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# PATTERNS, PATTERNS, PATTERAS... Developing Sequences of Numbers from Diagrams and Contexts 

## Learning Goals

In this lesson, you will:

- Write sequences of numbers generated from the creation of diagrams and written contexts.
- State varying growth patterns of sequences.

Key Terms

- sequence
- term
- ellipsis
egend tells us that when the inventor of the game of chess showed his work to the emperor, the emperor was so pleased that he allowed the inventor to choose any prize he wished. So the very wise inventor asked for the following: 1 gold coin for the first square on the chess board, 2 gold coins for the second square, 4 coins for the third, and so on up to the 64th square. The emperor, not as wise as the inventor, quickly agreed to such a cheap prize. Unfortunately, the emperor could not afford to pay even the amount for just the 32nd square: $2,147,483,648$ gold coins!

How many gold coins would the emperor have to pay for just the 10th square? 20th square? What pattern did you use to calculate your answers?

## Problem 1 Sequences

The inventor from the story used his knowledge of sequences to his advantage to gain riches.


A sequence is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects. A term in a sequence is an individual number, figure, or letter in the sequence.


Often, only the first few terms of a sequence are listed, followed by an ellipsis. An ellipsis is three periods, which stand for "and so on."

1. What is the next term in Sequence $A$ ?
2. What is the third term in Sequence $B$ ?
3. What is the twenty-fifth term in Sequence C?
4. What is the twelfth term in Sequence $D$ ?

## Problem 2 Designing a Bead Necklace

Emily is designing a necklace by alternating black and green beads. To create her necklace, she performs the following steps.

Step 1: She starts with one black bead.

Step 2: Next, she places one green bead on each side of the black bead.

Step 3: Then, she places two black beads on each side of the green beads.

Step 4: Then, she places three green beads on each side of the black beads.

Step 5 and 6: She continues this pattern two more times, alternating between black and green sets of beads.


1. Write the first six terms in the sequence that represents this situation. Make sure each term indicates the total number of beads on the necklace after Emily completes that step. Finally, explain how you determined the sequence.


## Problem 3 Crafting Toothpick Houses

Ross is crafting toothpick houses for the background of a diorama. He creates one house and then adds additional houses by adjoining them as shown.
 toothpicks used for one house. The second term should indicate the total number of toothpicks needed for two houses, and so on. Explain your reasoning.
2. How is the number of toothpicks needed to build each house represented in the sequence?

## Problem 4 Taking Apart a Card Trick

Matthew is performing a card trick. It is important that he collect the cards shown in a particular order. Each turn, he collects all of the cards in the right-most column, and all the cards in the bottom row.


1. Write a sequence to show the number of cards removed during each of the first five turns.
2. Write a sequence to show the number of cards remaining after each of the first five turns.
3. What pattern is shown in each sequence?

## Problem 5 Arranging Pennies

Lenny is making arrangements with pennies. He has made three penny arrangements and now he wants to make five more arrangements. Each time he adds another arrangement, he needs to add one more row to the base than the previous row in the previous arrangement.


1. Write the first eight terms in the sequence that represents this situation. Each term should indicate the total number of pennies in each arrangement.
Explain your reasoning.
2. Explain why the pattern does not increase by the same amount each time.

## Problem 6 Building Stairs



1. Write the first five terms in the sequence that represents this situation. Each term should indicate the number of faces shown from the cubes shown. The bottom faces are not shown. The first cube has 5 shown faces. Explain your reasoning.
2. Predict the number of shown faces in a stair configuration that is 7 cubes high. Show your work.

## Problem 7 Arranging Classroom Tables

Some schools purchase classroom tables that have trapezoid-shaped tops rather than rectangular tops. The tables fit together nicely to arrange the classroom in a variety of ways. The number of students that can fit around a table is shown in the first diagram. The second diagram shows how the tables can be joined at the sides to make one longer table.


1. Write the first 5 terms in the sequence that represents this situation. Each term should indicate the total number of students that can sit around one, two, three, four, and five tables. Explain your reasoning.
2. The first trapezoid table seats five students. Explain why each additional table does not have seats for five students.

## Problem 8 Drawing Flower Petals

Draw a flower in a series of stages. The figure shows a pair of flower petals as the starting point, Stage 0. In each stage, draw new petal pairs in the middle of every petal pair already drawn.

- In Stage 1, you will draw $\qquad$ petals.
- In Stage 2, you will draw $\qquad$ petals.
- In Stage 3, you will draw $\qquad$ petals.


1. Write the first 5 terms in the sequence that represents this situation. Each term should indicate the number of new petals drawn in that stage. Explain your reasoning.
[^0]
## Problem 9 Babysitting

Every Friday, Sarah earns $\$ 14$ for babysitting. Every Saturday, Sarah spends $\$ 10$ going out with her friends.

1. Write a sequence to show the amounts of money Sarah has every Friday after babysitting and every Saturday after going out with her friends for five consecutive weeks. The sequence should have 10 terms. Explain your reasoning.

## Problem 10 Recycling

The first week of school, Ms. Sinopoli asked her class to participate in collecting cans for recycling. The students started bringing in cans the second week of school. They collected 120 cans per week.

1. Write a sequence to show the running total of cans collected through the first nine weeks of school. Explain your reasoning.

## Problem 11 Selling Tickets

Sam is working at the ticket booth during a basketball game. His cash box has two $\$ 10$ bills, five $\$ 5$ bills, and twenty $\$ 1$ bills. Tickets cost $\$ 3$.

1. How much money does Sam have at the beginning of the basketball game?
2. Write a sequence to show the amount of cash Sam has available to start selling tickets, and the amounts available after selling one ticket, two tickets, three tickets, four tickets, and five tickets. Explain your reasoning.

## Talk the Talk



There are many different patterns that can generate a sequence.
Some possible patterns are:

- adding or subtracting by the same number each time,
- multiplying or dividing by the same number each time,
- adding by a different number each time, with the numbers being part of a pattern,
- alternating between adding and subtracting.

The next term in a sequence is calculated by determining the pattern of the sequence and then using that pattern on the last known term of the sequence.

Look back at Problems 2 through 11.

1. Describe the pattern of each sequence by completing the table shown.

| Sequence Name | Increases or <br> Decreases | Describe the Pattern |
| :---: | :---: | :---: |
| Designing a Bead Necklace |  |  |
| Crafting a Toothpick House |  |  |
| Taking Apart a Card Trick (1) |  |  |
| Arranging Pennies |  |  |
| Building Stairs |  |  |
| Arranging Classroom Tables |  |  |
| Drawing Flower Petals |  |  |
| Babysitting |  |  |
| Recycling |  |  |
| Selling Tickets |  |  |

2. Which sequences are similar? Explain your reasoning.

Be prepared to share your solutions and methods.

# EVERY GRAPH TELLS A STORY Describing Characteristics of Graphs 

## Learning Goals

In this lesson, you will:

- Describe characteristics of graphs using mathematical terminology.
- Describe a real-world situation that could be represented by a given graph.

Key Terms

- discrete graph
> continuous graph
- linear graph
- collinear points
- non-linear graph

Have you ever followed a trail of animal tracks? For expert animal trackers, there are many more signs to look for instead of just paw prints. Expert trackers look for rub, like when a deer scrapes velvet off its antlers. They look for chews-where a twig or section of grass has been eaten. If there is a clean cut on the plant, it may likely have been caused by an animal with incisors (like a rodent). If the plants have teeth marks all over them, those plants may likely have been eaten by a predator.

And of course, trackers look for scat, or droppings. From scat, trackers can tell an animal's shape and size and what the animal eats. Tubular scat may come from raccoons, bears, and skunks. Teardrop-shaped scat may come from an animal in the cat family. How do you follow clues in mathematics to solve problems?


## Problem 1 Characteristics of Graphs

There are many ways that data can be represented through graphical displays. In this lesson, you will explore many characteristics of graphs.

1. Graph the first four terms of Sequence $A: 0,2,4,6$. Let the term number represent the $x$-coordinate, and let the term value represent the $y$-coordinate. Then, list the coordinates of the points on your graph.


2. Would it make sense to connect the points on your graph? Why or why not?

A discrete graph is a graph of isolated points. Often, those points are counting numbers and do not consist of fractional numbers. A continuous graph is a graph with no breaks in it. The points in a continuous graph can have whole numbers and fractions to represent data points.
3. Is your graph from Question 1 discrete or continuous? Explain your reasoning.
4. Are the graphs of any sequence discrete or continuous? Explain your reasoning.
5. Carefully cut out Graphs $A$ through $L$ on the following pages.





D | y |
| :---: |



$x^{\circ}$

2




s<


2

2
6. Determine if the graphs you cut out are discrete or continuous.
a. Sort the graphs into two groups: those graphs that are discrete and those graphs that are continuous.
b. Record your findings in the table by writing the letter of each graph.

| Discrete Graphs | Continuous Graphs |
| :--- | :--- |
|  |  |
|  |  |

7. Determine if the graphs are increasing, decreasing, both increasing and decreasing, or neither increasing nor decreasing.
a. Analyze each graph from left to right.
b. Sort the graphs into four groups: those that are increasing, those that are decreasing, those that are both increasing and decreasing, and those that are neither increasing nor decreasing.
c. Record your findings in the table by writing the letter of each graph.

| Increasing | Decreasing | Both Increasing <br> and Decreasing | Neither Increasing <br> nor Decreasing |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
|  |  |  |  |

A linear graph is a graph that is a line or a series of collinear points. Collinear points are points that lie in the same straight line. A non-linear graph is a graph that is not a line and therefore not a series of collinear points.
8. Determine whether Graphs $A-L$ are linear or non-linear graphs.
a. Sort the graphs into two groups: those that are linear and those that are non-linear.
b. Record your findings in the table by writing the letter of each graph.


| Linear Graph | Non-linear Graph |
| :--- | :--- |
|  |  |
|  |  |

9. Clip Graphs $A-L$ together, and keep them for Lessons 2 and 3.


## Problem 2 Making Sense of Graphs



The graph shown represents Greg's distance from home after driving for $x$ hours.


1. Analyze the graph between 0 and 2 hours.
a. How far from home was Greg after driving for 2 hours?
b. How fast did Greg drive during this time? Explain your reasoning.
c. How do you know that Greg traveled at the same rate for the first two hours? Describe in terms of the graph.
2. Analyze the graph between 2 and 2.5 hours.
a. How far did Greg travel from home between 2 and 2.5 hours?
b. How fast did he travel during this time? Explain your reasoning.
c. Describe the shape of the graph between 2 and 2.5 hours.
3. Complete the table.

Label each segment of the graph with letters $A$ through $G$, beginning from the left. Record the time interval for each segment. Then, describe what happened in the problem situation represented by that segment of the graph. State how fast Greg traveled and in what direction (either from home or to home).

| Segment | Time Interval (hours) | Description of Greg's Trip |
| :---: | :---: | :---: |
| A | 0 to 2 | Greg traveled 120 miles from home at a rate of 60 mph . |
| $B$ | 2 to 2.5 | Greg took a half-hour break when he was 120 miles from home. |
| c |  |  |
| D |  |  |
| E |  |  |
| F |  |  |
| G |  |  |

4. The crew at the community swimming pool prepared the pool for opening day. The graph shows the depth of water in the swimming pool after $x$ hours.

a. Why do you think the pool was emptied and then refilled?
b. Complete the table.

Label each segment of the graph with letters $A$ through $E$, beginning from the left. Record the time interval for each segment. Then, describe what occurred in the problem situation represented by that segment in the graph. State how fast the water level in the pool changed and whether it was being drained or filled.

| Segment | Time Interval <br> (hours) |  |
| :---: | :---: | :---: |
| A |  |  |
| B |  |  |
| C |  |  |
| D |  |  |
| E |  |  |
|  |  |  |

c. Was the pool being emptied at the same rate the entire time? Explain using mathematics and the graph.
d. Why does it make sense for the graph of this situation to be continuous rather than discrete?

## Problem 3 Tell a Story



You and a friend go to the movies and decide to share a large bucket of popcorn．Write a story to describe each graph．
1.
2.


Be prepared to share your solutions and methods．

# TO BE OR MOT TO BE A FUNCTION? Defining and Recognizing Functions 

## Learning Goals

In this lesson, you will:

- Define relation and function.
- Determine whether a relation (represented as a mapping, set of ordered pairs, table, sequence, graph, equation, or context) is a function.


## Key Terms

- mapping
- set
- relation
- input
- output
- function
- domain
- range
- scatter plot
- vertical line test
n September 2009, museum volunteers in England began work on restoring the WITCH machine-regarded as the first modern computer still able to work. This huge computer, as long as an entire wall in a large room, was built starting in 1949 and was functional until 1957.

WITCH was used to perform mathematical calculations, but instead of typed input, the computer had to be fed paper tape for inputs. Then, the computer would produce its output on paper as well. Even though it was so huge, WITCH could only perform calculations as fast as a human with a modern calculator.

What types of inputs and outputs do modern computers use and produce? How does a modern computer turn inputs into outputs?

## Problem 1 Analyzing Ordered Pairs

As you learned previously, ordered pairs consist of an $x$-coordinate and a $y$-coordinate. You also learned that a series of ordered pairs on a coordinate plane can represent a pattern. You can also use a mapping to show ordered pairs. Mapping represents two sets of objects or items. An arrow connects the items together to represent a relationship between the two items.


1. Write the set of ordered pairs that represent a relationship in each mapping.
a.

b.

c.

d.

2. Create a mapping from the set of ordered pairs.
a. $\{(5,8),(11,9),(6,8),(8,5)\}$
b. $\{(3,4),(9,8),(3,7),(4,20)\}$

When you write out the ordered pairs for a mapping, you are writing a set of ordered pairs. A set is a collection of numbers, geometric figures, letters, or other objects that have some characteristic in common.
3. Write the set of ordered pairs to represent each table.
a.

| Input | Output |
| :---: | :---: |
| -10 | 20 |
| -5 | 10 |
| 0 | 0 |
| 5 | 10 |
| 10 | 20 |

b.

| $x$ | $y$ |
| :---: | :---: |
| 20 | -10 |
| 10 | -5 |
| 0 | 0 |
| 10 | 5 |
| 20 | 10 |

The mappings or ordered pairs shown in Questions 1 through 3 form relations. A relation is any set of ordered pairs or the mapping between a set of inputs and a set of outputs. The first coordinate of an ordered pair in a relation is the input, and the second coordinate is the output. A function maps each input to one and only one output. In other words, a function has no input with more than one output. The domain of a function is the set of all inputs of the function. The range of a function is the set of all outputs of the function.


In the mapping shown the domain is $\{1,2,3,4\}$ and the range is $\{1,3,5,7\}$.

This mapping represents a function because each input, or domain value, is mapped to only one output, or range value.


In the mapping shown the domain is $\{1,2,3,4,5\}$ and the range is $\{1,3,5,7\}$.

This mapping does not represent a function.
4. State why the relation in the example shown is not a function.

5. State the domain and range for each relation in Questions 2 and 3 . Then, determine which relations represent functions. If the relation is not a function, state why not.
6. Review and analyze Emil's work.

## Emil

My mapping represents
a function.


Explain why Emil's mapping is not an example of a function.

## Problem 2 Analyzing Contexts



Read each context and decide whether it fits the definition of a function.
Explain your reasoning.

1. Input: Sue writes a thank-you note to her best friend.

Output: Her best friend receives the thank-you note in the mail.
2. Input: A football game is being telecast.

Output: It appears on televisions in millions of homes.
3. Input: There are four puppies in a litter.

Output: One puppy was adopted by the Smiths, another by the Jacksons, and the remaining two by the Fullers.
4. Input: The basketball team has numbered uniforms.

Output: Each player wears a uniform with her assigned number.
5. Input: Beverly Hills, California, has the zip code 90210.

Output: There are 34,675 people living in Beverly Hills.
6. Input: A sneak preview of a new movie is being shown in a local theater. Output: 65 people are in the audience.
7. Input: Tara works at a fast food restaurant on weekdays and a card store on weekends.
Output: Tara's job on any one day.
8. Input: Janelle sends a text message to everyone in her contact list on her cell phone. Output: There are 41 friends and family on Janelle's contact list.
9. Create your own context problem, and decide whether it represents a function. Trade with a partner, and solve your partner's problem. Then, discuss your responses. Input:

Output:

## Problem 3 Analyzing Sequences


b. $1,0,1,0,1, \ldots$

Determine if each sequence represents a function.
Explain why or why not. If it is a function, identify its domain and range.
a. $2,4,6,8,10, \ldots$


2
c. $0,5,10,15,20, \ldots$
2. What do you notice about each answer in Question 1? What conclusion can you make about sequences?

## Problem 4 Analyzing Graphs



A relation can be represented as a graph. Graphs $A-L$ from Lesson 2.2 provide examples of graphical representations of relations.

A scatter plot is a graph of a collection of ordered pairs that allows an exploration of the relationship between the points.

1. Determine if these scatter plots represent functions. Explain your reasoning.
a.


b.


The vertical line test is a visual method used to determine whether a relation represented as a graph is a function. To apply the vertical line test, consider all of the vertical lines that could be drawn on the graph of a relation. If any of the vertical lines intersect the graph of the relation at more than one point, then the relation is not a function.

Review the scatter plot shown.


In this scatter plot, the relation is not a function. The input value 4 can be mapped to two different outputs, 1 and 4 . Those two outputs are shown as intersections to the vertical line segment drawn at $x=4$.
2. Use the vertical line test to determine if each graph represents a function. Explain your reasoning.
a.

b.



## Problem 5 Analyzing Equations



So far, you have determined whether a set of data points in a scatter plot represents a function. You can also determine whether an equation is a function.

The given equation can be used to convert yards into feet. Let $x$ represent the number of yards, and $y$ represent the number of feet.


1. Determine whether each equation is a function. List three ordered pairs that are solutions to each. Explain your reasoning.
a. $y=5 x+3$
b. $y=x^{2}$
c. $y=|x|$


## Problem 6 Function or Not?

1. Sorting Activity
a. Carefully cut out Relations $M$ through $X$ on the following pages.
b. Refer to Graphs $A$ through $L$ from Lesson 2.
c. Sort Relations $A$ through $X$ into two groups: those that are functions and those that are not functions.
d. Record your findings in the table by writing the letter of each relation.

Time to get

your graphs from the first lesson back out.


| Functions | Not Functions |
| :---: | :---: |
|  |  |
|  |  |



2

| S <br> The set of ordered pairs $\begin{aligned} & \{(2,3),(2,4),(2,5), \\ & (2,6),(2,7)\} \end{aligned}$ | T <br> The set of ordered pairs $\begin{aligned} & \{(2,1),(3,1),(4,1), \\ & (5,1),(6,1)\} \end{aligned}$ |
| :---: | :---: |
| U $y=x^{2}+1$ | v <br> This equation is used to calculate the number of inches in a foot: $y=12 x$ <br> Let $x$ represent the number of feet and $y$ represent the number of inches. |
| w <br> Input: <br> The morning announcements are read over the school intercom system during homeroom period. <br> Output: <br> All students report to homeroom at the start of the school day to listen to the announcements. | x <br> Input: <br> Each student goes through the cafeteria line. <br> Output: <br> Each student selects a lunch from the menu. |

2

## Talk the Talk

Choose the appropriate description to complete each sentence.

1. A relation is (always, sometimes, never) a function.
2. A function is (always, sometimes, never) a relation.

2

## SCALIMG A CLIFF Linear Functions

## Learning Goals

In this lesson, you will:

- Make input-output tables for linear functions.
- Graph linear functions.
- Determine characteristics of linear functions.

Key Term

- linear function

$C$an you draw a perfectly straight line without using a ruler or other straightedge? What about over a long distance?

Carpenters and other construction workers use what is called a chalk line to mark straight lines over long distances. A chalk line tool looks a bit like a tape measure. A cord that is coated in chalk is wound inside the tool. One person pulls the cord to the end of where the line will be, and the other person holds the tool at the beginning of the line. When they have the line where they want, both people pull the cord tight, and one person pulls up on the cord and lets go so that the cord "snaps" a straight line of chalk onto the surface below the cord.

Have you ever seen or used a chalk line tool? Does anyone in your class have one they could show?

## Problem 1 Climbing to the Top!

1. You and your friends are rock climbing a vertical cliff that is 108 feet tall along a beach. You have been climbing for a while and are currently 36 feet above the beach when you stop on a ledge to have a snack and then begin climbing again. You can climb about 12 feet in height each hour. If you maintain your pace after your break, how high will you have climbed in:
a. 1 hour?
b. 2 hours?
c. 180 minutes?
d. 210 minutes?
e. Which quantities are changing? Which quantities remain constant?
f. Which quantity depends on the other quantity?

2. Define a variable for the input quantity. Enter this variable in the "Expression" row at the bottom of Column 1.
3. Write an expression that you can use to represent the output quantity in terms of the input quantity. Enter this expression in the "Expression" row under output.

4. Use your expression to write an equation that you can solve to determine each answer. Then, write your answer in the appropriate place in table.
a. How long will it be until you have climbed to 84 feet above the beach?
b. How long will it be until you have climbed to 96 feet above the beach?
c. How long will it be until you have reached the top of the cliff?
5. Create a graph to represent the values in your table. Label the horizontal axis with the input quantity and the vertical axis with the output quantity. The axes are already numbered. Finally, plot the points on the coordinate plane.

6. Connect the points on your graphs.
7. Determine the domain and range of this situation.

8. Does your table represent the same domain and range? Why or why not?
9. Is the relation shown in the graph a function? Explain why or why not.


Drawing a line through the data set of a graph is a way to model or represent relationships. The points on your graph represent equivalent ratios because the climbing time per height remained constant. In some problem situations, when you draw a line all the points will make sense. In other problem situations, not all the points on the line will make sense. For example, if a graph displayed the cost per ticket, you cannot purchase a fractional part of a ticket, but the line you would draw on the graph would help you model the relationship and see how the cost changes as more tickets are purchased. So, when you graph relations and model that relationship with a line, it is up to you to consider each situation and interpret the meaning of the data values from a line drawn on a graph.

## Talk the Talk

When you graph the input and output values of some functions, the graph forms a straight line. A function whose graph is a straight line is a linear function.

The relation shown in the graph in this lesson is a linear function. The graph is a line segment.

Let's think about the problem situation, your table, and your graph.


1. Which variable is the dependent variable?
2. Which variable is the independent variable?
3. Describe what happens to the value of the dependent variable each time the independent variable increases by 1.
4. Describe what happens to the value of the dependent variable when the independent variable increases by 2 .
5. Compare the values of the dependent variable when the independent variable is 1 and 6 . Describe how the dependent variable changes in relation to the independent variable.
6. Describe how the independent and dependent values change in linear functions.


Be prepared to share your solutions and methods.

## U.S. SHIRTS Using Tables, Graphs, and Equations, Part 1

## Learning Goals

In this lesson, you will:

- Use different models to represent a problem situation.
- Determine an initial value when given a final result.
- Identify the advantages and disadvantages of using a particular representation.

Have you ever wondered where your clothes come from? Who actually makes the clothes you wear? For the most part, clothes are made in countries like Vietnam, India, Pakistan, and Mexico, just to name a few. However, only 40 to 50 years ago, clothes were created here in the United States. It was common for people to seek employment creating clothes. Well, the trend of creating clothes in the United States is slowly on the rise. The opening of boutiques and American clothes designers have stressed creating unique and cutting edge fashion, but also not to mass produce clothing-and this idea of creating clothes in the United States has reinvented itself. Why do you think clothing began being made in other countries? Do you think the United States will one day become a clothing creating powerhouse that it once was?

## Problem 1 Cost Analysis



This past summer you were hired to work at a custom T-shirt shop, U.S. Shirts. One of your responsibilities is to calculate the total cost of customers' orders. The shop charges $\$ 8$ per shirt plus a one-time charge of $\$ 15$ to set up a T-shirt design.

1. Describe the problem situation and your responsibility in your own words.
2. What is the total cost of an order for:
a. 3 shirts?

3. Explain how you calculated each total cost.
4. How many shirts can a customer buy if they have:
a. $\$ 50$ to spend?
b. $\$ 60$ to spend?
c. $\$ 220$ to spend?

5. Explain how you calculated the number of shirts that each customer can buy.

6. Complete the table of values for the problem situation.

7. What are the variable quantities in this problem situation? Define the variables that can represent these quantities including each quantity's units.
8. What are the constant quantities in this problem situation? Include the units that are used to measure these quantities.
9. Which variable quantity depends on the other variable quantity?
10. Which of the variables from Question 7 is the independent variable, and which is the dependent variable?

11. Create a graph of the data from your table in Question 6 on the grid shown.

First, choose your bounds and intervals by completing the table shown. Remember to label your graph clearly and name your graph.

| Variable Quantity | Lower Bound | Upper Bound | Interval |
| :---: | :---: | :---: | :---: |
| Number of shirts |  |  |  |
| Total cost |  |  |  |


12. Draw a line to model the relationship between the number of shirts and the total cost of the shirts.
13. Do all the points on the line make sense in terms of this problem situation? Why or why not?
14. Define the variables and write an algebraic equation for the problem situation.

15. Define the domain and range for this problem situation.

## Talk the Talk

So far in this chapter, you have represented problem situations in four different ways: as a sentence, as a table, as a graph, and as an equation.


1. Complete the graphic organizer to explain the advantages and disadvantages of each representation.



2

## HOT SHIRTS Using Tables, Graphs, and Equations, Part 2

## Learning Goals

In this lesson, you will:

- Use different methods to represent a problem situation.
- Estimate values of expressions that involve decimals.
- Determine an initial value when given a final result.

Key Terms

- estimation
- point of intersection

You might be surprised to know that the word " $T$-shirt" wasn't really used until the 1920s. And, until the 1950s, people thought of T-shirts as underwear. Popular actors like John Wayne and James Dean surprised audiences in the mid-1950s by wearing this underwear on screen!

Since then, T-shirts have become one of the most popular items of clothing in the world.

## Problem 1 Analyzing the Competition

Previously, you explored a job at U.S. Shirts. One of U.S. Shirts' competitors, Hot Shirts, advertises that it makes custom T-shirts for $\$ 5.50$ each with a one-time setup fee of $\$ 49.95$. Your boss brings you the advertisement from Hot Shirts and asks you to figure out how the competition might affect business.

1. Describe the problem situation and how it will affect business in your own words.

2. What is the total customer cost of an order for:
a. 3 shirts from Hot Shirts?
b. 10 shirts from Hot Shirts?

c. 50 shirts from Hot Shirts?
d. 100 shirts from Hot Shirts?
3. Explain how you calculated the total customer costs.

Remember, you can use estimation to determine the approximate values before you do actual calculations to get a sense of the answer.

For example, you can estimate the difference of 125.35 and 84.95.
So, you could round 125.35 down to 125 , and round 84.95 up to 85 . Then, calculate the difference of 125 and 85 , to $125-85=40$.

You can write this as $125.35-84.95 \approx 40$.
The symbol $\approx$ means "is approximately equal to."
4. Estimate the value of each expression.
a. $748.75+60.22$
b. $345-214$
c. $45.13(20.44)$
5. Estimate the number of shirts that a customer can purchase from Hot Shirts for:
a. $\$ 50$.
b. $\$ 60$.
c. $\$ 220$.
6. Explain how you used estimation to efficiently determine the number of shirts that can be purchased.

7. Complete the table of values for the problem situation.


| Number of Shirts Ordered | Total Cost <br> (dollars) |
| :--- | :--- |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

8. Create a graph of the data from the table on the grid shown. First, choose your bounds and intervals by completing the table shown. Remember to label your graph clearly and name your graph.

| Variable Quantity | Lower Bound | Upper Bound | Interval |
| :---: | :---: | :---: | :---: |
| Number of shirts |  |  |  |
| Total cost |  |  |  |


|  |  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
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9. Define the variables and write an algebraic equation for this problem situation.


## Problem 2 Which Is the Better Buy?

You have explored the costs of ordering T-shirts from two companies, U.S. Shirts and Hot Shirts. Your boss asked you to determine which company has the better price for T-shirts in different situations.

1. Would you recommend U.S. Shirts or Hot Shirts as the better buy for an order of five or fewer T-shirts? What would each company charge for exactly five shirts? Describe how you calculated your answer.
2. For an order of 18 shirts, which company's price is the better buy? How much better is the price? Explain your reasoning.
3. For an order of 80 shirts, which company's price is better? How much better is the price? Explain your reasoning.
4. Create the graphs for the total cost for U.S. Shirts and Hot Shirts on the grid shown. First, determine the bounds and intervals for the grid by completing the table shown.

5. Estimate the number of T-shirts for which the total costs are the same. Explain how you determined the number of T-shirts.
6. For how many T-shirts is it more expensive to order from U.S. Shirts?
7. For how many T-shirts is it more expensive to order from Hot Shirts?
8. Look at your graph. Describe the graphs of the lines in your own words.

Notice that the graphs intersect at about $(14,127)$. This point of intersection indicates where the total cost for each company is the same. So, when U.S. Shirts sells 14 shirts, the total cost is $\$ 127$, and when Hot Shirts sells 14 shirts, the total cost is $\$ 127$.
9. Write a response to your boss that compares the costs of ordering from each company. Try to answer your boss's question, "Will Hot Shirts' prices affect the business at U.S. Shirts?"

Be prepared to share your solutions and methods.

- Chapter 2 Linear Functions


## WHAT, MOT LIKES? Introduction to Non-Linear Functions

## Learning Goals

In this lesson, you will:

- Define, graph, and analyze non-linear functions, including:
- absolute value
- area of a square
- volume of a cube


## Key Terms

- absolute value function
- quadratic function
- cube or cubic function

Have someone in your class think of a whole number from 1 to 20 . Ask each other student in the class to guess what the number is. Record all the guesses without revealing the mystery number.

On the graph shown, have the recorder determine each guess on the $x$-axis and plot its distance (shown on the $y$-axis) from the mystery number.

What is the mystery number? Did you graph a function?


## Problem 1 The $\mathbf{V}$



Recall that the absolute value of a number is defined as the distance from the number to zero on a number line. The symbol for absolute value is $|x|$.

1. Evaluate each expression shown.
a. $|-3|=$
b. $|11|=$
c. $\left|-5 \frac{2}{3}\right|=$
d. $|110.89|=$
2. Use the function $y=|x|$, to complete the table.


| $x$ | $y=\|x\|$ |
| :---: | :---: |
| -7 |  |
| -3 |  |
| -1 |  |
| -0.5 |  |
| 0 |  |
| 2 |  |
| 4 |  |
| 7 |  |

3. Graph the values from the table on the coordinate plane.

4. Connect the points to model the relationship of the equation $y=|x|$.
5. What is the domain of this function? Do all the points on the graph make sense in terms of the equation $y=|x|$. Explain your reasoning.
6. Does the graph of these points form a straight line? Explain your reasoning.

7. What is the minimum, or least value of $y$ ? How do you know? State the range of this function.
8. Is this a linear function? Explain your reasoning.

You have just graphed an absolute value function. An absolute value function is a function that can be written in the form $f(x)=|x|$, where $x$ is any number. Function notation can be used to write functions such that the dependent variable is replaced with the name of the function, such as $f(x)$.

## Problem 2 Not V but U


b. 5 feet.
c. 2.4 centimeters.
d. $12 \frac{5}{8}$ inches.

In the equation $A=s^{2}$ the side length of a square, $s$, is the independent variable and the area of a square, $A$, is the dependent variable. This formula can also be modeled by the equation $y=x^{2}$, where $x$ represents the side length of a square and $y$ represents the area of a square.
2. Use the equation, $y=x^{2}$, to complete the table.

| $x$ | $y=x^{2}$ |
| :---: | :---: |
| -3 |  |
| -2 |  |
| -1 |  |
| -0.5 |  |
| 0 |  |
| 2 |  |
| 2.3 |  |
| 3 |  |

3. Graph the values from the table on the coordinate plane.

4. Connect the points to model the relationship of the equation $y=x^{2}$.
5. What is the domain of this function? Do all the points on the graph make sense in terms of the equation $y=x^{2}$. Explain your reasoning.
6. What is the minimum, or least value of $y$ ? How do you know? State the range of this function.
7. Does the graph of these points form a straight line? Explain your reasoning.

8. Is this a linear function? Explain your reasoning.

You have just graphed a quadratic function. A quadratic function is a function that can be written in the form $f(x)=a x^{2}+b x+c$, where $a, b$, and $c$ are any numbers and $a$ is not equal to zero.

## Problem 3 Not V or U

Recall that the volume of a cube is defined as the product of the length of one edge times itself 3 times and is written as $V=s^{3}$.

1. Calculate the volume of cubes with an edge length that is:
a. 2 inches.
b. 1.5 feet.
c. 2.1 centimeters.
d. $1 \frac{3}{4}$ inches.

In the equation $V=s^{3}$, the side length of a cube, $s$, is the independent variable and the volume of the cube, $V$ is the dependent variable. This formula can also be modeled by the equation $y=x^{3}$, where $x$ represents the side length of a cube and $y$ represents the volume of a cube.
2. Use the equation, $y=x^{3}$, to complete the table.

| $x$ | $y=x^{3}$ |
| :---: | :---: |
| -2 |  |
| -1.5 |  |
| -1 |  |
| -0.5 |  |
| 0 |  |
| 1.5 |  |
| 2 |  |
| 2.1 |  |


3. Graph the values from the table on the coordinate plane.

4. Connect the points to model the relationship of the equation $y=x^{3}$.
5. What is the domain of this function? Do all the points on the graph make sense in terms of the equation $y=x^{3}$. Explain your reasoning.
6. What is the minimum value of $y$ ? How do you know? State the range of this function.
7. Does the graph of these points form a straight line? Explain your reasoning.

8. Is this a linear function? Explain your reasoning.

You have just graphed a cubic function. A cubic function is a function that can be written in the form $f(x)=a_{3} x^{3}+a_{2} x^{2}+a_{1} x+a_{0}$.

## Talk the Talk



You have just completed tables of values and graphs for three different non-linear functions.

Name each equation and explain how it represents a function.

- $y=|x|$
- $y=x^{2}$
- $y=x^{3}$

Be prepared to share your solutions and methods.

## Chapter 2 Summary

## Key Terms

| - sequence (2.1) | $\checkmark$ set (2.3) | - estimation (2.6) |
| :---: | :---: | :---: |
| - term (2.1) | - relation (2.3) | - point of |
| - ellipsis (2.1) | - input (2.3) | intersection (2.6) |
| - discrete graph (2.2) | - output (2.3) | - absolute value |
| - continuous | - function (2.3) | function (2.7) |
| graph (2.2) | - domain (2.3) | $\checkmark$ quadratic function (2.7) |
| - linear graph (2.2) | - range (2.3) | - cube or cubic |
| - collinear points (2.2) | - scatter plot (2.3) | function (2.7) |
| - non-linear graph (2.2) | - vertical line test (2.3) |  |
| - mapping (2.3) | - linear function (2.4) |  |

## 2.1 <br> Writing Sequences of Numbers Generated from the Creation of Diagrams and Written Contexts

A sequence is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects. A term in a sequence is an individual number, figure, or letter in the sequence. Often, a diagram can be used to show how each term changes as the sequence progresses.

## Example

The first three terms in this sequence show how many total squares are in each set of steps as new steps are added.


If two more figures were drawn, the sequence would be $1,3,6,10,15$, with each term indicating the total number of squares in each figure.

### 2.1 Stating Varying Growth Patterns of Sequences

There are many different patterns that can generate a sequence of numbers.
Some possible patterns are:

- adding or subtracting by the same number each time,
- multiplying or dividing by the same number each time,
- adding by a different number each time, with the numbers being part of a pattern,
- alternating between adding and subtracting.


## Example

Fletcher is starting a new job delivering the newspaper. He will make $\$ 20$ every Friday and will put $\$ 8$ into his savings account every Monday morning. You can write a sequence for the amount of money that Fletcher earns and how much he has left after putting money in his savings.

The sequence is $\$ 0, \$ 20, \$ 12, \$ 32, \$ 24, \$ 44, \$ 36, \$ 56, \$ 48, \$ 68$. The pattern in the sequence is add 20 , then subtract 8 .


## Describing Characteristics of Graphs Using Mathematical

 TerminologyA discrete graph is a graph that consists of isolated points. A continuous graph is a graph with no breaks in it. A linear graph is a graph that is a line or a series of collinear points. A non-linear graph is a graph that is not a line and not a series of collinear points.

## Example

Two different graphs are shown.


The graph is discrete, non-linear, and neither increasing nor decreasing.


The graph is continuous, linear, and increasing.

## Example

The graph shown describes a situation that could represent student math scores throughout the school year.


The student's math score from Week 4 was 55.
The student's math score went up 30 points from Week 6 to Week 12.
The score went up possibly because the student studied harder or found a helpful tutor.
The score dropped in Week 17, possibly because the student didn't study that week.
It makes sense not to connect the points because scores are discrete values, they do not continually change by the second.

### 2.3 Determining Whether a Relation Is a Function

A relation is any set of ordered pairs or the mapping between a set of inputs and a set of outputs. The first coordinate of an ordered pair in a relation is the input, and the second coordinate is the output. A function maps each input to one and only one output.
Relations that are not functions will have more than one output for each input.

## Example

| Input | Output |
| :---: | :---: |
| -10 | 20 |
| -5 | 10 |
| 0 | 0 |
| 5 | 10 |
| 10 | 20 |

The ordered pairs are $(-10,20),(-5,10)$, $(0,0),(5,10)$, and $(10,20)$. Even though there are outputs, or $y$-values, with more than one input, there are no inputs, or $x$-values, with more than one output. So, this relation is a function.

## 2.3 <br> Determining Whether a Graph or Scatter Plot Is a Function

A scatter plot is a graph of a collection of ordered pairs that allows an exploration of the relationship between the points. The vertical line test is a visual method of determining whether a relation represented on a coordinate plane is a function. To apply the vertical line test, consider all of the vertical lines that could be drawn on the graph of a relation. If any of the vertical lines intersects the graph of the relation at more than one point, then the relation is not a function.

## Example



The vertical line test intersects the graph at two points, $(4,1)$ and $(4,6)$. So, the relation is not a function.

### 2.3 Determining Whether an Equation Is a Function

To test if an equation is a function, first substitute values for $x$ into the equation, and then determine if any $x$-value can be mapped to more than one $y$-value. If each $x$-value has exactly one $y$-value, then it is a function. Otherwise, it is not a function.

## Example

$y=5 x+12$ is a function because no $x$-value can be mapped to more than one $y$-value. Some examples of ordered pairs are $(0,12),(1,17),(2,22)$, and $(3,27)$.

### 2.3 Determining Whether a Context Describes a Function

If each input in a context has exactly one output, then it is a function. Otherwise it is not a function.

## Example

Input: A garden nursery sends a catalog to each customer on its preferred customer list.
Output: Each preferred customer gets one catalog.
The relation is a function because each customer gets one catalog.

### 2.4 Graphing Linear Functions

When graphing the input and output values of some functions, the graph forms a straight line. Such functions are called linear functions.

## Example

Tristan is filling his swimming pool with water. The water depth increases by 4 inches every hour. The input-output table displays the water depth in the pool for the first 8 hours.

| Time <br> (hours) | Water Depth <br> (inches) |
| :---: | :---: |
| 0 | 0 |
| 1 | 4 |
| 2 | 8 |
| 3 | 12 |
| 4 | 16 |
| 5 | 20 |
| 7 | 28 |
| 8 | 32 |

The equation $d=4 t$ can be used to model the water depth, $d$, after $t$ hours.

## Water Depth in Tristan's Pool



The graph displays the water depth for the first 8 hours. The variable, $d$, is the dependent variable, because the depth of the water in the pool depends on the amount of time which has passed. The variable, $t$, is the independent variable.

## 2.5

## Using Tables, Graphs, and Equations

Tables, graphs, and equations can provide different representations of the same problem situation. In a problem, when one variable depends on another variable it is called the dependent variable. The other variable is called the independent variable because it does not depend on the dependent variable.

## Example

Chanise wants to record some CDs of her music. CD Cutz studio charges a start-up fee of $\$ 50$ and then $\$ 3$ for every CD produced.

The cost of producing 30 CDs is $30(3)+50=90+50=\$ 140$.
The cost of producing 40 CDs is $40(3)+50=120+50=\$ 170$.
The cost of producing 50 CDs is $50(3)+50=150+50=\$ 200$.
If Chanise has $\$ 250$ to spend on CDs, the number of CDs she can produce can be determined by working backwards.

$$
\text { Number of CDs: } \frac{250-50}{3}=\frac{200}{3} \approx 66.67
$$

Chanise can produce 66 CDs with $\$ 250$.
The table of values shown represents the problem situation.

| Number of CDs Produced | Total Cost <br> (dollars) |
| :---: | :---: |
| 30 | 140 |
| 40 | 170 |
| 50 | 200 |
| 60 | 230 |
| 80 | 290 |
| 100 | 350 |

Letters can represent the variables in the problem and determine which variable is the independent variable and which is the dependent variable.

The variable $x$ represents the number of CDs produced. The variable $C$ represents the total cost to produce the CDs in dollars. The variable $C$ is the dependent variable because its value depends on the number of CDs produced, $x$. The variable $x$ is the independent variable.

A graph can be created based on the data in your table.


The equation $C=3 x+50$ represents the problem situation. In the equation, $C$ represents the total cost, in dollars, of producing $x$ CDs.

Estimating Values of Expressions that Involve Decimals
When an exact value of an expression is not needed, estimation can be used to determine an approximate value. One way to estimate is to use rounding.

## Example

The estimated value of the expression is shown.

$$
\begin{gathered}
83.90-48.05+14.22 \\
84-48+14=50 \\
83.90-48.05+14.22 \approx 50
\end{gathered}
$$

## 2.6

## Using the Point of Intersection to Compare Two Models

In a graph of two lines, the point of intersection is the point at which the two lines cross. When graphing two cost models, the point of intersection is the point at which the two costs are equal.

## Example

Chanise is trying to decide where to produce her newest music CD. The cost, in dollars, of producing CDs at CD Cutz is represented by the equation $C=3 x+50$. The cost, in dollars, of producing CDs at The CD Barn is represented by the equation $C=2 x+100$. In each equation, $C$ represents the total cost of producing $x$ CDs. The graph shows the total cost of producing CDs at each studio.


According to the graph, the cost of producing CDs at each studio is the same when Chanise produces 50 CDs. Each studio would charge $\$ 200$ to produce 50 CDs. CD Cutz costs less than The CD Barn when Chanise produces fewer than 50 CDs. The CD Barn costs less than CD Cutz when Chanise produces more than 50 CDs.

## 2.7

## Graphing Non-Linear Functions

There are many different types of non-linear functions. Several common non-linear functions are absolute value functions, square or quadratic functions, and cube or cubic functions. Each of these types of functions has a very distinctive shape.

## Examples


absolute value function

quadratic function

cubic function

2


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