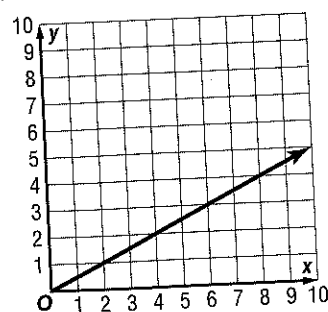**51.** Patricia pays \$1.19 each to download songs to her digital media player. If n is the number of downloaded songs, which equation represents the cost C in dollars?

A $C = 1.19n$

B $n = 1.19C$

C $C = 1.19 \div n$

D $C = n + 1.19$

52. Suppose that y varies directly as x, and $y = 8$ when $x = 6$. What is the value of y when $x = 8$?

F 6

G 12

H $10\frac{2}{3}$

J 16

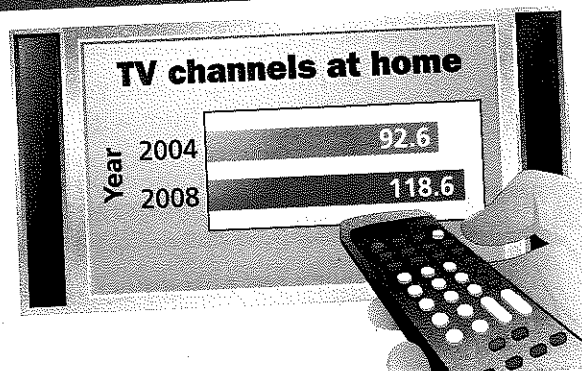**53.** What is the relationship between the input (x) and output (y)?

A The output is two more than the input.

B The output is two less than the input.

C The output is twice the input.

D The output is half the input.

54. SHORT RESPONSE A telephone company charges \$40 per month plus \$0.07 per minute. How much would a month of service cost a customer if the customer talked for 200 minutes?

55. TELEVISION The graph shows the average number of television channels American households receive. What was the annual rate of change from 2004 to 2008? Explain the meaning of the rate of change. (Lesson 3-3)

TV channels at home

Year	
2004	92.6
2008	118.6

Solve each equation. (Lesson 3-2)

56. $0 = 18 - 9x$

57. $2x + 14 = 0$

58. $-4x + 16 = 0$

59. $-5x - 20 = 0$

60. $8x - 24 = 0$

61. $12x - 144 = 0$

Evaluate each expression if $a = 4$, $b = -2$, and $c = -4$. (Lesson 2-5)

62. $|2a + c| + 1$

63. $4a - |3b + 2|$

64. $-|a + 1| + |3c|$

65. $-a + |2 - a|$

66. $|c - 2b| - 3$

67. $-2|3b - 8|$

Find each difference.

68. $13 - (-1)$

69. $4 - 16$

70. $-3 - 3$

71. $-8 - (-2)$

72. $16 - (-10)$

73. $-8 - 4$

Arithmetic Sequences as Linear Functions

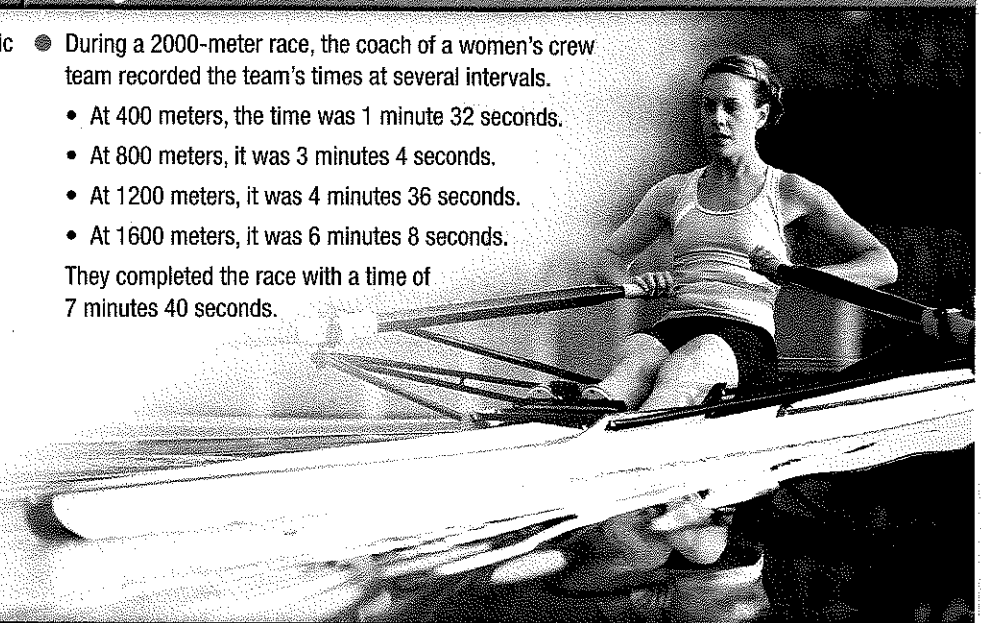| ::Then	::Now	::Why?
● You indentified linear functions.	● **1** Recognize arithmetic sequences. **2** Relate arithmetic sequences to linear functions.	● During a 2000-meter race, the coach of a women's crew team recorded the team's times at several intervals. • At 400 meters, the time was 1 minute 32 seconds. • At 800 meters, it was 3 minutes 4 seconds. • At 1200 meters, it was 4 minutes 36 seconds. • At 1600 meters, it was 6 minutes 8 seconds. They completed the race with a time of 7 minutes 40 seconds.

NewVocabulary
sequence
terms of the sequence
arithmetic sequence
common difference

Common Core State Standards

Content Standards
F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Mathematical Practices
8 Look for and express regularity in repeated reasoning.

1 Recognize Arithmetic Sequences You can relate the pattern of team times to linear functions. A **sequence** is a set of numbers, called the **terms of the sequence**, in a specific order. Look for a pattern in the information given for the women's crew team. Make a table to analyze the data.

Distance (m)	400	800	1200	1600	2000
Time (min : sec)	1:32	3:04	4:36	6:08	7:40

$+1:32 \quad +1:32 \quad +1:32 \quad +1:32$

As the distance increases in regular intervals, the time increases by 1 minute 32 seconds. Since the difference between successive terms is constant, this is an **arithmetic sequence**. The difference between the terms is called the **common difference** d.

KeyConcept Arithmetic Sequence

Words	An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate called the *common difference*.
Examples	$3, \quad 5, \quad 7, \quad 9, \quad 11, \ldots \qquad 33, \quad 29, \quad 25, \quad 21, \quad 17, \ldots$ $\quad +2 \; +2 \; +2 \; +2 \qquad\qquad -4 \; -4 \; -4 \; -4$ $\qquad\qquad d = 2 \qquad\qquad\qquad\qquad\qquad d = -4$

The three dots used with sequences are called an *ellipsis*. The ellipsis indicates that there are more terms in the sequence that are not listed.

Example 1 Identify Arithmetic Sequences

Determine whether each sequence is an arithmetic sequence. Explain.

a. $-4, -2, 0, 2, \ldots$

$$-4 \quad -2 \quad 0 \quad 2$$
$$+2 \quad +2 \quad +2$$

The difference between terms in the
sequence is constant. Therefore, this
sequence is arithmetic.

b. $\frac{1}{2}, \frac{5}{8}, \frac{3}{4}, \frac{13}{16}, \ldots$

$$\frac{1}{2} \quad \frac{5}{8} \quad \frac{3}{4} \quad \frac{13}{16}$$
$$+\frac{1}{8} \quad +\frac{1}{8} \quad +\frac{1}{16}$$

This is not an arithmetic sequence.
The difference between terms is
not constant.

▶ **Guided**Practice

1A. $-26, -22, -18, -14, \ldots$

1B. $1, 4, 9, 25, \ldots$

You can use the common difference of an arithmetic sequence to find the next term.

Example 2 Find the Next Term

Find the next three terms of the arithmetic sequence 15, 9, 3, −3, … .

Step 1 Find the common difference by
subtracting successive terms.

$$15 \quad 9 \quad 3 \quad -3$$
$$-6 \quad -6 \quad -6$$

The common
difference is −6.

Step 2 Add −6 to the last term of the
sequence to get the next term.

$$-3 \quad -9 \quad -15 \quad -21$$
$$-6 \quad -6 \quad -6$$

The next three terms in the sequence are −9, −15, and −21.

▶ **Guided**Practice

2. Find the next four terms of the arithmetic sequence 9.5, 11.0, 12.5, 14.0, … .

Each term in an arithmetic sequence can be expressed in terms of the first term a_1 and the common difference d.

Term	Symbol	In Terms of a_1 and d	Numbers
first term	a_1	a_1	8
second term	a_2	$a_1 + d$	$8 + 1(3) = 11$
third term	a_3	$a_1 + 2d$	$8 + 2(3) = 14$
fourth term	a_4	$a_1 + 3d$	$8 + 3(3) = 17$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
nth term	a_n	$a_1 + (n-1)d$	$8 + (n-1)(3)$

KeyConcept nth Term of an Arithmetic Sequence

The nth term of an arithmetic sequence with first term a_1 and common difference d is given by
$a_n = a_1 + (n-1)d$, where n is a positive integer.

Example 3 Find the nth Term

a. Write an equation for the nth term of the arithmetic sequence $-12, -8, -4, 0, \ldots$.

Step 1 Find the common difference.

$$-12 \quad -8 \quad -4 \quad 0$$
$$+4 \quad +4 \quad +4$$

The common difference is 4.

Step 2 Write an equation.

$$a_n = a_1 + (n-1)d \qquad \text{Formula for the } n\text{th term}$$
$$= -12 + (n-1)4 \qquad a_1 = -12 \text{ and } d = 4$$
$$= -12 + 4n - 4 \qquad \text{Distributive Property}$$
$$= 4n - 16 \qquad \text{Simplify.}$$

b. Find the 9th term of the sequence.

Substitute 9 for n in the formula for the nth term.

$$a_n = 4n - 16 \qquad \text{Formula for the } n\text{th term}$$
$$a_9 = 4(9) - 16 \qquad n = 9$$
$$a_9 = 36 - 16 \qquad \text{Multiply.}$$
$$a_9 = 20 \qquad \text{Simplify.}$$

c. Graph the first five terms of the sequence.

n	$4n - 16$	a_n	(n, a_n)
1	$4(1) - 16$	-12	$(1, -12)$
2	$4(2) - 16$	-8	$(2, -8)$
3	$4(3) - 16$	-4	$(3, -4)$
4	$4(4) - 16$	0	$(4, 0)$
5	$4(5) - 16$	4	$(5, 4)$

d. Which term of the sequence is 32?

In the formula for the nth term, substitute 32 for a_n.

$$a_n = 4n - 16 \qquad \text{Formula for the } n\text{th term}$$
$$32 = 4n - 16 \qquad a_n = 32$$
$$32 + 16 = 4n - 16 + 16 \qquad \text{Add 16 to each side.}$$
$$48 = 4n \qquad \text{Simplify.}$$
$$12 = n \qquad \text{Divide each side by 4.}$$

▶ **Guided**Practice

Consider the arithmetic sequence $3, -10, -23, -36, \ldots$.

3A. Write an equation for the nth term of the sequence.

3B. Find the 15th term in the sequence.

3C. Graph the first five terms of the sequence.

3D. Which term of the sequence is −114?

2 Arithmetic Sequences and Functions

As you can see from Example 3, the graph of the first five terms of the arithmetic sequence lie on a line. An arithmetic sequence is a linear function in which n is the independent variable, a_n is the dependent variable, and d is the slope. The formula can be rewritten as the function $f(n) = (n - 1)d + a_1$, where n is a counting number.

While the domain of most linear functions are all real numbers, in Example 3 the domain of the function is the set of counting numbers and the range of the function is the set of integers on the line.

Real-World Example 4 Arithmetic Sequences as Functions

INVITATIONS Marisol is mailing invitations to her quinceañera. The arithmetic sequence $0.42, $0.84, $1.26, $1.68, ... represents the cost of postage.

a. Write a function to represent this sequence.

The first term, a_1, is 0.42. Find the common difference.

0.42 0.84 1.26 1.68
   +0.42 +0.42 +0.42

The common difference is 0.42.

$$a_n = a_1 + (n - 1)d \qquad \text{Formula for the } n\text{th term}$$
$$= 0.42 + (n - 1)0.42 \qquad a_1 = 0.42 \text{ and } d = 0.42$$
$$= 0.42 + 0.42n - 0.42 \qquad \text{Distributive Property}$$
$$= 0.42n \qquad \text{Simplify.}$$

The function is $f(n) = 0.42n$.

b. Graph the function and determine the domain.

The rate of change of the function is 0.42. Make a table and plot points.

n	$f(n)$
1	0.42
2	0.84
3	1.26
4	1.68
5	2.10

The domain of a function is the number of invitations Marisol mails. So, the domain is {0, 1, 2, 3, ...}.

GuidedPractice

4. TRACK The chart below shows the length of Martin's long jumps.

Jump	1	2	3	4
Length (ft)	8	9.5	11	12.5

A. Write a function to represent this arithmetic sequence.

B. Then graph the function.

Check Your Understanding

Example 1 Determine whether each sequence is an arithmetic sequence. Write *yes* or *no*. Explain.

1. 18, 16, 15, 13, ...

2. 4, 9, 14, 19, ...

Example 2 Find the next three terms of each arithmetic sequence.

3. 12, 9, 6, 3, ...

4. −2, 2, 6, 10, ...

Example 3 Write an equation for the nth term of each arithmetic sequence. Then graph the first five terms of the sequence.

5. 15, 13, 11, 9, ...

6. −1, −0.5, 0, 0.5, ...

Example 4 7. **SAVINGS** Kaia has $525 in a savings account. After one month she has $580 in the account. The next month the balance is $635. The balance after the third month is $690. Write a function to represent the arithmetic sequence. Then graph the function.

Practice and Problem Solving

Example 1 Determine whether each sequence is an arithmetic sequence. Write *yes* or *no*. Explain.

8. −3, 1, 5, 9, ...

9. $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{7}{16}, ...$

10. −10, −7, −4, 1, ...

11. −12.3, −9.7, −7.1, −4.5, ...

Example 2 Find the next three terms of each arithmetic sequence.

12. 0.02, 1.08, 2.14, 3.2, ...

13. 6, 12, 18, 24, ...

14. 21, 19, 17, 15, ...

(15) $-\frac{1}{2}, 0, \frac{1}{2}, 1, ...$

16. $2\frac{1}{3}, 2\frac{2}{3}, 3, 3\frac{1}{3}, ...$

17. $\frac{7}{12}, 1\frac{1}{3}, 2\frac{1}{12}, 2\frac{5}{6}, ...$

Example 3 Write an equation for the nth term of the arithmetic sequence. Then graph the first five terms in the sequence.

18. −3, −8, −13, −18, ...

19. −2, 3, 8, 13, ...

20. −11, −15, −19, −23, ...

21. −0.75, −0.5, −0.25, 0, ...

Example 4 22. **AMUSEMENT PARKS** Shiloh and her friends spent the day at an amusement park. In the first hour, they rode two rides. After 2 hours, they had ridden 4 rides. They had ridden 6 rides after 3 hours.

a. Write a function to represent the arithmetic sequence.

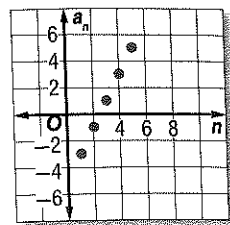b. Graph the function and determine the domain.

23. **CCSS MODELING** The table shows how Ryan is paid at his lumber yard job.

Linear Feet of 2×4 Planks Cut	10	20	30	40	50	60	70
Amount Paid in Commission ($)	8	16	24	32	40	48	56

a. Write a function to represent Ryan's commission.

b. Graph the function and determine the domain.

24. The graph is a representation of an arithmetic sequence.

  a. List the first five terms.

  b. Write the formula for the nth term.

  c. Write the function.

25. NEWSPAPERS A local newspaper charges by the number of words for advertising. Write a function to represent the advertising costs.

DAILY NEWS ADVERTISING	
10 words $7.50	20 words $10.00
15 words $8.75	25 words $11.25

26. The fourth term of an arithmetic sequence is 8. If the common difference is 2, what is the first term?

27. The common difference of an arithmetic sequence is -5. If a_{12} is 22, what is a_1?

28. The first four terms of an arithmetic sequence are 28, 20, 12, and 4. Which term of the sequence is -36?

29. CARS Jamal's odometer of his car reads 24,521. If Jamal drives 45 miles every day, what will the odometer reading be in 25 days?

30. YEARBOOKS The yearbook staff is unpacking a box of school yearbooks. The arithmetic sequence 281, 270, 259, 248 ... represents the total number of ounces that the box weighs as each yearbook is taken out of the box.

  a. Write a function to represent this sequence.

  b. Determine the weight of each yearbook.

  c. If the box weighs at least 11 ounces empty and 292 ounces when it is full, how many yearbooks were in the box?

31. SPORTS To train for an upcoming marathon, Olivia plans to run 3 miles per day for the first week and then increase the daily distance by a half mile each of the following weeks.

  a. Write an equation to represent the nth term of the sequence.

  b. If the pattern continues, during which week will she run 10 miles per day?

  c. Is it reasonable to think that this pattern will continue indefinitely? Explain.

H.O.T. Problems   Use Higher-Order Thinking Skills

32. OPEN ENDED Create an arithmetic sequence with a common difference of -10.

33. CCSS PERSEVERANCE Find the value of x that makes $x + 8$, $4x + 6$, and $3x$ the first three terms of an arithmetic sequence.

34. REASONING Compare and contrast the domain and range of the linear functions described by $Ax + By = C$ and $a_n = a_1 + (n - 1)d$.

35. CHALLENGE Determine whether each sequence is an arithmetic sequence. Write yes or no. Explain. If yes, find the common difference and the next three terms.

  a. $2x + 1, 3x + 1, 4x + 1...$

  b. $2x, 4x, 8x, ...$

36. WRITING IN MATH How are graphs of arithmetic sequences and linear functions similar? different?

Standardized Test Practice

37. GRIDDED RESPONSE The population of Westerville is about 35,000. Each year the population increases by about 400. This can be represented by the following equation, where n represents the number of years from now and p represents the population.

$$p = 35,000 + 400n$$

In how many years will the Westerville population be about 38,200?

38. Which relation is a function?

  A $\{(-5, 6), (4, -3), (2, -1), (4, 2)\}$

  B $\{(3, -1), (3, -5), (3, 4), (3, 6)\}$

  C $\{(-2, 3), (0, 3), (-2, -1), (-1, 2)\}$

  D $\{(-5, 6), (4, -3), (2, -1), (0, 2)\}$

39. Find the formula for the nth term of the arithmetic sequence.

$$-7, -4, -1, 2, ...$$

  F $a_n = 3n - 4$

  G $a_n = -7n + 10$

  H $a_n = 3n - 10$

  J $a_n = -7n + 4$

40. STATISTICS A class received the following scores on the ACT. What is the difference between the median and the mode in the scores?
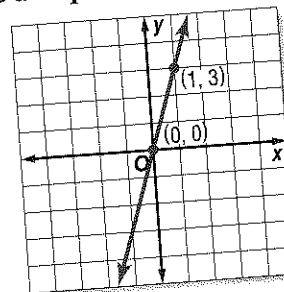
18, 26, 20, 30, 25, 21, 32, 19, 22, 29, 29, 27, 24

  A 1   **C** 3
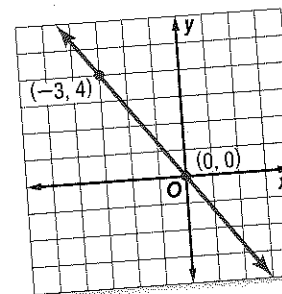
  B 2   **D** 4

Spiral Review

Name the constant of variation for each direct variation. Then find the slope of the line that passes through each pair of points. (Lesson 3-4)

41.

42.

Find the slope of the line that passes through each pair of points. (Lesson 3-3)

43. $(5, 3), (-2, 6)$

44. $(9, 2), (-3, -1)$

45. $(2, 8), (-2, -4)$

Solve each equation. Check your solution. (Lesson 2-3)
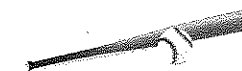
46. $5x + 7 = -8$

47. $8 = 2 + 3n$

48. $12 = \dfrac{c - 6}{2}$

49. SPORTS The most popular sports for high school girls are basketball and softball. Write and use an equation to find how many more girls play on basketball teams than on softball teams. (Lesson 2-1)

Basketball
453,000 girls

Softball
369,000 girls

Skills Review

Graph each point on the same coordinate plane.

50. $A(2, 5)$

51. $B(-2, 1)$

52. $C(-3, -1)$

53. $D(0, 4)$

54. $F(5, -3)$

55. $G(-5, 0)$