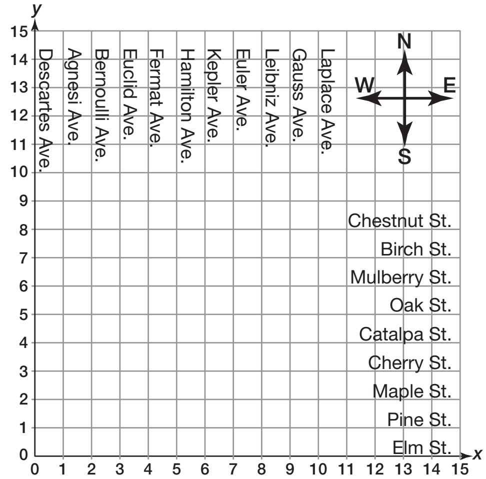
2. How many blocks does Tamara walk to get to the bookstore?

3. Determine the distance, in blocks, Tamara would walk if she traveled from her house to the bookstore and then to Shawn's house.

4. Determine the distance, in blocks, Tamara would walk if she traveled in a straight line from her house to Shawn's house. Explain your calculation. Round your answer to the nearest tenth of a block.

6

5. Don, a friend of Shawn and Tamara, lives three blocks east of Descartes Avenue and five blocks north of Elm Street. Freda, another friend, lives seven blocks east of Descartes Avenue and two blocks north of Elm Street. Plot the location of Don's house and Freda's house on the grid. Label each location and label the coordinates of each location.



a. Name the streets that Don lives on.

b. Name the streets that Freda lives on.

6. Another friend, Bert, lives at the intersection of the avenue that Don lives on and the street that Freda lives on. Plot the location of Bert's house on the grid in Question 5 and label the coordinates. Describe the location of Bert's house with respect to Descartes Avenue and Elm Street.

7. How do the coordinates of Bert's house compare to the coordinates of Don's house and Freda's house?

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8. Use the house coordinates to write and evaluate an expression that represents the distance between Don's and Bert's houses.

9. How far, in blocks, does Don have to walk to get to Bert's house?

10. Use the house coordinates to write an expression that represents the distance between Bert's and Freda's houses.

11. How far, in blocks, does Bert have to walk to get to Freda's house?

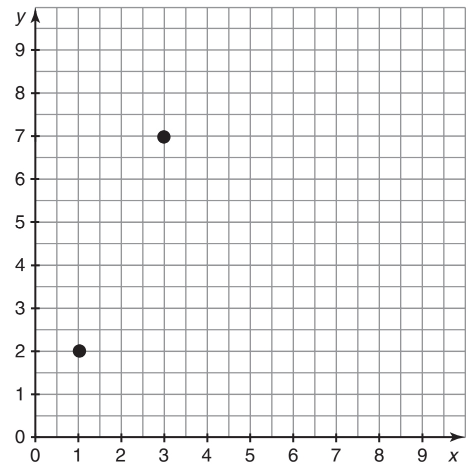
12. All three friends meet at Don's house to study geometry. Freda walks to Bert's house, and then they walk together to Don's house. Use the coordinates to write and evaluate an expression that represents the distance from Freda's house to Bert's house and from Bert's house to Don's house.

13. How far, in blocks, does Freda walk altogether?

14. Draw the direct path from Don's house to Freda's house on the coordinate plane in Question 5. If Freda walks to Don's house on this path, how far, in blocks, does she walk? Explain how you determined your answer.

Problem 2 The Distance between Two Points

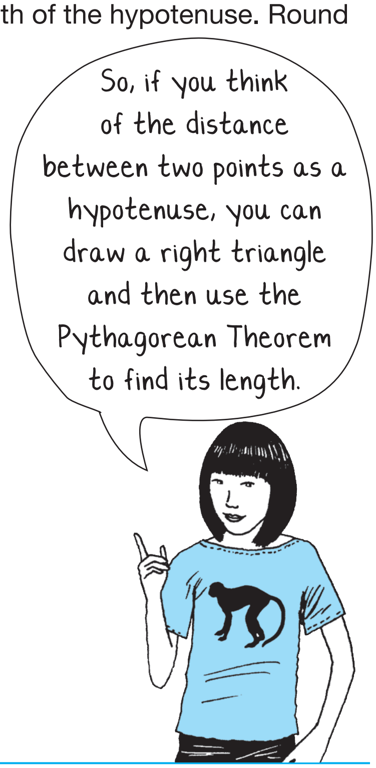
1. The points (1, 2) and (3, 7) on are shown on the coordinate plane. You can calculate the distance between these two points by drawing a right triangle. When you think about this line segment as the hypotenuse of the right triangle, you can use the Pythagorean Theorem.



a. Connect the points with a line segment. Draw a right triangle with this line segment as the hypotenuse.

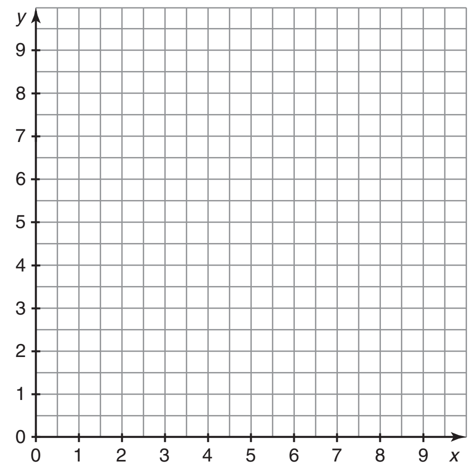
b. What are the lengths of each leg of the right triangle?

c. Use the Pythagorean Theorem to determine the length of the hypotenuse. Round your answer to the nearest tenth.



Determine the distance between each pair of points by graphing and connecting the points, creating a right triangle, and applying the Pythagorean Theorem.

2. (3, 4) and (6, 8)



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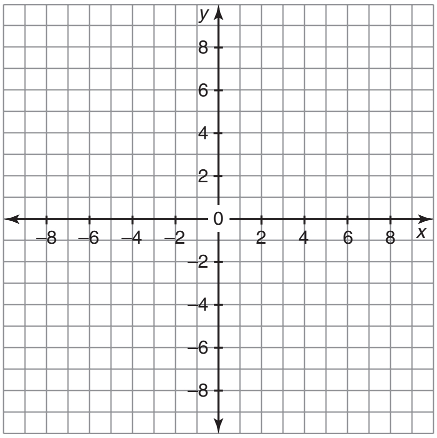
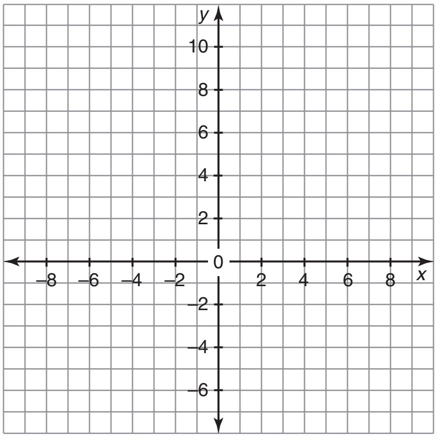
3. (−6, 4) and (2, −8)



4. (−5, 2) and (−6, 10)



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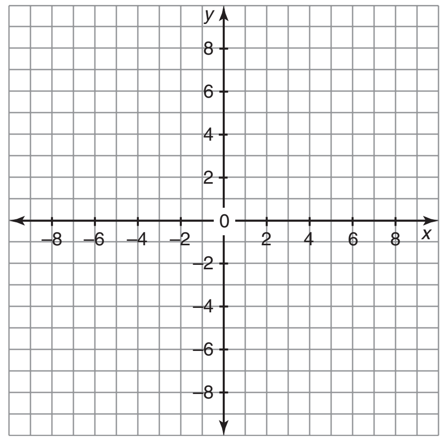
5. $(-1, -4)$ and $(-3, -6)$



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Be prepared to share your solutions and methods.

6.5 DIAGONALLY

Diagonals in Two Dimensions

Learning Goal

In this lesson, you will:

- Use the Pythagorean Theorem to determine the length of diagonals in two-dimensional figures.

You have certainly seen signs like this one.

This sign means "no parking." In fact, a circle with a diagonal line through it (from top left to bottom right) is considered the universal symbol for "no." This symbol is used on street signs, on packaging, and on clothing labels, to name just a few.

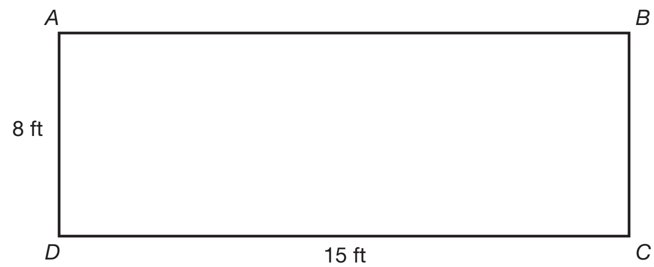
What other examples can you name?

6

Problem 1 Diagonals of a Rectangle and a Square

Previously, you have drawn or created many right triangles and used the Pythagorean Theorem to determine side lengths. In this lesson, you will explore the diagonals of various shapes.

1. Rectangle *ABCD* is shown.



   a. Draw diagonal *AC* in rectangle *ABCD*. Then, determine the length of diagonal *AC*.

   b. Draw diagonal *BD* in rectangle *ABCD*. Then, determine the length of diagonal *BD*.

   c. What can you conclude about the diagonals of this rectangle?

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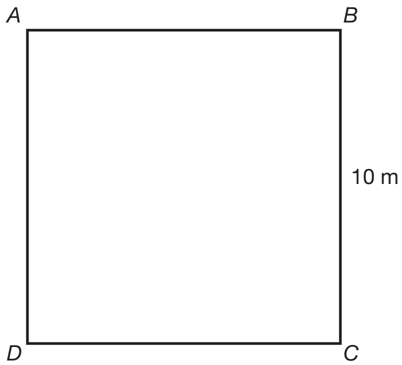
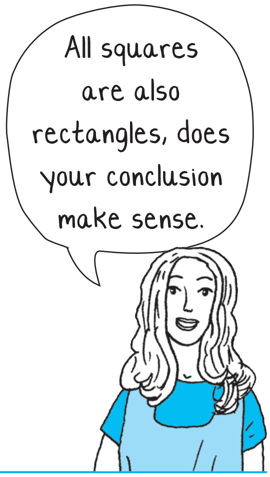
2. Square *ABCD* is shown.



a. Draw diagonal *AC* in square *ABCD*. Then, determine the length of diagonal *AC*.

b. Draw diagonal *BD* in square *ABCD*. Then, determine the length of diagonal *BD*.

c. What can you conclude about the diagonals of this square?



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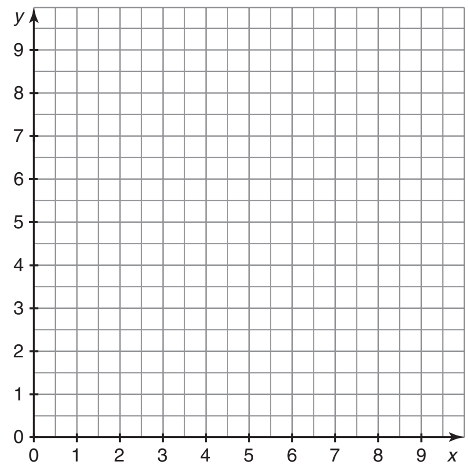
Problem 2 Diagonals of Trapezoids

1. Graph and label the coordinates of the vertices of trapezoid *ABCD*.
   A(1, 2), *B*(7, 2), *C*(7, 5), *D*(3, 5)



   a. Draw diagonal *AC* in trapezoid *ABCD*.

   b. What right triangle can be used to determine the length of diagonal *AC*?

   c. Determine the length of diagonal *AC*.

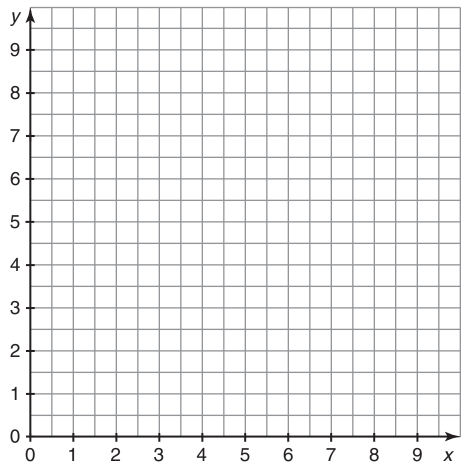
   d. Draw diagonal *BD* in trapezoid *ABCD*.

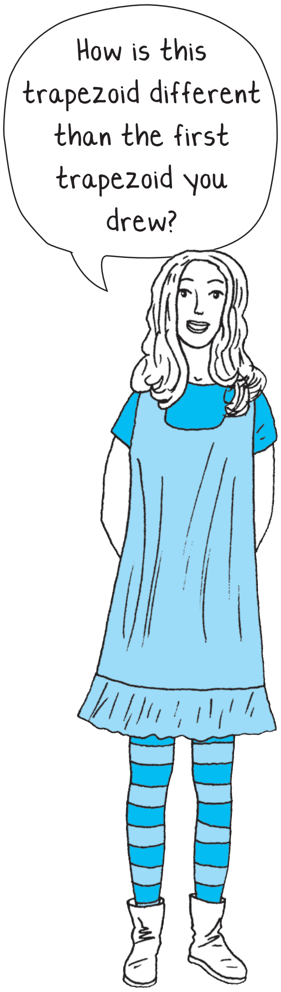
   e. What right triangle can be used to determine the length of diagonal *BD*?

f. Determine the length of diagonal *BD*.

g. What can you conclude about the diagonals of this trapezoid?

2. Graph and label the coordinates of the vertices of isosceles trapezoid *ABCD*.
A(1, 2), *B*(9, 2), *C*(7, 5), *D*(3, 5)





a. Draw diagonal *AC* in trapezoid *ABCD*.

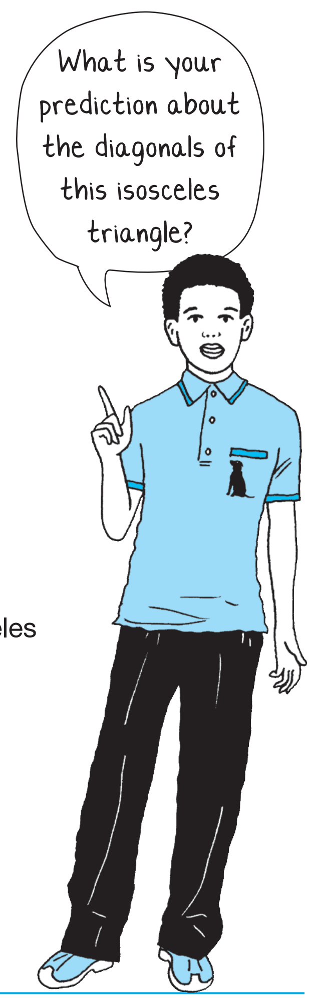
b. What right triangle can be used to determine the length of diagonal *AC*?

c. Determine the length of diagonal AC.

d. Draw diagonal BD in trapezoid $ABCD$.

e. What right triangle can be used to determine the length of diagonal BD?

f. Determine the length of diagonal BD.

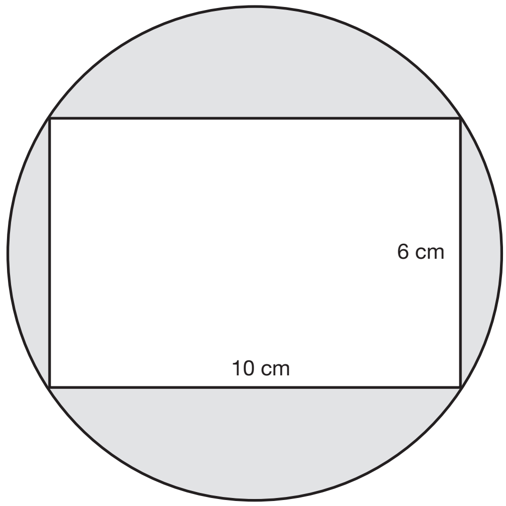


g. What can you conclude about the diagonals of this isosceles trapezoid?

Problem 3 Composite Figures

Use your knowledge of right triangles, the Pythagorean Theorem, and areas of shapes to determine the area of each shaded region. Use 3.14 for π.

1. A rectangle is inscribed in a circle as shown.



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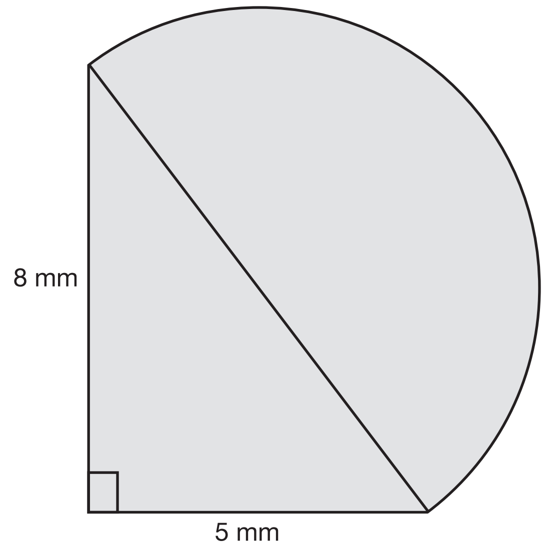
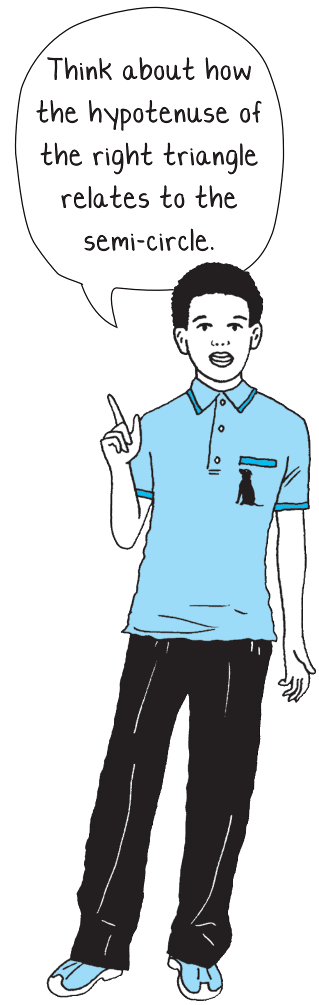
2. The figure is composed of a right triangle and a semi-circle.





Be prepared to share your solutions and methods.

6.6 Two Dimensions Meet Three Dimensions

Diagonals in Three Dimensions

Learning Goals

In this lesson, you will:

- Use the Pythagorean Theorem to determine the length of a diagonal of a solid.
- Use a formula to determine the length of a diagonal of a rectangular solid given the lengths of three perpendicular edges.
- Use a formula to determine the length of a diagonal of a rectangular solid given the diagonal measurements of three perpendicular sides.

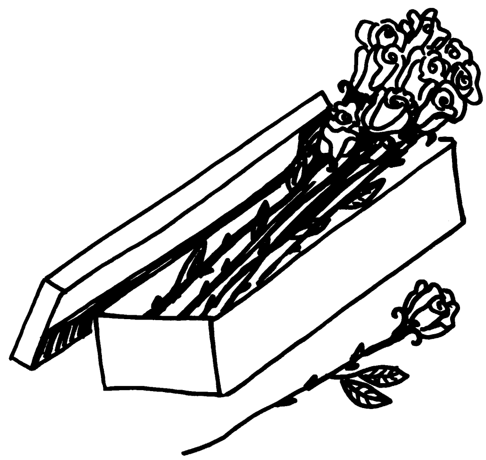
Harry Houdini was one of the most famous escapologists in history. What is an escapologist? He or she is a person who is an expert at escaping from restraints—like handcuffs, cages, barrels, fish tanks, and boxes.

On July 7, 1912, Houdini performed an amazing box escape. He was handcuffed, and his legs were shackled together. He was then placed in a box which was nailed shut, roped, weighed down with 200 pounds of lead, and then lowered into the East River in New York!

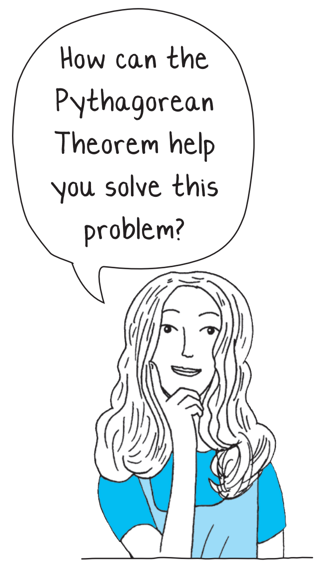
Houdini managed to escape in less than a minute. But he was a professional. So don't try to become an escapologist at home!

Problem 1 A Box of Roses

A rectangular box of long-stem roses is 18 inches in length, 6 inches in width, and 4 inches in height.

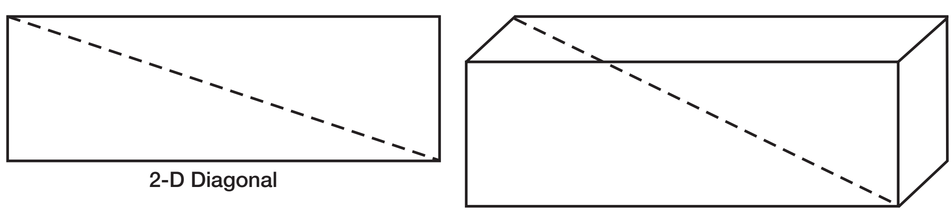


Without bending a long-stem rose, you are to determine the maximum length of a rose that will fit into the box.



1. What makes this problem different from all of the previous applications of the Pythagorean Theorem?

2. Compare a two-dimensional diagonal to a three-dimensional diagonal. Describe the similarities and differences.



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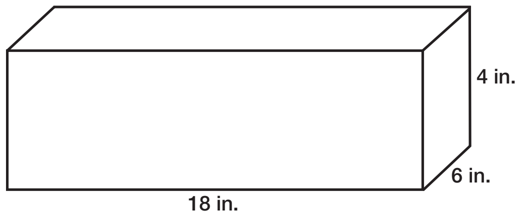
3. Which diagonal represents the maximum length of a rose that can fit into a box?

4. Draw all of the sides in the rectangular solid you cannot see using dotted lines.



5. Draw a three-dimensional diagonal in the rectangular solid shown.

6. Let's consider that the three-dimensional diagonal you drew in the rectangular solid is also the hypotenuse of a right triangle. If a vertical edge is one of the legs of that right triangle, where is the second leg of that same right triangle?

7. Draw the second leg using a dotted line. Then lightly shade the right triangle.

8. Determine the length of the second leg you drew.

9. Determine the length of the three-dimensional diagonal.

10. What does the length of the three-dimensional diagonal represent in terms of this problem situation.

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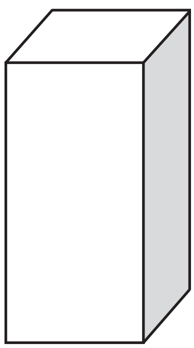
11. Describe how the Pythagorean Theorem was used to solve this problem.

Problem 2 Drawing Diagonals

Draw all of the sides you cannot see in each rectangular solid using dotted lines. Then draw a three-dimensional diagonal using a solid line.

1.



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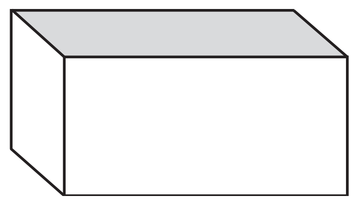

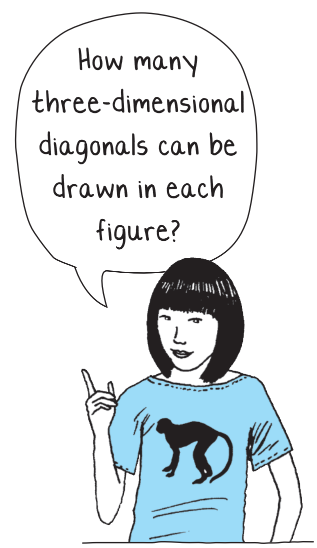

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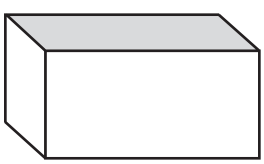

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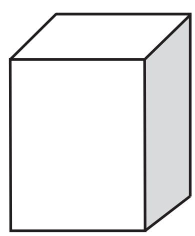

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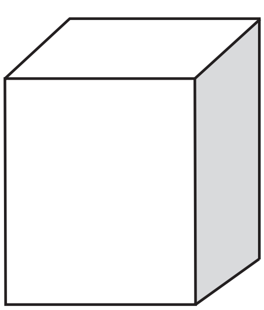

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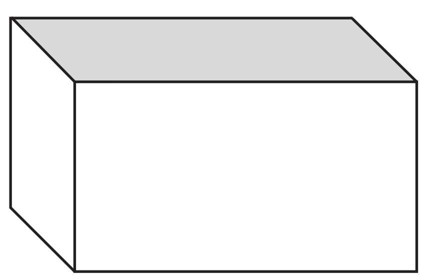
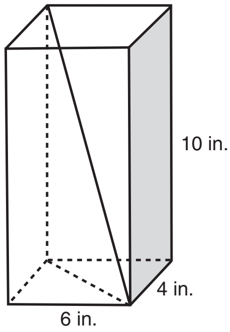
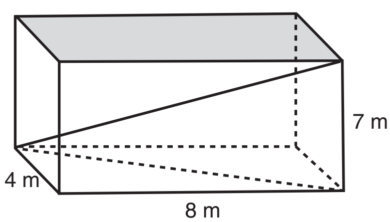


Problem 3 Applying the Pythagorean Theorem

Determine the length of the diagonal of each rectangular solid.

1. 

2. 

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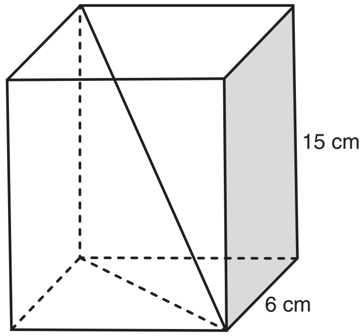
3.



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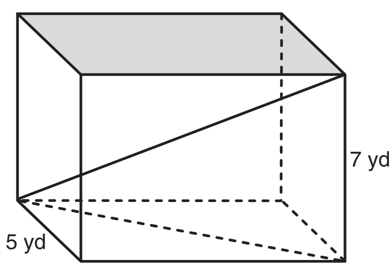

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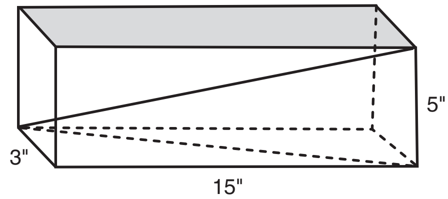

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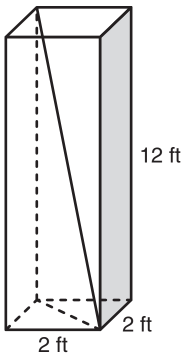

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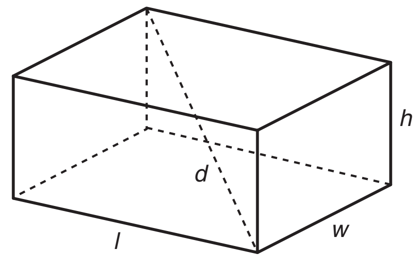
Problem 4 Student Discovery

1. Norton thought he knew a shortcut for determining the length of a three-dimensional diagonal. He said, "All you have to do is calculate the sum of the squares of the rectangular solids' 3 perpendicular edges (the length, the width, and the height) and that sum would be equivalent to the square of the three-dimensional diagonal." Does this work? Use the rectangular solid in Problem 1 to determine if Norton is correct. Explain your reasoning.

2. Use Norton's strategy to calculate the length of the diagonals of each rectangular solid in Problem 3. How do these answers compare to the answers in Problem 3?

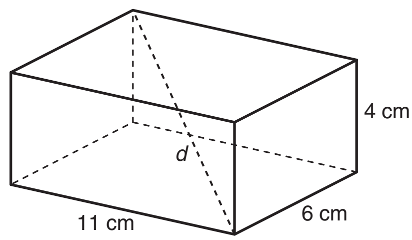
The square of a three-dimensional diagonal is equal to the sum of the squares of each dimension of the rectangular solid.



$$d^2 = \ell^2 + w^2 + h^2$$

$$d = \sqrt{\ell^2 + w^2 + h^2}$$

3. Use the formula $d = \sqrt{\ell^2 + w^2 + h^2}$ to determine the length of a three-dimensional diagonal of the rectangular prism shown.



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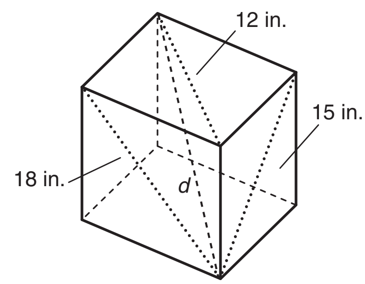
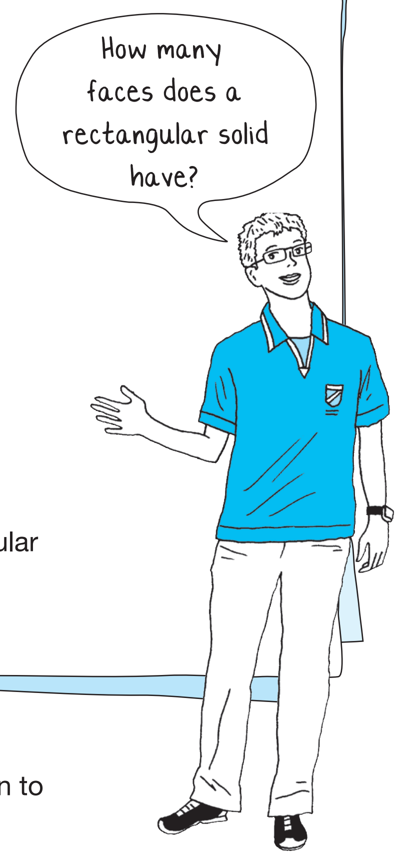
If you know the diagonal lengths of each face of a rectangular solid, you can determine the length of a three-dimensional diagonal.

Let d represent the length of a three dimensional diagonal.

$d^2 = \frac{1}{2}$ (sum of the squares of the diagonals of each unique face)





$$d^2 = \frac{1}{2}(18^2 + 15^2 + 12^2)$$

$$d^2 = \frac{1}{2}(324 + 225 + 144)$$

$$d^2 = 346.5$$

$$d = \sqrt{346.5}$$

$$d \approx 18.6$$

The length of the three-dimensional diagonal of this rectangular prism is about 18.6 inches.

Use your knowledge of diagonals and the two formulas given to answer each question.

4. A rectangular box has a length of 6 feet and a width of 2 feet. The length of a three-dimensional diagonal of the box is 7 feet. What is the height of the box?

5. The length of the diagonal across the front of a rectangular box is 20 inches, and the length of the diagonal across the side of the box is 15 inches. The length of a three-dimensional diagonal of the box is 23 inches. What is the length of the diagonal across the top of the box?

6. Pablo is packing for a business trip. He is almost finished packing when he realizes that he forgot to pack his umbrella. Before Pablo takes the time to repack his suitcase, he wants to know if the umbrella will fit in the suitcase. His suitcase is in the shape of a rectangular prism and has a length of 2 feet, a width of 1.5 feet, and a height of 0.75 foot. The umbrella is 30 inches long. Will the umbrella fit in Pablo's suitcase? Explain your reasoning.

Be prepared to share your solutions and methods.

6

Chapter 6 Summary

Key Terms

- right triangle (6.1)
- right angle (6.1)
- leg (6.1)
- hypotenuse (6.1)
- diagonal of a square (6.1)
- Pythagorean Theorem (6.1)
- theorem (6.1)
- postulate (6.1)
- proof (6.1)
- converse (6.2)
- Converse of the Pythagorean Theorem (6.2)
- Pythagorean triple (6.2)

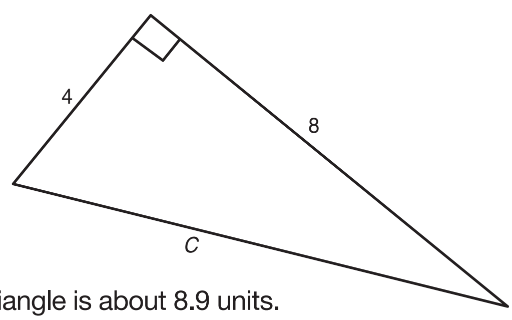
6.1 Applying the Pythagorean Theorem

A right triangle is a triangle with a right angle. A right angle is an angle with a measure of 90° and is indicated by a square drawn at the corner formed by the angle. A leg of a right triangle is either of the two shorter sides. Together, the two legs form the right angle of a right triangle. The hypotenuse of a right triangle is the longest side and is opposite the right angle. The Pythagorean Theorem states that if a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$.

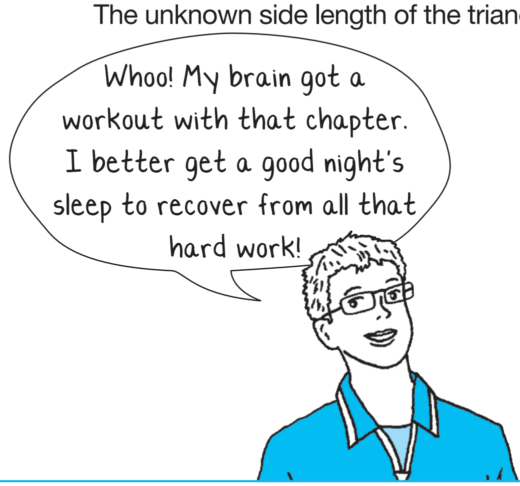
Example

Determine the unknown side length of the triangle.

$$a^2 + b^2 = c^2$$
$$4^2 + 8^2 = c^2$$
$$16 + 64 = c^2$$
$$80 = c^2$$
$$\sqrt{80} = c$$
$$c \approx 8.9$$



The unknown side length of the triangle is about 8.9 units.



6

6.2 Applying the Converse of the Pythagorean Theorem

The Converse of the Pythagorean Theorem states that if a, b, and c are the side lengths of a triangle and $a^2 + b^2 = c^2$, then the triangle is a right triangle.

Example

Determine whether a triangle with side lengths 5, 9, and 10 is a right triangle.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 5^2 + 9^2 &\stackrel{?}{=} 10^2 \\ 25 + 81 &\stackrel{?}{=} 100 \\ 106 &\neq 100 \end{aligned}$$

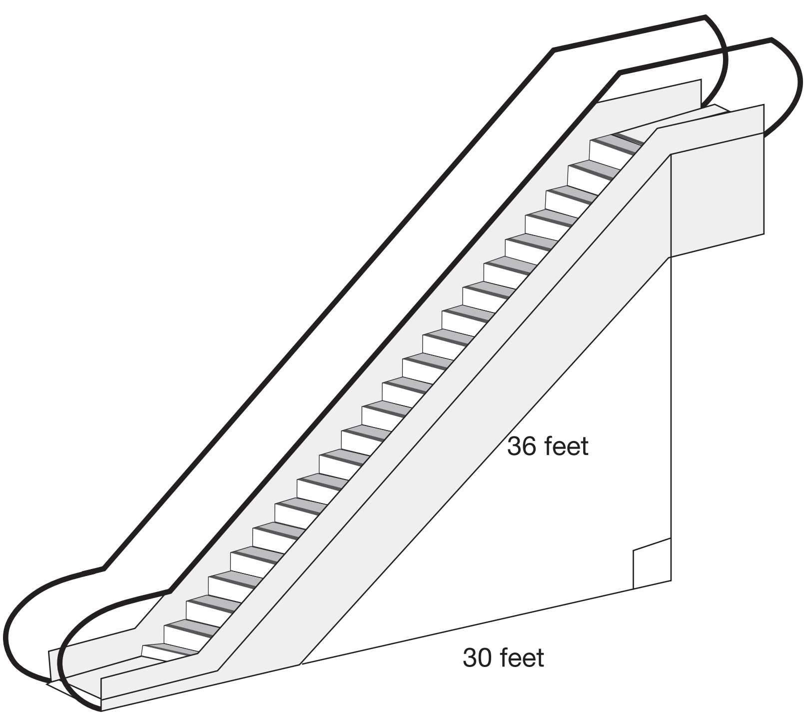
A triangle with side lengths 5, 9, and 10 is not a right triangle because $5^2 + 9^2 \neq 10^2$.

6.3 Applying the Pythagorean Theorem to Solve Real-World Problems

The Pythagorean Theorem can be used to solve a variety of real-world problems which can be represented by right triangles.

Example

An escalator in a department store carries customers from the first floor to the second floor. Determine the distance between the two floors.



$$\begin{aligned} a^2 + b^2 &= c^2 \\ 30^2 + b^2 &= 36^2 \\ 900 + b^2 &= 1296 \\ b^2 &= 396 \\ b &= \sqrt{396} \\ b &\approx 19.90 \end{aligned}$$

The distance between the two floors is 19.90 feet.

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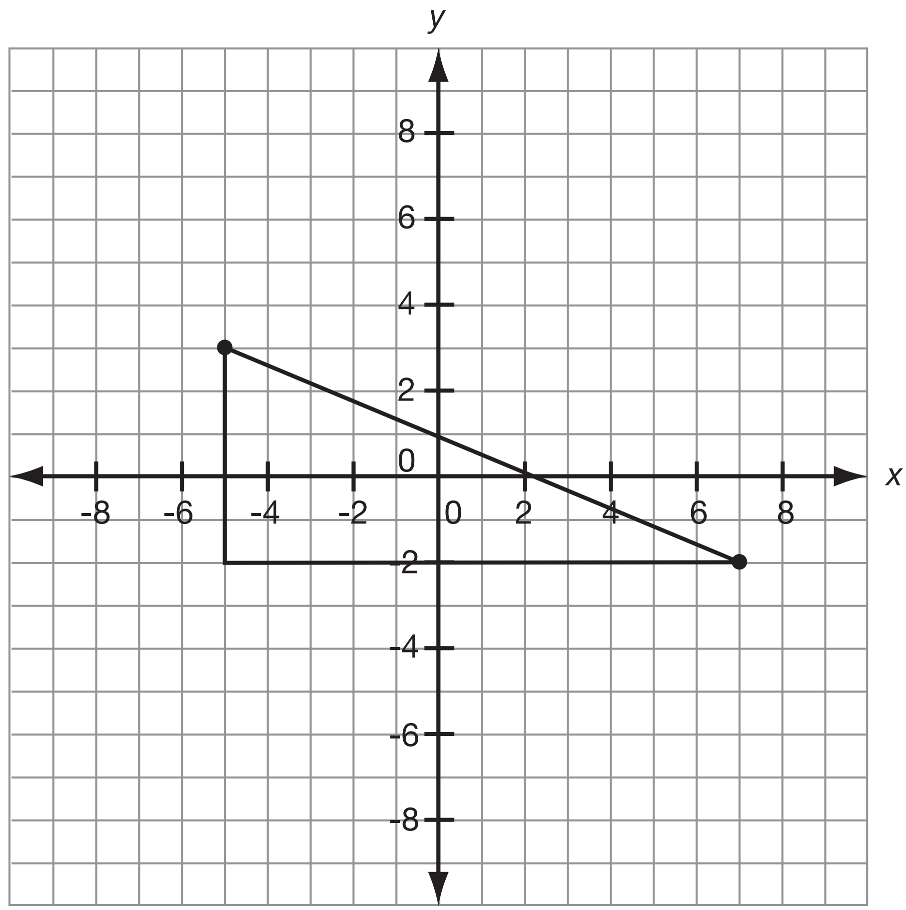
6.4 Determining the Distance Between Two Points in a Coordinate System

The distance between two points, which do not lie on the same horizontal or vertical line, on a coordinate plane can be determined using the Pythagorean Theorem.

Example

Determine the distance between points $(-5, 3)$ and $(7, -2)$.



A line segment is drawn between the two points to represent the hypotenuse of a right triangle. Two line segments are drawn (one horizontal and one vertical) to represent the legs of the right triangle. The lengths of the legs are 5 units and 12 units.

$$a^2 + b^2 = c^2$$
$$5^2 + 12^2 = c^2$$
$$25 + 144 = c^2$$
$$c^2 = 169$$
$$c = \sqrt{169}$$
$$c = 13$$

The distance between $(-5, 3)$ and $(7, -2)$ is 13 units.

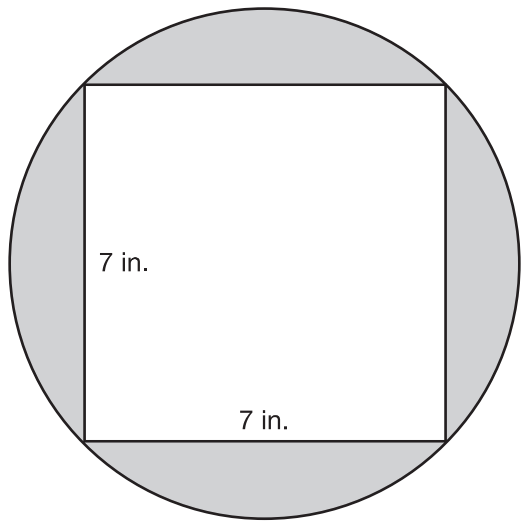
6.5

Determining the Lengths of Diagonals Using the Pythagorean Theorem

The Pythagorean Theorem can be a useful tool for determining the length of a diagonal in a two-dimensional figure.

Example

Determine the area of the shaded region.



The area of the square is:

$A = s^2$
$A = 7^2$
$A = 49$ square inches

The diagonal of the square is the same length as the diameter of the circle. The diagonal of the square can be determined using the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$
$$7^2 + 7^2 = c^2$$
$$49 + 49 = c^2$$
$$c^2 = 98$$
$$c = \sqrt{98}$$
$$c \approx 9.90 \text{ inches}$$

So, the radius of the circle is $\frac{1}{2}(9.90) = 4.95$ inches
The area of the circle is:

$A = \pi r^2$
$A = (3.14)(4.95)^2$
$A \approx 76.94$ square inches

The area of the shaded region is $76.94 - 49 \approx 27.94$ square inches.

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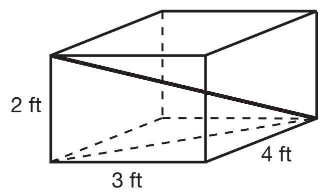
6.6 Determining the Lengths of Diagonals in Three-Dimensional Solids

The Pythagorean Theorem can be used to determine the length of a diagonal in a geometric solid. An alternate formula derived from the Pythagorean Theorem can also be used to determine the length of a diagonal in a geometric solid. In a right rectangular prism with length ℓ, width w, height h, and diagonal length d, $d^2 = \ell^2 + w^2 + h^2$.

Example

Determine the length of a diagonal in a right rectangular prism with a length of 4 feet, a width of 3 feet, and a height of 2 feet.



The diagonal is the hypotenuse of a triangle with one leg being the front left edge of the prism and the other leg being the diagonal of the bottom face.

The length of the diagonal of the bottom face is:

$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 = c^2$$
$$9 + 16 = c^2$$
$$c^2 = 25$$
$$c = \sqrt{25}$$
$$c = 5 \text{ feet}$$

The length of the prism's diagonal is:

$$a^2 + b^2 = c^2$$
$$2^2 + 5^2 = c^2$$
$$4 + 25 = c^2$$
$$c^2 = 29$$
$$c = \sqrt{29}$$
$$c \approx 5.39 \text{ feet}$$

Using the alternate formula, the length of the prism's diagonal is:

$$d^2 = \ell^2 + w^2 + h^2$$
$$d^2 = 4^2 + 3^2 + 2^2$$
$$d^2 = 16 + 9 + 4$$
$$d^2 = 29$$
$$d = \sqrt{29}$$
$$d \approx 5.39 \text{ feet}$$

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