1 Linear Equations

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1.1 A Park Ranger's Work Is Never Done

Solving Problems Using Equations

Learning Goal

In this lesson, you will:

- Write and solve two-step equations to represent problem situations.
- Solve two-step equations.

Key Terms

- inverse operations
- two-step equation
- solution
- coefficient
- constant
- Properties of Equality

The first equations to ever be written down using symbolic notation are much like the equations you will study in this lesson. They first appeared in a book called the Whetstone of Witte by Robert Recorde in 1557. This book is notable because it contained the first recorded use of the equals sign. Recorde got tired of writing the words "is equal to" in all his equations so he decided that a pair of parallel lines of the same length sitting sideways would be the perfect symbol because, as he said, "no two things can be more equal." Here are the first two equations ever written:

$$14x + 15 = 71$$

$$20x - 18 = 102$$

Can you solve the world's oldest symbolic equations?

Problem 1 Building a Walkway

Many situations can be modeled by equations that need more than one operation to solve them.

1. At a local national park, the park rangers decide that they want to extend a wooden walkway through the forest to encourage people to stay on the path. The existing walkway is 150 feet long. The park rangers believe that they can build the additional walkway at a rate of about 5 feet per hour.

   a. How many total feet of walkway will there be after the park rangers work for 5 hours?

   b. How many total feet of walkway will there be after the park rangers work for 7 hours?

   c. Define a variable for the amount of time that the rangers will work. Then, use the variable to write an expression that represents the total number of feet of walkway built, given the amount of time that the rangers will work.

   d. How many hours will the rangers need to work to have a total of 500 feet of walkway completed?

   e. Explain the reasoning you used to solve part (d).

   f. What mathematical operations did you perform to calculate your answer to part (d)?

g. Write an equation that you can use to determine the amount of time the rangers need to build a total of 500 feet of walkway. Then, determine the value of the variable that will make this equation true.

h. Interpret your answer in terms of this problem situation.

2. How many hours will the rangers need to work to build a total of 270 feet of walkway? Explain your reasoning.

a. What mathematical operations did you perform to determine your answer?

b. Explain why using these mathematical operations gives you the correct answer.

c. Write an equation that you can use to determine the amount of time it will take to have a total of 270 feet of walkway completed.

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d. Determine the value of the variable that will make this equation true.

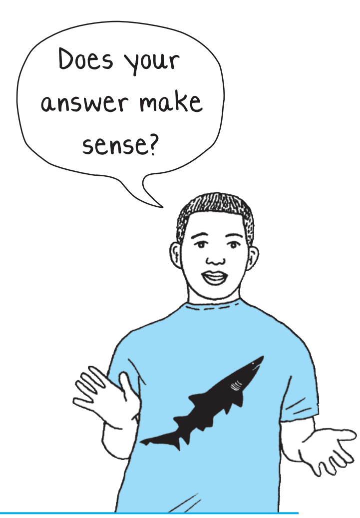
e. Interpret your answer in terms of this problem situation.

3. How many hours will the rangers need to work to build a total of 100 feet of walkway? Explain your reasoning.

a. What mathematical operations did you perform to determine your answer?

b. Write an equation that you can use to determine the amount of time it will take to have a total of 100 feet of walkway completed.

c. Determine the value of the variable that will make this equation true.



d. Interpret your answers in terms of this problem situation.

Problem 2 Rescuing a Bear Cub

1. Part of a park ranger's job is to perform rescue missions for people and animals. Suppose that a bear cub has fallen into a steep ravine on the park grounds. The cub is 77 feet below the surface of the ground in the ravine. A ranger coaxed the cub to climb into a basket attached to a rope and is pulling up the cub at a rate of 7 feet per minute.

   a. How many feet below the surface of the ground will the cub be in 6 minutes?

   b. How many feet below the surface of the ground will the cub be in 11 minutes?

   c. Define a variable for the amount of time spent pulling the cub up the ravine. Then use the variable to write an expression that represents the number of feet below the surface of the ground the cub is, given the number of minutes that the ranger has spent pulling up the cub.

   d. In how many minutes will the cub be 14 feet from the surface?

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e. Explain your reasoning you used to solve part (d).

f. What mathematical operations did you perform to determine your answer to part (d)?

g. Explain why using these mathematical operations gives you the correct answer.

h. Write an equation that you can use to determine the number of minutes it takes for the cub to be 14 feet below the surface of the ground by setting the expression you wrote in part (c) equal to 14. Then determine the value of the variable that will make the equation true.

2. In how many minutes will the cub be 28 feet from the surface?

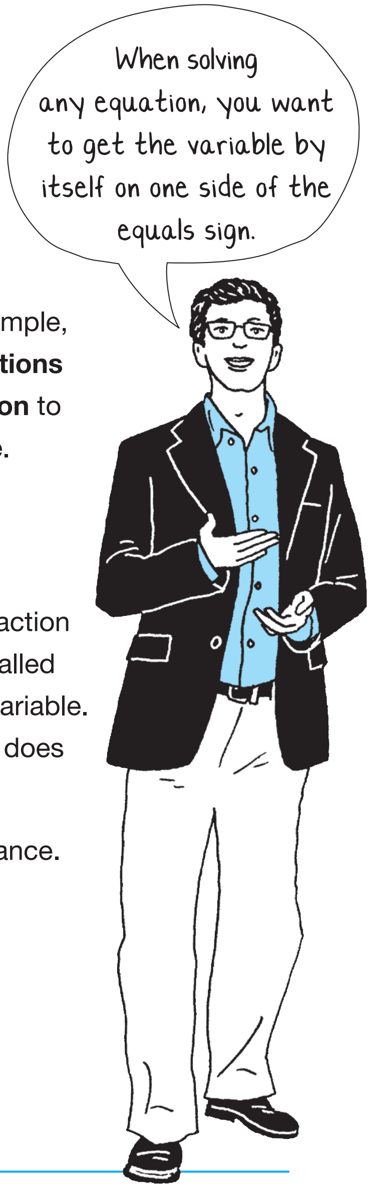
a. Explain your reasoning.

b. What mathematical operations did you perform to determine your answer?

c. Explain why using these mathematical operations gives you the correct answer.

d. Write an equation that you can use to determine in how many minutes the cub will be 28 feet from the surface. Then, determine the value of the variable that will make the equation true.

Problem 3 Solving Two-Step Equations



In Problems 1 and 2, you were solving two-step equations. To solve two-step equations you need to perform *inverse operations*. **Inverse operations** are operations that "undo" each other. For example, adding 3 and subtracting 3 are inverse operations. **Two-step equations** are equations that require two inverse operations to solve. A **solution** to an equation is any value for a variable that makes the equation true.

Let's consider the equation:

$$2m - 6 = 22$$

The left side of this equation has two terms separated by the subtraction operation. The 2 in the first term of the left side of the equation is called the *coefficient*. A **coefficient** is the number that is multiplied by a variable. The terms 6 and 22 are called *constants*. A **constant** is a term that does not change in value.

Remember, when you are solving equations you must maintain balance.

Two different examples of ways to solve the same two-step equation are shown.

	Method 1	Method 2
	$2m - 6 = 22$	$2m - 6 = 22$
Step 1:	$2m - 6 + 6 = 22 + 6$	$\underline{\quad + 6 = + 6}$
	$2m = 28$	$2m = 28$
Step 2:	$\frac{2m}{2} = \frac{28}{2}$	$\frac{2m}{2} = \frac{28}{2}$
	$m = 14$	$m = 14$

1. Describe the inverse operations used in each step.

   Step 1:

   Step 2:

2. What is the difference between the strategies used to solve the equation?

3. Verify the solution is $m = 14$.

4. Solve each two-step equation. Show your work.

   a. $5v - 34 = 26$

   b. $3x + 7 = 37$

c. $23 + 4x = 83$

d. $2.5c - 12 = 13$

e. $\frac{3}{4}x + 2 = 4\frac{2}{3}$

f. $-\frac{2}{3}b + \frac{2}{5} = 6\frac{4}{5}$

g. $-\frac{t}{5} - 9 = 21$

h. $2 = 2.27 - \frac{s}{4}$

i. $12m - 17 = 139$

j. $121.1 = -19.3 - 4d$

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k. $-23z + 234 = 970$

l. $7685 = 345 - 5d$

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Talk the Talk

The **Properties of Equality** allow you to balance and solve equations involving any number.

Properties of Equality	For all numbers *a*, *b*, and *c*, ...
Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $ac = bc$.
Division Property of Equality	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

1. Describe the strategies you can use to solve any two-step equation.

2. Describe a solution to any equation.

Be prepared to share your solutions and methods.

1.2 Why Doesn't This Work?

Equations with Infinite or No Solutions

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Learning Goals

In this lesson, you will:

- Identify and solve equations that have infinite solutions.
- Identify and solve equations that have no solutions.

Some things are always true, and some things are never true. And sometimes, using math, we can discover things that are always true or never true that are somewhat surprising.

The Meteorologists' Theorem is a good example of a surprising result that is always true. This theorem tells us that there are always two places directly opposite each other on Earth that have the exact same temperature.

What other things can you think of that are always true in math? How about never true?

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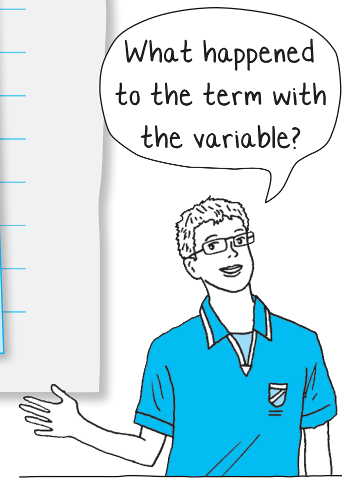
Problem 1 Interpreting Solutions

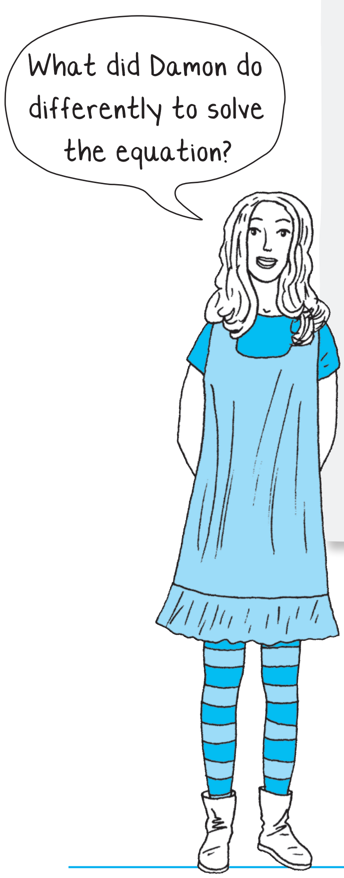
Amy and Damon were solving an equation from their math homework. They came across the equation shown.

$$3x + 7 = 5x + 2(3 - x) + 1$$

Amy

$$3x + 7 = 5x + 2(3 - x) + 1$$
$$3x - 5x + 7 = 5x - 5x + 2(3 - x) + 1$$
$$-2x + 7 = 2(3 - x) + 1$$
$$-2x + 7 = 6 + (-2x) + 1$$
$$-2x + 7 = 7 + (-2x)$$
$$-2x + 2x + 7 = 7 + (-2x) + 2x$$
$$7 = 7$$





Damon

$$3x + 7 = 5x + 2(3 - x) + 1$$
$$3x + 7 = 5x + 6 + (-2x) + 1$$
$$3x + 7 = 5x + (-2x) + 6 + 1$$
$$3x + 7 = 3x + 7$$
$$3x + 7 + (-7) = 3x + 7 + (-7)$$
$$\frac{3x}{3} = \frac{3x}{3}$$
$$x = x$$

1. Explain why both Amy's and Damon's methods are correct, but have different solutions.

2. How would you interpret the final equation in each solution? Is the final equation always true, sometimes true, or never true? Explain your reasoning.

3. Complete the table by substituting the given values of x into the expressions from each side of the equation and evaluate them. Show your work.

x	$3x + 7$	$5x + 2(3 - x) + 1$
-5		
-3		
1		
4		

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4. Based on the results from evaluating each row, does it appear that the expressions are equivalent? Explain your reasoning.

You can also use a graphing calculator to determine if the expressions are equivalent. Follow the steps provided to graph the expression $3x + 7$.

Step 1: Press **Y=**. Your cursor should be blinking on the line \$Y_1 =$. Enter the equation. To enter a variable like *x*, Press **X, T, θ, *n*** once.

Step 2: Press **WINDOW** to set the bounds and intervals you want displayed.

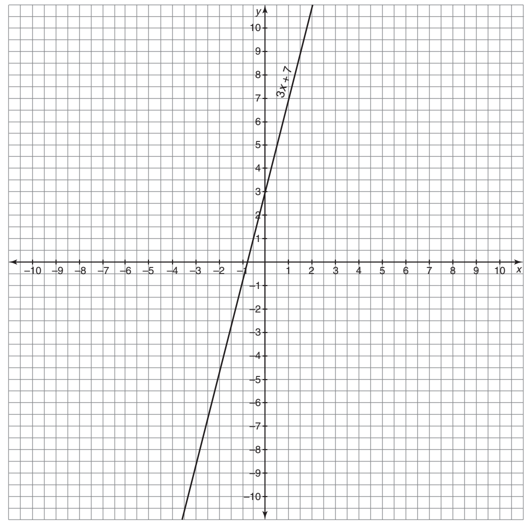
```
WINDOW
  Xmin  = -10
  Xmax  =  10
  Xscl  =  1
  Ymin  = -10
  Ymax  =  10
  Yscl  =  1
```

The **Xmin** represents the least point on the *x*-axis that will be seen on the screen. The **Xmax** represents the greatest point that will be seen on the *x*-axis. Lastly, the **Xscl** represents the intervals.

These same intervals are also used for the *y*-axis (**Ymin**, **Ymax**, and **Yscl**). Set the **Xmin** to -10. Set the **Xmax** to 10 and lastly set the interval to 1. Use the same settings for the *y*-axis.

Step 3: Press GRAPH.

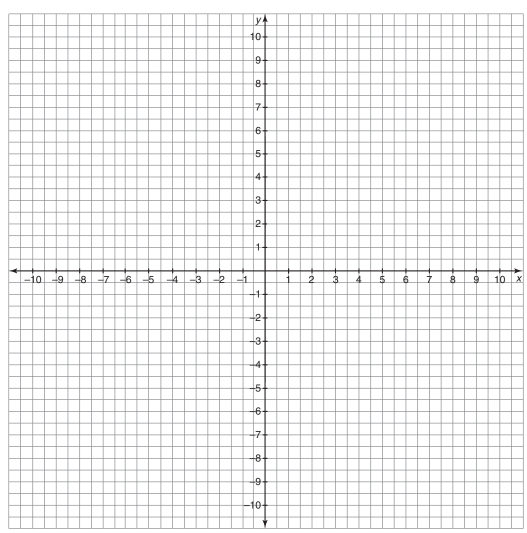
Your graphing calculator should display the screen shown.

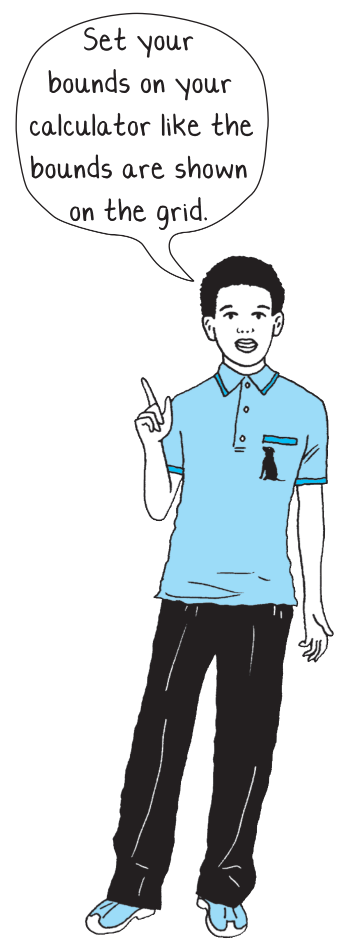


Next, graph the second expression from the table.

Press Y= and go to Y_2 and enter $5x + 2(3 - x) + 1$. Then, move your cursor to the left of Y_2 and press ENTER one time to change the way the line will be displayed. Finally, press GRAPH.

5. Sketch the graph on the coordinate plane shown.





6. What do you notice about each graph? What does this mean in terms of a value of *x* for which these expressions are equivalent?

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7. Amy says, "Since the expressions are equivalent, then the equation is true for all values of the variable. This means that there are an infinite number of solutions to this equation." Is Amy correct? Explain your reasoning.

Problem 2 Okay, But What About This One?

Consider this new equation:

$$3(x - 5) + 11 = x + 2(x + 5)$$

Amy

$3(x - 5) + 11 = x + 2(x + 5)$

$3x + (-15) + 11 = x + 2x + 10$

$3x + (-4) = 3x + 10$

$3x - 3x + (-4) = 3x - 3x + 10$

$-4 \neq 10$

Damon

$3(x - 5) + 11 = x + 2(x + 5)$

$3x + (-15) + 11 = x + 2x + 10$

$3x + (-4) + 4 = 3x + 10 + 4$

$3x = 3x + 14$

$3x + (-3x) = 3x + (-3x) + 14$

$0 \neq 14$

1. Explain why both Amy's and Damon's methods are correct, but have different solutions.

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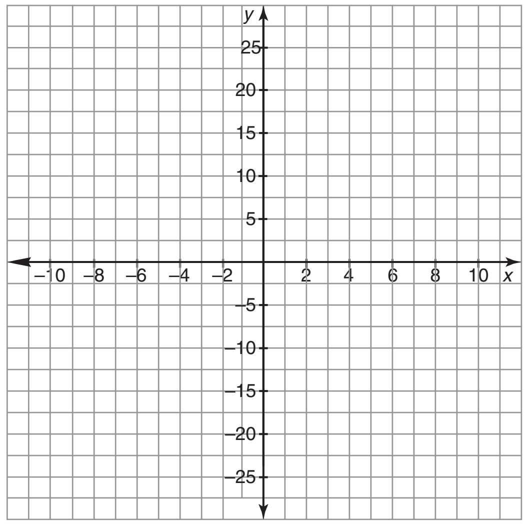
2. How would you interpret the final equation in each solution? Is the final equation always true, sometimes true, or never true? Explain your reasoning.

3. Complete the table by substituting the given values of x into the expressions from each side of the equation and evaluate them. Show your work.

x	$3(x - 5) + 11$	$x + 2(x + 5)$
-4		
-2		
1		
5		

4. Based on the results of evaluating each row, does it appear that the expressions are equivalent based on the values in the table? Explain your reasoning.

5. Graph each expression on your graphing calculator and sketch each graph on the coordinate plane shown.



6. What do you notice about each graph? What does this mean in terms of a value of x for which these expressions are equivalent?

7. Damon says, “Since the graphs never intersect, there are no solutions to this equation.” Is Damon correct? Explain your reasoning.

Talk the Talk

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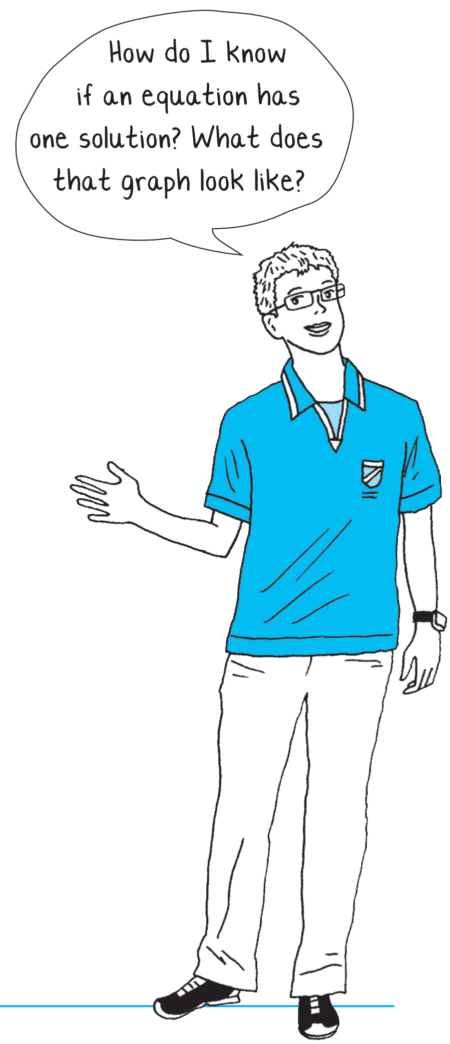
1. Solve each equation and determine if there are no solutions, one solution, or an infinite number of solutions. Check your answers.

   a. $2x - 7 + 3x = 4x + 2$

   b. $3(x - 1) + x = 4(x + 2)$



   c. $5(2x - 1) + x + 17 = 5x + 6(x + 2)$

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2. When you solve any equation, describe how you know when there will be:

   a. one solution.

   b. no solution.

   c. infinitely many solutions.

3. When you use a graphing calculator and you graph the left side of an equation as y_1 and the right side of the same equation as y_2, describe what the display will look like if there is:

   a. one solution.

   b. no solution.

   c. infinitely many solutions.

Be prepared to share your solutions and methods.

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1.3 Who Has the Most?

Solving Linear Equations

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Learning Goal

In this lesson, you will:

- Write and solve linear equations.

A magic square is an arrangement of numbers such that the sum of the numbers in each row and each column is the same.

A magic square has n rows and n columns. For example, the magic square shown has 3 rows and 3 columns.

1	5	9
8	3	4
6	7	2

This magic square is special. It is called a *normal* magic square because it contains each counting number from 1 to n^2. That is, it contains each counting number from 1 to 3^2, or 1 to 9.

There is an equation you can use to determine the sum of each row and column in a normal magic square. It is $s = \frac{n(n^2 + 1)}{2}$, where s stands for the sum.

Can you create a 4×4 normal magic square? What is the sum of each row and column in a 4×4 normal magic square?

Problem 1 How Many DVDs Do I Have?

Sometimes, you are asked to determine the value of unknown quantities using only information you have for a quantity. For example, inventory managers can determine how much product was sold and how much product to order using algebraic equations.

Five friends have a certain number of DVDs.

- Dan has the fewest.
- Donna has 7 more than Dan.
- Betty has twice as many as Donna.
- Jerry has 3 times as many as Dan.
- Kenesha has 6 less than Donna.

1. Define a variable for the number of DVDs that Dan has.

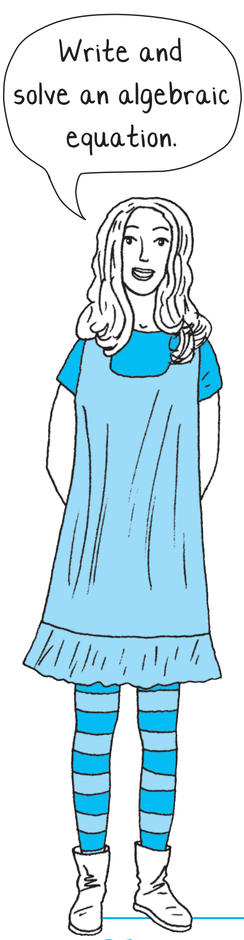
2. Use your defined variable to write algebraic expressions to represent the number of DVDs each person has.

   a. DVDs that Donna owns:

   b. DVDs that Betty owns:

   c. DVDs that Jerry owns:

   d. DVDs that Kenesha owns:



3. If the friends have a total of 182 DVDs altogether, then how many does each person have? Make sure to check your work.

   a. DVDs that Dan owns:

   b. DVDs that Donna owns:

   c. DVDs that Betty owns:

   d. DVDs that Jerry owns:

   e. DVDs that Kenesha owns:

Donna says that the sum of the number of her DVDs and Kenesha's DVDs is the same as the number of DVDs that Betty owns.

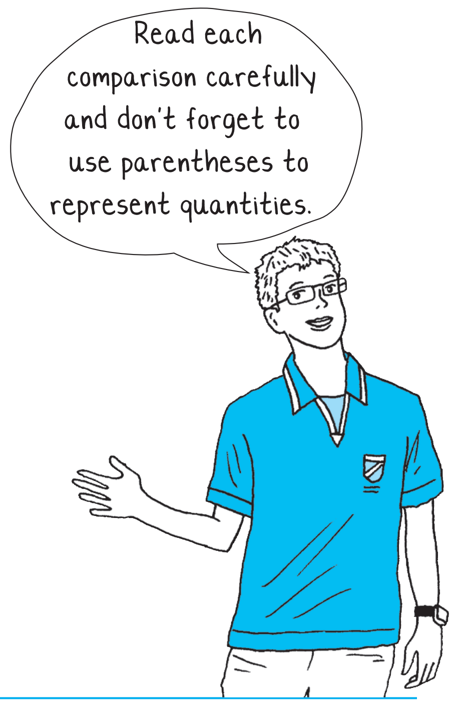
4. Write and solve an algebraic equation to show why Donna's reasoning is incorrect.

Problem 2 Raising Money!

A group of club members raised money for a club trip.

- Henry raised $7.50 less than Harry.
- Helen raised twice as much as Henry.
- Heddy raised a third as much as Helen.
- Hailey raised $4 less than 3 times as much as Helen.

1. Define a variable for the amount of money Harry raised.



2. Use your defined variable to write algebraic expressions to represent the amount each person raised.

   a. The amount of money Henry raised:

   b. The amount of money Helen raised:

   c. The amount of money Heddy raised:

   d. The amount of money Hailey raised:

3. If Harry raised $55, how much money did each person raise?

   a. The amount of money Henry raised:

   b. The amount of money Helen raised:

   c. The amount of money Heddy raised:

   d. The amount of money Hailey raised:

4. If Heddy raised $40, how much money did each person raise?

a. The amount of money Harry raised:

b. The amount of money Henry raised:

c. The amount of money Helen raised:

d. The amount of money Hailey raised:

5. If Henry, Helen, and Hailey raised $828.50 altogether, how much money did each person raise?

a. The amount of money Harry raised:

b. The amount of money Henry raised:

c. The amount of money Helen raised:

d. The amount of money Heddy raised:

e. The amount of money Hailey raised:

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6. Could Harry and Henry together have raised the same amount of money as Helen? Explain your reasoning.

7. If Heddy and Hailey raised $126 altogether, how much money did each person raise?

a. The amount of money Harry raised:

b. The amount of money Henry raised:

c. The amount of money Helen raised:

d. The amount of money Heddy raised:

e. The amount of money Hailey raised:

Be prepared to share your solutions and methods.

1.4 Games and Practice
Solving More Linear Equations

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Learning Goal

In this lesson, you will:

- Solve linear equations.

You probably have played your share of games. You either have played games in a sports setting, or perhaps you played board games or video games. However, game playing isn't something that is a new invention. In fact, throughout history people have enjoyed playing games. For example, many experts feel that the oldest board game originated from Egypt in approximately 3500 B.C. And games go beyond just board games. Another type of game people have loved to participate in is solving riddles.

To solve riddles, it takes using the information known to determine the information that is unknown. Have you ever tried solving riddles? What strategies have you used to solve riddles?

Problem 1 How Many MP3s Do I Have?

Terry, Trudy, Tom, and Trevor have challenged their friends with this riddle.

- Terry said: "If you add 150 to the number of MP3 downloads Tom has and then double that number and finally divide it by three, you have the number of MP3 downloads I have."
- Trudy said: "If you take the number of MP3 downloads Tom has, subtract 30, multiply that difference by 5, and finally divide that product by 4, the result will be the number of MP3 downloads I have."
- Trevor said: "Well, if you take twice the number of MP3 downloads Tom has, add 30, multiply the sum by 4, and finally divide that product by 3, you will have the number of MP3 downloads I have."

1. Do you have enough information to determine how many MP3 downloads each person has?

2. What do you need to know to determine the number of MP3 downloads each person has?

3. Define a variable for the number of MP3 downloads Tom has, and then write expressions for the number each of the other people has.

a. The number of MP3 downloads Terry has:

b. The number of MP3 downloads Trudy has:

c. The number of MP3 downloads Trevor has:

4. Suppose Tom has 150 MP3 downloads. Determine how many MP3 downloads each person has.

   a. Terry

   b. Trudy

   c. Trevor

5. What if Terry and Trevor have the same number of MP3 downloads? How many MP3 downloads would each person have?

   a. The number of MP3 downloads Tom has:

   b. The number of MP3 downloads Trudy has:

   c. The number of MP3 downloads Trevor and Terry have:

6. What if the sum of Trudy's and Trevor's MP3 downloads is 39 more than the number Terry has? How many would each person have?

a. The number of MP3 downloads Tom has:

b. The number of MP3 downloads Trudy has:

c. The number of MP3 downloads Trevor has:

d. The number of MP3 downloads Terry has:

Problem 2 Practice

Solve each equation shown. Make sure to check your work.

1. $\frac{3}{4}(2x + 5) = 14$

2. $\frac{-7(3x + 6)}{3} = 7$

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3. $\frac{-3(-2x - 5)}{4} = -5(3x + 5) + \frac{5}{4}$

4. $\frac{2}{3}(6x - 5) = 2 - \frac{1}{3}(3x - 2)$

Be prepared to share your solutions and methods.

Chapter 1 Summary

Key Terms

- two-step equation (1.1)
- inverse operations (1.1)
- solution (1.1)
- coefficient (1.1)
- constant (1.1)
- Properties of Equality (1.1)

1.1 Solving Two-Step Equations

Many situations can be modeled by equations that need more than one operation to solve. Perform inverse operations to get the variable by itself on one side of the equals sign to solve the equation.

Example

Follow the steps of inverse operations to calculate the solution for the equation $6x + 9 = 33$

$$6x + 9 = 33$$

$$6x + 9 - 9 = 33 - 9$$

$$6x = 24$$

$$\frac{6x}{6} = \frac{24}{6}$$

$$x = 4$$

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1.2 Solving Equations That Have Infinite Solutions

When solving an equation with infinite solutions, the final equation after simplifying is always true for any value of the variable.

Example

The equation shown has infinite solutions because the constants are equivalent to each other.

$$\begin{aligned}
2(x-3) + x - 5 &= 4(x-5) + 9 - x \\
2x + (-6) + x + (-5) &= 4x + (-20) + 9 + (-x) \\
2x + x + (-6) + (-5) &= 4x + (-x) + (-20) + 9 \\
3x + (-11) &= 3x + (-11) \\
3x + (-3x) + (-11) &= 3x + (-3x) + (-11) \\
-11 &= -11
\end{aligned}$$

1.2 Solving Equations That Have No Solutions

When solving an equation that has no solutions, the final equation after simplifying is never true for any value of the variable.

Example

The equation shown has no solutions because the two constants are not equivalent to each other.

$$\begin{aligned}
6(2x+3) - x &= 5(3x-2) - 4x + 1 \\
12x + 18 + (-x) &= 15x + (-10) + (-4x) + 1 \\
12x + (-x) + 18 &= 15x + (-4x) + (-10) + 1 \\
11x + 18 &= 11x + (-9) \\
11x + (-11x) + 18 &= 11x + (-11x) + (-9) \\
18 &\neq -9
\end{aligned}$$

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1.3 Writing and Solving Linear Equations

Define a variable and use the variable to write algebraic expressions to represent the different amounts in the problem situation. Combine the expressions to write and solve an algebraic equation.

Example

The ages of four siblings equals 27. Let x represent Paige's age.

- Peter is 2 years older than Paige: $x + 2$
- Perry is 3 years older than twice Paige's age: $2x + 3$
- Pippa is 11 years younger than Perry: $(2x + 3) - 11$

$$x + (x + 2) + (2x + 3) + [(2x+3) - 11] = 27$$
$$x + x + 2x + 2x + 2 + 3 + 3 + (-11) = 27$$
$$6x + (-3) = 27$$
$$6x + (-3) + 3 = 27 + 3$$
$$6x = 30$$
$$\frac{6x}{6} = \frac{30}{6}$$
$$x = 5$$

Paige $= 5$

Peter $= 5 + 2$, or 7

Perry $= 2(5) + 3$; or 13

Pippa $= 2(5) + 3 - 11$, or 2

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1.4 Solving Linear Equations

Combine terms and perform inverse operations to get the variable by itself on one side of the equals sign to solve the equation. Check your solution by substituting the solution for the variable in the original equation.

Example

You must perform operations and inverse operations to determine the value of the variable. Once the variable's value is determined, the value is substituted into the original equation.

$$\frac{2(-4x-6)}{5} = \frac{7}{15}(3x+9) - \frac{3}{5}$$
$$5\left(\frac{2(-4x-6)}{5}\right) = 5\left(\frac{7}{15}(3x+9) - \frac{3}{5}\right)$$
$$2(-4x-6) = \frac{7}{3}(3x+9) - 3$$
$$-8x - 12 = 7x + 21 - 3$$
$$-8x - 12 - 18 = 7x + 18 - 18$$
$$-8x + 8x - 30 = 7x + 8x$$
$$-30 = 15x$$
$$\frac{-30}{15} = \frac{15x}{15}$$
$$-2 = x$$

Check:

$$\frac{2(-4(-2)-6)}{5} \stackrel{?}{=} \frac{7}{15}(3(-2)+9) - \frac{3}{5}$$
$$\frac{2(8-6)}{5} \stackrel{?}{=} \frac{7}{15}(-6+9) - \frac{3}{5}$$
$$\frac{4}{5} \stackrel{?}{=} \frac{7}{5} - \frac{3}{5}$$
$$\frac{4}{5} = \frac{4}{5}$$