



2-6 Spreadsheet Lab Descriptive Modeling

Common Core State Standards
Content Standards
 N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.

When using numbers to model a real-world situation, it is often helpful to have a metric. A metric is a rule for assigning a number to some characteristic or attribute. For example, teachers use metrics to determine grades. Each teacher determines an appropriate metric for assessing a student's performance and assigning a grade. You can use a spreadsheet to calculate different metrics.

Activity



Dorrie wants to buy a house. She has the following expenses: rent of \$650, credit card monthly bills of \$320, a car payment of \$410, and a student loan payment of \$115. Dorrie has a yearly salary of \$46,500. Use a spreadsheet to find Dorrie's debt-to-income ratio.

- Step 1** Enter Dorrie's debts in column B.
- Step 2** Add her debts using a function in cell B6. Go to Insert and then Function. Then choose Sum. The sum of 1495 appears in B6.
- Step 3** Now insert Dorrie's salary in column C. Remember to find her monthly salary by dividing the yearly salary by 12.

A mortgage company will use the debt-to-income ratio as a metric to determine if Dorrie qualifies for a loan. The **debt-to-income ratio** is calculated as *how much she owes per month* divided by *how much she earns each month*.

Lab 2-6 B Spreadsheet.xls

	A	B	C
1	Type of Debt	Expenses	Salary
2	Rent	650	3875
3	Credit Cards	320	
4	Car Payment	410	
5	Student Loan	115	
6		1495	0.385806

Sheet 1 / Sheet 2 / Sheet 3

Step 4 Enter a formula to find the debt-to-income ratio in cell C6. In the formula bar, enter $=B6/C2$.

The ratio of about 0.39 appears. An ideal ratio would be 0.36 or less. A ratio higher than 0.36 would cause an increased interest rate or may require a higher down payment.

The spreadsheet shows a debt-to-income ratio of about 0.39. Dorrie should try to eliminate or reduce some debts or try to earn more money in order to lower her debt-to-income ratio.

Exercises

1. How could Dorrie improve her debt-to-income ratio?
2. Another metric mortgage companies use is the ratio of monthly mortgage to total monthly income. An ideal ratio is 0.28. Using this metric, how much could Dorrie afford to pay for a mortgage each month?
3. How effective are each of these metrics as measures of whether Dorrie can afford to buy a house? Explain your reasoning.
4. **MODELING** Metrics are used to compare athletes. For example, ERAs are used to compare pitchers. Find a metric and evaluate its effectiveness for modeling. Compare it to other metrics, and then define your own metric.

2-7 Percent of Change

Then

- You solved proportions.

Now

- 1 Find the percent of change.
- 2 Solve problems involving percent of change.

Why?

- Every year, millions of people volunteer their time to improve their community. The difference in the number of volunteers from one year to the next can be used to determine a percent to represent the increase or decrease in volunteers.

New Vocabulary

percent of change
 percent of increase
 percent of decrease

Common Core State Standards

Content Standards
 N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Mathematical Practices
 8 Look for and express regularity in repeated reasoning.

1 Percent of Change Percent of change is the ratio of the change in an amount to the original amount expressed as a percent. If the new number is greater than the original number, the percent of change is a **percent of increase**. If the new number is less than the original number, the percent of change is a **percent of decrease**.

Example 1 Percent of Change

Determine whether each percent of change is a percent of *increase* or a percent of *decrease*. Then find the percent of change.

- a. original: 20
final: 23
- b. original: 25
final: 17

Subtract the original amount from the final amount to find the amount of change: $23 - 20 = 3$.

Since the new amount is greater than the original, this is a percent of increase.

Use the original number, 20, as the base.

$$\begin{array}{rcl} \text{change} & \longrightarrow & 3 \\ \text{original amount} & \longrightarrow & 20 \end{array} \quad \frac{3}{20} = \frac{r}{100}$$

$$3(100) = r(20)$$

$$300 = 20r$$

$$\frac{300}{20} = \frac{20r}{20}$$

$$15 = r$$

The percent of increase is 15%.

Guided Practice

- 1A. original: 66
new: 30
- 1C. original: 24
new: 40

- 1B. original: 9.8
new: 12.1

- 1D. original: 500
new: 131

Subtract the original amount from the final amount to find the amount of change: $17 - 25 = -8$.

Since the new amount is less than the original, this is a percent of decrease.

Use the original number, 25, as the base.

$$\begin{array}{rcl} \text{change} & \longrightarrow & -8 \\ \text{original amount} & \longrightarrow & 25 \end{array} \quad \frac{-8}{25} = \frac{r}{100}$$

$$-8(100) = r(25)$$

$$-800 = 25r$$

$$\frac{-800}{25} = \frac{25r}{25}$$

$$-32 = r$$

The percent of decrease is 32%.

Real-World Example 2 Percent of Change

CRUISE The total number of passengers on cruise ships increased 10% from 2007 to 2009. If there were 17.22 million passengers in 2009, how many were there in 2007?

Let f = the number of passengers in 2007. Since 10% is a percent of increase, the number of passengers in 2007 is less than the number of passengers in 2009.

$$\begin{aligned} \text{change} &\rightarrow \frac{17.22 - f}{f} = \frac{10}{100} \\ \text{original amount} &\rightarrow \frac{17.22 - f}{100} = \frac{10f}{100} \\ (17.22 - f)100 &= 10f \\ 1722 - 100f &= 10f \\ 1722 - 100f + 100f &= 10f + 100f \\ 1722 &= 110f \\ \frac{1722}{110} &= \frac{110f}{110} \\ 15.65 &\approx f \end{aligned}$$

Percent proportion
Find the cross products.
Distributive Property
Add 100f to each side.
Simplify.
Divide each side by 110.
Simplify.

There were approximately 15.65 million passengers in 2007.

Guided Practice

- 2. TUITION** A recent percent of increase in tuition at Northwestern University, in Evanston, Illinois, was 5.4%. If the new cost is \$33,408 per year, find the original cost per year.

2 Solve Problems Two applications of percent of change are sales tax and discounts. Sales tax is an example of a percent of increase. Discount is an example of a percent of decrease.

Example 3 Sales Tax

SHOPPING Marta is purchasing wire and beads to make jewelry. Her merchandise is \$28.62 before tax. If the tax is 7.25% of the total sales, what is the final cost?

Step 1 Find the tax.
The tax is 7.25% of the price of the merchandise.
 $7.25\% \text{ of } \$28.62 = 0.0725 \times 28.62$
 $= 2.07495$
Use a calculator.

Step 2 Find the cost with tax.
Round \$2.07495 to \$2.07 since tax is always rounded to the nearest cent.
Add this amount to the original price: $\$28.62 + \$2.07 = \$30.69$.
The total cost of Marta's jewelry supplies is \$30.69.

Guided Practice

- 3. SHOPPING** A new DVD costs \$24.99. If the sales tax is 6.85%, what is the total cost?

To find a discounted amount, you will follow similar steps to those for sales tax.



Example 4 Discounts

DISCOUNT Since Tyrell has earned good grades in school, he qualifies for the Good Student Discount on his car insurance. His monthly payment without the discount is \$85. If the discount is 20%, what will he pay each month?

Step 1 Find the discount.
The discount is 20% of the original payment.
 $20\% \text{ of } \$85 = 0.20 \times 85$
 $= 17$
Use a calculator.

Step 2 Find the cost after discount.
Subtract \$17 from the original payment: $\$85 - \$17 = \$68$.
With the Good Student Discount, Tyrell will pay \$68 per month.

Guided Practice

- 4. SALES** A picture frame originally priced at \$14.89 is on sale for 40% off. What is the discounted price?

Check Your Understanding

Example 1

State whether each percent of change is a percent of increase or a percent of decrease. Then find the percent of change. Round to the nearest whole percent.

- original: 78
new: 125
- original: 41
new: 24
- original: 6 candles
new: 8 candles
- original: 35 computers
new: 32 computers
- GEOGRAPHY** The distance from Phoenix to Tucson is 120 miles. The distance from Phoenix to Flagstaff is about 21.7% longer. To the nearest mile, what is the distance from Phoenix to Flagstaff?

Example 2

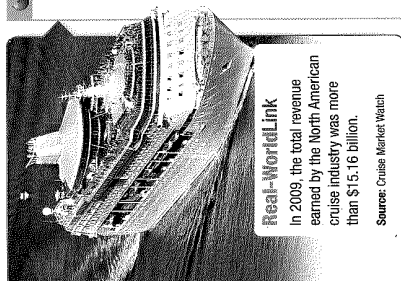
Find the total price of each item.

- dress: \$22.50
sales tax: 7.5%
- video game: \$35.99
sales tax: 6.75%
- PROM** A limo costs \$85 to rent for 3 hours plus a 7% sales tax. What is the total cost to rent a limo for 6 hours?
- GAMES** A computer game costs \$49.95 plus a 6.25% sales tax. What is the total cost of the game?

Example 3

Find the discounted price of each item.

- guitar: \$95.00
discount: 15%
- DVD**: \$22.95
discount: 25%
- SKATEBOARD** A skateboard costs \$99.99. If you have a coupon for 20% off, how much will you save?
- MODELING** Tickets to the county fair are \$8 for an adult and \$5 for a child. If you have a 15% discount card, how much will 2 adult tickets and 2 child tickets cost?



Real-World Link

In 2009, the total revenue earned by the North American cruise industry was more than \$15.16 billion.

Source: Cruise Market Watch

State whether each percent of change is a percent of *increase* or a percent of *decrease*. Then find the percent of change. Round to the nearest whole percent.

14. original: 35
new: 40
15. original: 16
new: 10
16. original: 27
new: 73
17. original: 92
new: 21
18. original: 21.2 grams
new: 10.8 grams
19. original: 11 feet
new: 25 feet
20. original: \$68
new: \$76
21. original: 21 hours
new: 40 hours

22. GASOLINE The average cost of regular gasoline in North Carolina increased by 73% from 2006 to 2007. If the average cost of a gallon of gas in 2006 was \$2.069, what was the average cost in 2007? Round to the nearest cent.

- 223. CARS** Beng is shopping for a car. The cost of a new car is \$15,500. This is 25% greater than the cost of a used car. What is the cost of the used car?

Find the total price of each item.

- | | | | | | |
|--|------------------------------------|--------------------------------|---------------------------------|------------------------------------|----------------------------------|
| 24. messenger bag: \$28.00
tax: 7.25% | 25. software: \$45.00
tax: 5.5% | 26. vase: \$5.50
tax: 6.25% | 27. book: \$25.95
tax: 5.25% | 28. magazine: \$3.50
tax: 5.75% | 29. pillow: \$9.99
tax: 6.75% |
|--|------------------------------------|--------------------------------|---------------------------------|------------------------------------|----------------------------------|

Find the discounted price of each item.

- | | | |
|--|--|--|
| 30. computer: \$1099.00
discount: 25% | 31. CD player: \$89.99
discount: 15% | 32. athletic shoes: \$59.99
discount: 40% |
| 33. jeans: \$24.50
discount: 33% | 34. jacket: \$125.00
discount: 25% | 35. belt: \$14.99
discount: 20% |
| Find the final price of each item. | | |
| 36. sweater: \$14.99
discount: 12%
tax: 6.25% | 37. printer: \$60.00
discount: 25%
tax: 6.75% | 38. board game: \$25.00
discount: 15%
tax: 7.5% |

39. CONSUMER PRICE INDEX An *index* measures the percent change of a value from a base year. An index of 115 means that there was a 15% increase from the base year. In 2000, the consumer price index of dairy products was 160.7. In 2007, it was 194.0. Determine the percent of change.

- 40. FINANCIAL LITERACY** The current price of each share of a technology company is \$135. If this represents a 16.2% increase over the past year, what was the price per share a year ago?

41. CSS MODELING A group of girls are shopping for dresses to wear to the spring dance. One finds a dress priced \$75 with a 20% discount. A second girl finds a dress priced \$85 with a 30% discount.

- a. Find the amount of discount for each dress.
 - b. Which girl is getting the better price for the dress?
- 42. RECREATIONAL SPORTS** In 1995, there were 73,567 youth softball teams. By 2007, there were 86,049. Determine the percent of increase.

Average Retail Prices of Selected Grocery Items		
Grocery Item	Cost in 2000 (\$ per pound)	Cost in 2007 (\$ per pound)
milk (gallon)	2.79	3.87
turkey (whole)	0.99	1.01
chicken (whole)	1.08	1.17
ground beef	1.63	2.23
apples	0.82	1.12
iceberg lettuce	0.65	0.95
peanut butter	1.96	1.88

Source: Statistical Abstract of the United States

- 44. 🍄 MULTIPLE REPRESENTATIONS** In this problem, you will explore patterns in percentages.

- a. Tabular Copy** and complete the following table.

1% of	500	is 5.	100% of	is 20.	% of 80 is 20.
2% of		is 5.	50% of	is 20.	% of 40 is 20.
4% of		is 5.	25% of	is 20.	% of 20 is 20.
8% of		is 5.	12.5% of	is 20.	% of 10 is 20.

- b. Verbal** Describe the patterns in the second and fifth columns.

H.O.T. Problems

- 45. OPEN ENDED** Write a real-world problem to find the total price of an item including sales tax.
- 46. REASONING** If you have 75% of a number n , what percent of decrease is it from the number n ? If you have 40% of a number a , what percent of decrease do you have from the number a ? What pattern do you notice? Is this always true?
- 47. ERROR ANALYSIS** Maddie and Xavier are solving for the percent change if the original amount was \$25 and the new amount is \$28. Is either of them correct? Explain your reasoning.

Maddie

$$\frac{3}{28} = \frac{v}{100}$$
$$3(100) = 28v$$
$$300 = 28v$$
$$10.7 = v$$

Xavier

$$\frac{3}{25} = \frac{y}{100}$$
$$3(100) = 25y$$
$$300 = 25y$$
$$12 = y$$

- 48. CHALLENGE** Determine whether the following statement is *sometimes, always, or never* true. *The percent of change is less than 100%.*
- 49. WRITING IN MATH** When is percent of change used in the real world? Explain how to find a percent of change between two values.

Standardized Test Practice

50. **GEOMETRY** The rectangle has a perimeter of P centimeters. Which equation could be used to find the length ℓ of the rectangle?



- A $P = 2.4\ell$
 B $P = 4.8 + \ell$
 C $P = 2.4 + 2\ell$
 D $P = 4.8 + 2\ell$

51. **SHORT RESPONSE** Henry is painting a room with four walls that are 12 feet by 14 feet. A gallon of paint costs \$18 and covers 350 square feet. If he uses two coats of paint, how much will it cost him to paint the room?

52. The number of students at Franklin High School increased from 840 to 910 over a 5-year period. What was the percent of increase?

- F 8.3%
 G 14.0%
 H 18.5%
 J 92.3%

53. **PROBABILITY** Two dice are rolled. What is the probability that the sum is 10?

- A $\frac{1}{3}$
 B $\frac{1}{6}$
 C $\frac{1}{12}$
 D $\frac{1}{36}$

Spiral Review

54. **TRAVEL** The Chan's minivan requires 5 gallons of gasoline to travel 120 miles. How many gallons of gasoline will they need to travel 360 miles? (Lesson 2-6)

Evaluate each expression if $x = -2$, $y = 6$, and $z = 4$. (Lesson 2-5)

55. $|3 - x| + 7$ 56. $12 - |z + 9|$ 57. $|y + x| - z + 4$

Solve each equation. Round to the nearest hundredth. Check your solution. (Lesson 2-4)

58. $1.03p - 4 = -2.15p + 8.72$ 59. $18 - 3.8t = 7.36 - 1.9t$

60. $5.4w + 8.2 = 9.8w - 2.8$ 61. $2[d + 3(d - 1)] = 18$

Solve each equation. Check your solution. (Lesson 2-3)

62. $5n + 6 = -4$ 63. $-11 = 7 + 3c$

64. $15 = 4a - 5$ 65. $-14 + 7g = -63$

66. **RIVERS** The Congo River in Africa is 2900 miles long. That is 310 miles longer than the Niger River, which is also in Africa. (Lesson 2-2)

- a. Write an equation you could use to find the length of the Niger River.
 b. What is the length of the Niger River?

67. **FOOD** Cameron purchased x pounds of apples for \$0.99 per pound and y pounds of oranges for \$1.29 per pound. Write an algebraic expression that represents the cost of the purchase. (Lesson 1-1)

Skills Review

Translate each equation into a sentence.

68. $d - 14 = 5$

69. $2f + 6 = 19$

70. $y - 12 = y + 8$

71. $3a + 5 = 27 - 2a$

72. $-6c^2 - 4c = 25$

73. $d^4 + 64 = 3d^3 + 77$

2-7 Algebra Lab Percentiles

A **percentile** is a measure that is often used to report test data, such as standardized test scores. It tells us what percent of the total scores were below a given score.

- Percentiles measure rank from the bottom.
- There is no 0 percentile rank. The lowest score is at the 1st percentile.
- There is no 100th percentile rank. The highest score is at the 99th percentile.

Activity

A talent show was held for the twenty finalists in the Teen Idol contest. Each performer received a score from 0 through 30 with 30 being the highest. What is Victor's percentile rank?

- Step 1** Write one score on each of 20 slips of paper.
Step 2 Arrange the slips vertically from greatest to least score.
Step 3 Find Victor's percentile rank.

Victor had a score of 28. There are 18 scores below his score. To find his percentile rank, use the following formula.

$$\text{number of scores below } \frac{28}{20} \cdot 100 = \frac{18}{20} \cdot 100 \text{ or } 90$$

Victor scored at the 90th percentile in the contest.

Name	Score	Name	Score
Arnold	17	Ishi	27
Benito	9	James	20
Brooke	25	Kat	16
Carmen	21	Malik	10
Daniel	14	Natalie	26
Della	29	Pearl	4
Fernando	15	Tyyla	6
Heather	12	Victor	28
Horatio	5	Warren	22
Ingrid	11	Yolanda	18

Analyze the Results

- Find the median, lower quartile, and upper quartile of the scores.
- Which performer was at the 50th percentile? the 25th percentile? the 75th percentile?
- Compare and contrast the values for the median, lower quartile, and upper quartile and the scores for the 25th, 50th, and 75th percentiles.
- While Victor scored at the 90th percentile, what percent of the 30 possible points did he score?
- CS ARGUMENTS** Compare and contrast the percentile rank and the percent score.
- Are there any outliers in the data that could alter the results of our computations?
- Deciles** are values that divide a set of data into ten equal-sized parts. The 1st decile contains data up to but not including the 10th percentile; the 2nd decile contains data from the 10th percentile up to but not including the 20th percentile; and so on.
 - Which contestants' scores fall in the 6th decile?
 - In which decile are Heather and Daniel?

