3

ANALYZING LINEAR EQUATIONS

3

3.1 Hitting the Slopes

Determining Rate of Change from a Graph

Learning Goals

In this lesson, you will:

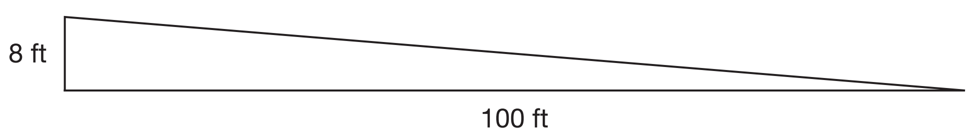
- Determine the rate of change from a graph.
- Create a scenario, a table, and an equation from a graph.
- Connect the rate of change represented in a graph to the rate of change in other representations.
- Use $\frac{\text{rise}}{\text{run}}$ to calculate the rate of change from a graph.
- Determine if a graph has a rate of change that is increasing, decreasing, zero, or undefined.
- Compare unit rates of change in the same graph.

Key Terms

- rate
- rate of change
- per
- unit rate
- rise
- run
- $\frac{\text{rise}}{\text{run}}$

You may have seen road grade signs before. These signs often show a car going down a road at an angle. Below this picture is a percent. What does this percent mean?

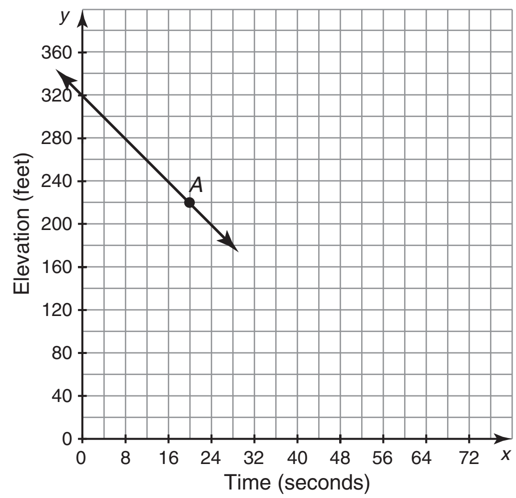
Well, if you see a sign that reads "8%," that means that the road you are on is going down (or up) 8 feet for every 100 feet you drive.



How much would you go up or down if the road stayed at 8% for one mile?

Problem 1 Hit the Slopes

The linear graph shown is a model of a skier's elevation, over time, while skiing down a hill.



1. What does point A on the graph represent?

2. At what elevation did the skier start? Label the point on the graph representing your answer with the letter B.

3. Why do you think the graph extends beyond the y-axis?

4. About how many seconds would it take for the skier to reach the bottom of the hill? Explain your reasoning.

5. How many feet did the skier descend down the hill each second? Explain your reasoning.

6. Label (24, 200) with *C*. How could you use points *A* and *C* to calculate the number of feet the skier descended each second?

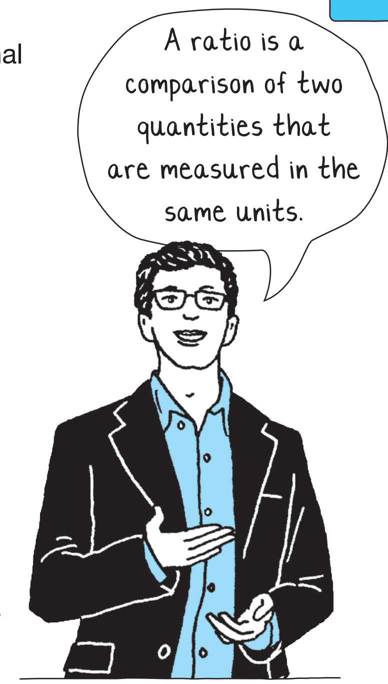
A **rate** is a ratio in which the two quantities being compared are measured in different units. Rates are commonly written in fractional form, with the dependent variable as the numerator, and the independent variable as the denominator.

A **rate of change** is used to describe the rate of increase or decrease.

For this problem, you can write a rate to compare the change in elevation to the change in time:

$$\frac{\text{change in elevation}}{\text{change in time}} \begin{array}{l} \leftarrow \\ \leftarrow \end{array} \frac{\text{dependent variable}}{\text{independent variable}}$$

This rate is read as "change in elevation *per* change in time." **Per** means "for each" or "for every."



7. Write a rate to compare the change in elevation to the change in time at point *A*. Describe what the rate means. Make sure to state whether the rate is a rate of increase or a rate of decrease.

8. Write a rate to compare the change in elevation to the change in time at point *C*. Describe what the rate means. Make sure to state whether the rate is a rate of increase or a rate of decrease.

A **unit rate** is a comparison of two measurements in which the denominator has a value of one unit.

9. Write the rates of change at points *A* and *C* as unit rates.

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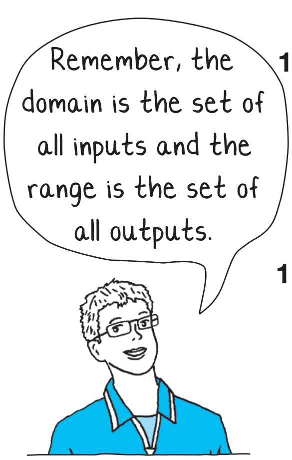
10. What do you notice about these unit rates? Explain your observation.

11. What are the independent and dependent variables in the graph?



12. What is the domain of the problem situation? Include units in your response.

13. What is the range of the problem situation? Include units in your response.

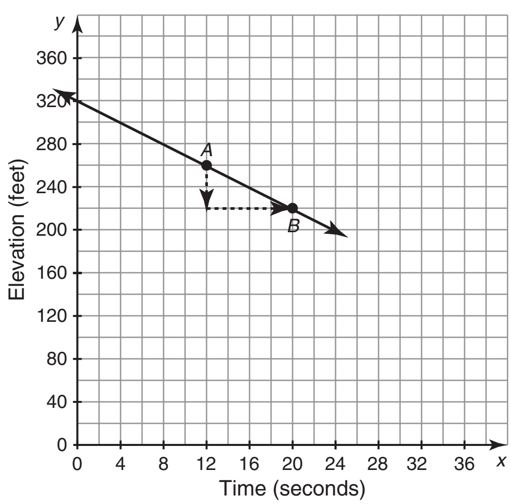
14. What is the unit rate of change modeled in the graph? Use numerical values and units. State whether it is a rate of increase or a rate of decrease.

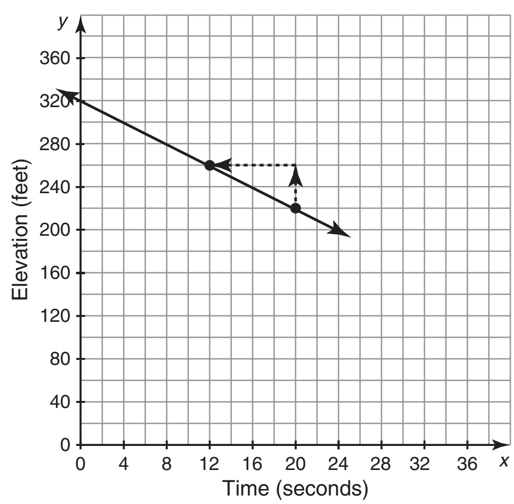
15. Write in sentence form what is happening in the problem. Include:

- the initial values of the independent and dependent variables in the context of the problem;
- a sentence explaining the rate of change in terms of the context of the problem; and
- the final values of the independent and dependent variables in the context of the problem.

You can calculate the rate of change from a graph.

Complete these steps to determine the rate of change shown in the graph.





Step 1: Choose two points on the line. Choose points with coordinate values that can be read without estimation. The two points shown are Points *A* and *B*.

Step 2: Determine the *rise*. The **rise** is the vertical change from the first point to the second point.

Use the scale when counting. If you are determining the rise from point A to point B, the rise will be a negative value. If you are determining the rise from point B to point A, the rise will have a positive value.

The rise from A to B is -40.
The rise from B to A is 40.

Step 3: Continue from the point where you left off on the graph to complete the next step. Determine the *run*. The **run** is the horizontal change from the first point to the second point.

Use the scale when counting. If you are determining the run from point A to point B, then the run will have a positive value.

If you are determining the run from point B to point A, then the run will have a negative value.

The run from A to B is 8.
The run from B to A is -8.

Step 4: Write a rate in the format $\frac{rise}{run}$.

The unit rate from point A to point B is $\frac{-40}{8} = \frac{-5}{1}$.

The unit rate from point B to point A is $\frac{40}{-8} = \frac{5}{-1}$.

Notice that the rate of change is equal no matter which point you start from.

The ratio $\frac{\textbf{rise}}{\textbf{run}}$ is a representation of the rate of change shown in the graph. The ratio is read as "rise over run."

16. Why is the rise the value for the numerator and the run the value for the denominator?

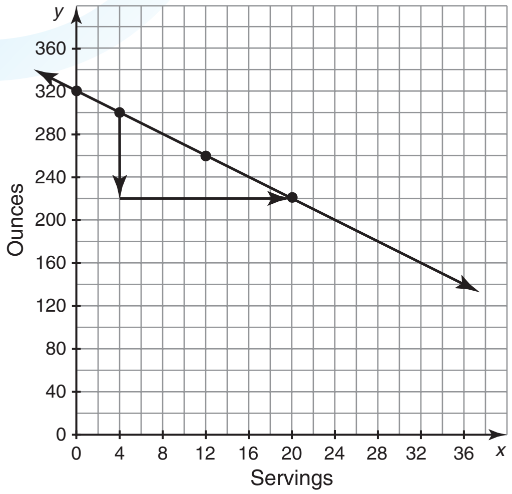
17. What does it mean if a rate of change is negative?

Problem 2 Rise and Then Run

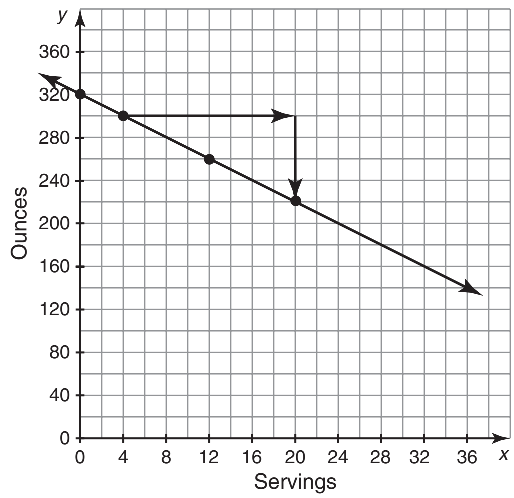
The ratio $\frac{\text{rise}}{\text{run}}$ was used either correctly or incorrectly to determine the rate of change in the following graphs shown. Each graph models the same problem.

- Follow the arrows to write the rate of change.
- Explain any errors in the process of drawing the arrows.

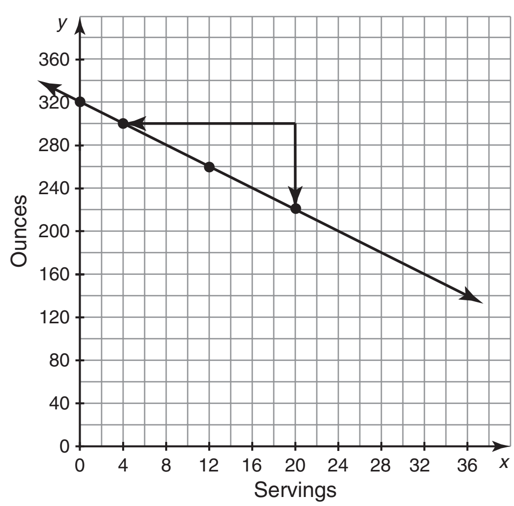
1.



2.


y
360
320
280
240
200
160
120
80
40
0
Ounces
0
4
8
12
16
20
24
28
32
36
x
Servings

3.


y
360
320
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120
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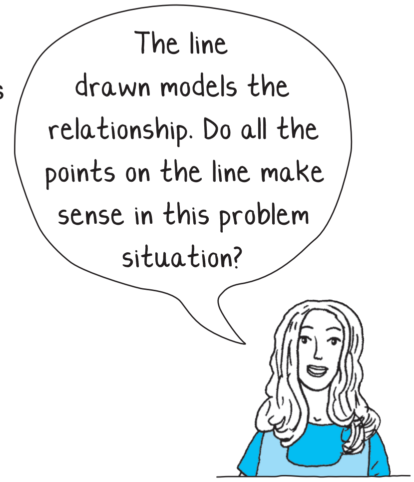
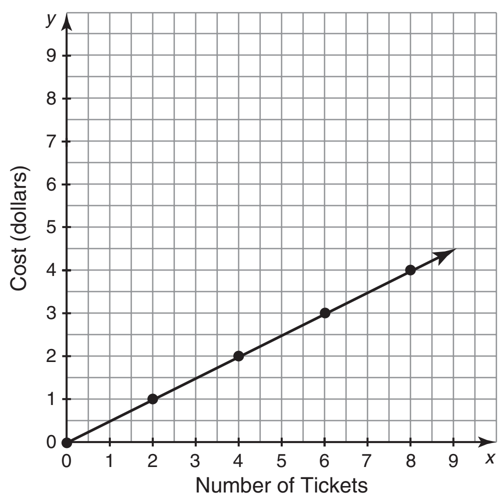
36
x
Servings

Problem 3 Selecting Coordinate Points



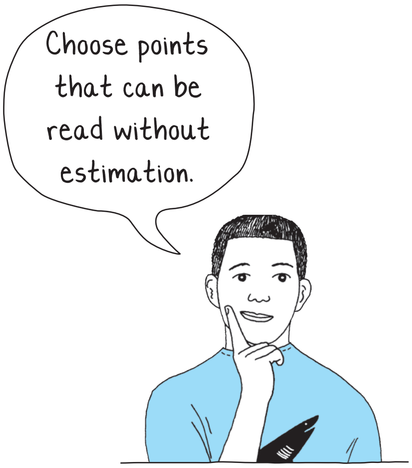
1. Shelley read the rate of change from the graph shown as $\frac{1 \text{ dollar}}{2 \text{ tickets}}$, or \$1 for every 2 tickets. She plotted points on the graph to show the values she used to determine the rate of change.

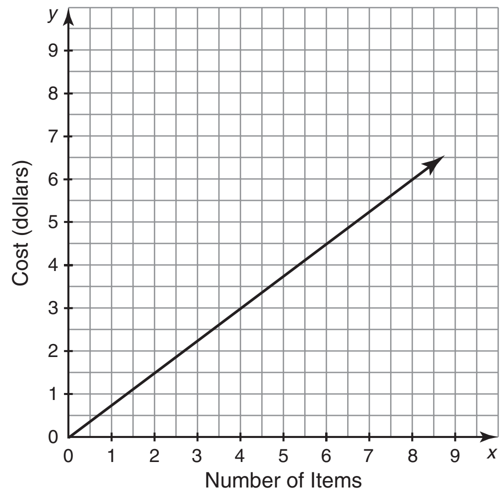


   a. Restate the rate of change as a unit rate. Explain its meaning. Show your work.

   b. Why do you think Shelley did not read the unit rate of change from the graph?

2. Determine the rate of change from the graph shown. Plot points on the graph to show the values you used to determine the rate.



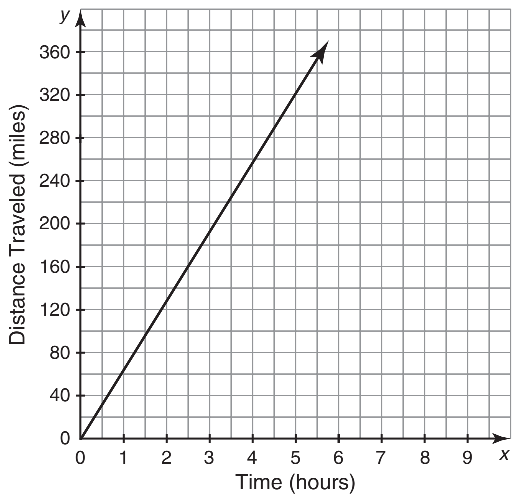


3. Restate the rate of change as a unit rate. Explain its meaning.

4. What is the cost of 10 items? Show your work.

5. Do you prefer to use the original rate of change determined from the graph, or the unit rate when making calculations? Explain your reasoning.

6. Calculate the rate of change from the graph shown. Plot points on the graph to show the values you used to determine the rate.

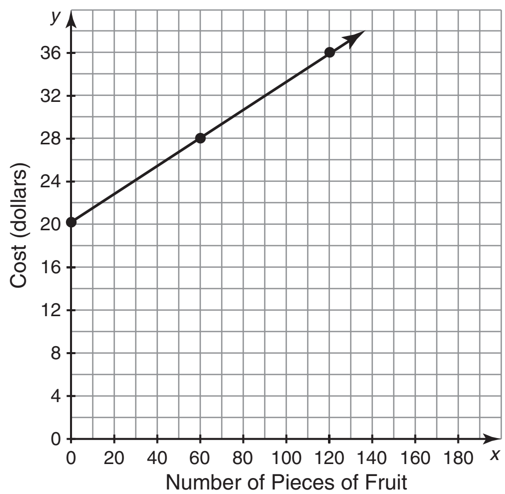


7. Restate the rate as a unit rate. Explain its meaning.

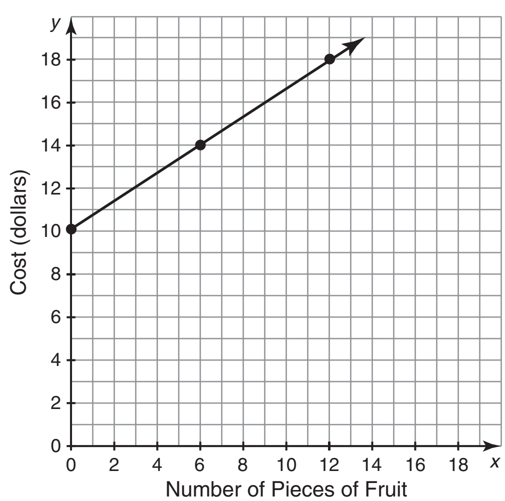
Problem 4 Investigating Rate of Change from Graphs

1. Determine the rate of change from each graph.

a.



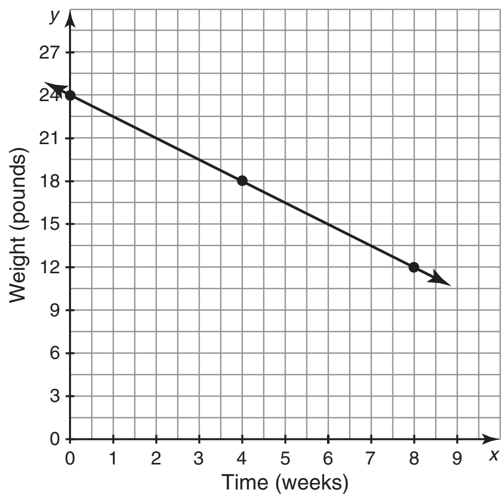
b.



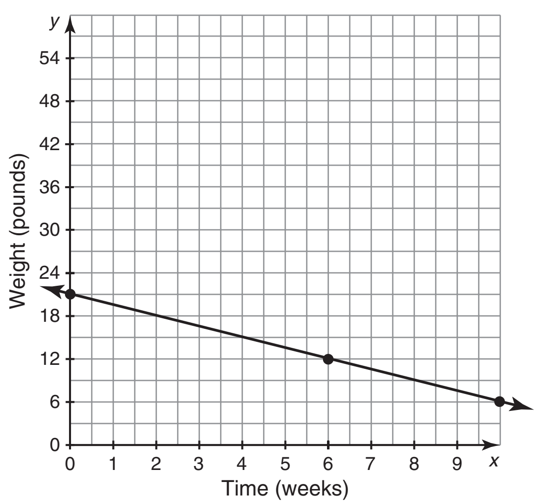
c. The two graphs look exactly alike. How could they show different rates of change?

2. Determine the rate of change from each graph.

a.



b.



c. The two graphs look different. How could they show the same rate of change?

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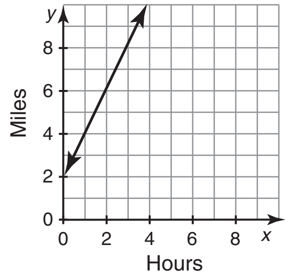
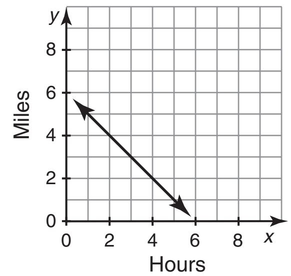
3. Determine the unit rate of change for each graph.

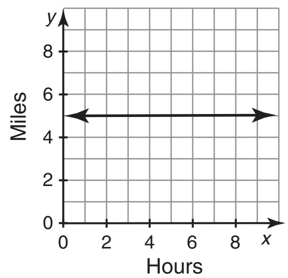
a.



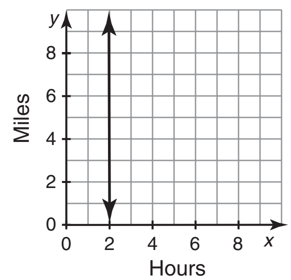
b.



c.



d.



e. How can you tell from the graph whether the rate of change will be positive or negative before determining $\frac{\text{rise}}{\text{run}}$?

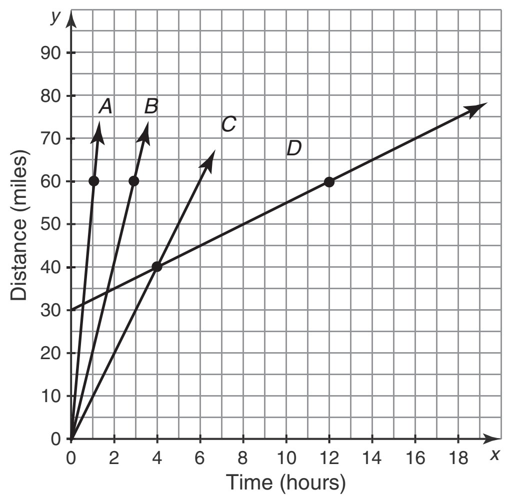
f. Describe the graph's direction if the rate of change is equal to zero.

g. Describe the graph's direction if the rate of change is undefined.

Talk the Talk

The graph shown represents the distance four cars travel over time.

1. Calculate the unit rate for each car. Show your work.



2. Describe how the steepness of the line is related to the rate of change.

Be prepared to share your solutions and methods.

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3.2 At the Arcade

Determining Rate of Change from a Table

Learning Goals

In this lesson, you will:

- Determine the rate of change from a table of values.
- Create a graph, a context, and an equation from a table of values.
- Connect the rate of change represented in a table of values to the rate of change in other representations.
- Use $\frac{y_2 - y_1}{x_2 - x_1}$ to calculate the rate of change from a table of values or two coordinate pairs.
- Determine whether a table of values will make a straight line if graphed.

Key Term

- first differences

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Some call them crane games or teddy pickers or grab machines or claw games. These machines can be found inside restaurants, arcades, supermarkets, and even movie theaters. Although there are many different kinds, these games usually involve controlling a claw to pick up a prize, like a stuffed toy, a doll, or candy.

Have you ever won a prize from one of these games?

Problem 1 At the Arcade

Ron has a player's card for the arcade at the mall. His player's card keeps track of the number of credits he earns as he wins games. Each winning game earns the same number of credits, and those credits can be redeemed for various prizes. Ron has been saving his credits to collect a prize worth 500 credits.

The table shows the number of credits Ron had on his game card at various times today when he checked his balance at the arcade.

Number of Games Ron Won Today	Number of Credits on Ron's Player's Card
0	120
12	216
18	264
25	320
40	440

1. Explain the meaning of the ordered pair (0, 120) listed in the table.

2. Write a rate to compare the change in credits earned to the change in games won. Show your work.

3. Write the rate as a unit rate and explain its meaning.

4. Recalculate the unit rate by using different values from the table. Show your work.

5. Analyze Rhonda's calculations shown.

$$\frac{440 \text{ credits}}{40 \text{ games won}} = \frac{11 \text{ credits}}{1 \text{ game won}}$$

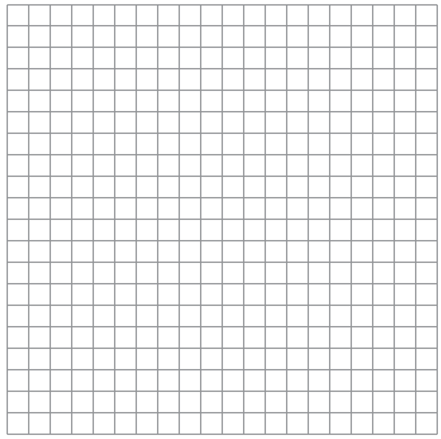
I used the last listing in the table and wrote a rate: $\frac{\text{credits}}{\text{games won}}$. Then, I divided both the first and second terms by 40 to write the rate as a unit rate. I got $\frac{11}{1}$. The unit rate is 11 credits per each game won.

Explain to Rhonda why her calculations are incorrect.

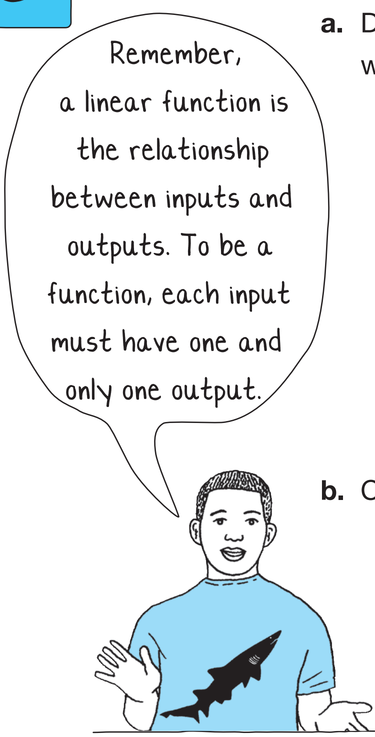
6. Before Ron started winning games today, how many games had he won for which he had saved the credits on his player's card? Show your work.

7. After Ron won his fortieth game today, how many more games does he need to win to collect a prize worth 500 credits? Show your work and explain your reasoning.

8. Create a graph to represent the information from the previous table. Include scales and labels.



a. Does the graph represent a linear function? Explain why or why not.



b. Calculate the rate of change from the graph. Show your work.

9. What is the domain of the problem situation? Include units in your response.

10. What is the range of the problem situation? Include units in your response.

11. What is the unit rate of change modeled in the graph? Use numerical values and units. State whether it is a rate of increase or a rate of decrease.

12. Write in sentence form what is happening in the problem. Include:

- the initial values of the independent and dependent variables in the context of the problem;
- a sentence explaining the rate of change in terms of the context of the problem; and
- the final values of the independent and dependent variables in the context of the problem.

Problem 2 Calculating Rate of Change from a Table

So far, you have determined the rate of change from a graph using the $\frac{\text{rise}}{\text{run}}$ method. However, you can also determine the rate of change from a table.

1. Complete the steps to determine the rate of change from a table.

Number of Games Ron Won Today	Number of Credits on Ron's Player's Card
0	120
12	216
18	264
25	320
40	440

Step 1: Choose any two values of the independent variable. Calculate their difference.

Step 2: Calculate the difference between the corresponding values of the dependent variable. It is important that the order of values you used for determining the difference of the independent variables be followed for the dependent variables.

Step 3: Write a rate to compare the change in the dependent variable to the change in the independent variable.

Step 4: Rewrite the rate as a unit rate.

2. This method was used either correctly or incorrectly to determine the rate of change in the three tables shown, each modeling the same problem.

- Follow the arrows to calculate the rate of change. Show your work.
- Explain any errors that may have occurred when the arrows were drawn.

Example 1

Number of Games Ron Won Today	Number of Credits on Ron's Player's Card
Games	Credits
0	120
12	216
18	264
25	320
40	440

Example 2

Number of Games Ron Won Today	Number of Credits on Ron's Player's Card
Games	Credits
0	120
12	216
18	264
25	320
40	440

Example 3

Number of Games Ron Won Today	Number of Credits on Ron's Player's Card
Games	Credits
0	120
12	216
18	264
25	320
40	440

3

This method of determining a rate of change is not a formal method. It can be referred to as an informal method for determining a rate of change.

There is a formal mathematical process that can be used to calculate the rate of change of a linear function from a table of values with at least two coordinate pairs.

The rate of change of the linear function can be calculated using two ordered pairs and the formula:

$$\text{rate of change of a linear function} = \frac{y_2 - y_1}{x_2 - x_1},$$

where the first point is at (x_1, y_1) and the second point is at (x_2, y_2).

For example, let's consider the table that shows the number of credits Ron had on his game card at various times when he checked his balance at the arcade.

Number of Games Ron Won Today	Number of Credits on Ron's Player's Card
0	120
12	216
18	264
25	320
40	440

Step 1: From the table of values, use (12, 216) as the first point and (25, 320) as the second point.

Step 2: Label the points with the variables.

$$\begin{array}{cc} (12,\ 216) & (25,\ 320) \\ \downarrow\ \ \downarrow & \downarrow\ \ \downarrow \\ (x_1,\ y_1) & (x_2,\ y_2) \end{array}$$

Step 3: Use the formula for the rate of change of a linear function and substitution.

By substitution: $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{320 - 216}{25 - 12}$

$$= \frac{104}{13}$$

$$= \frac{8}{1}$$

The rate of change is $\dfrac{8 \text{ credits}}{1 \text{ game}}$.

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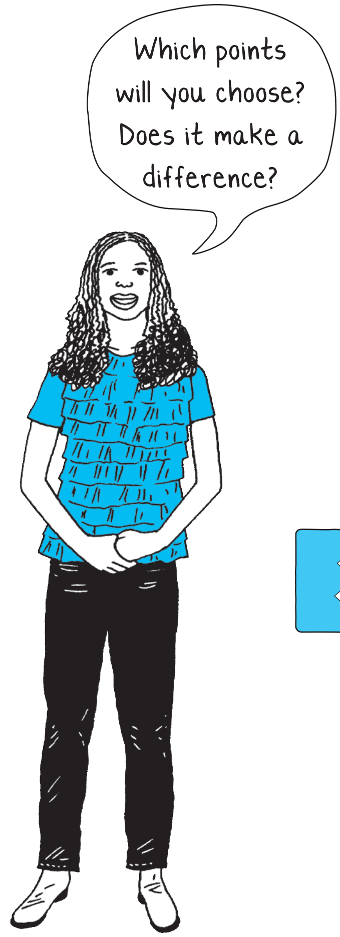
3. Repeat the process to calculate the rate of change using two different values from the table. Show all work.



4. How is using the formula for a table related to using $\frac{\text{rise}}{\text{run}}$ for a graph?

5. Calculate the unit rate of change of each linear function using the formula.
Show all work.

a.

Number of Carnival Ride Tickets	Cost (dollars)
4	9
8	12
16	18
32	30

b.

x	y
−1	13
0	−2
4	−62
10	−152

c.

Days Passed	Vitamins Remaining in Bottle
7	25
8	23
9	21
10	19

d.

x	*y*
7	9
18	9
29	9
40	9

3

6. Only two points are necessary to use the informal method or the formula to calculate the rate of change of a linear function. Given two points:

- use the informal method to determine the rate of change; and
- use the formula to determine the rate of change.

a. (10, 25) and (55, 40)

x	y
10	25
55	40

3

b. (4, 19) and (16, 5)

x	y
4	19
24	3

c. Which method do you prefer, the informal one or the formula? Explain your choice.

Problem 3 Is that Relation Linear?

If the rate of change between every pair of ordered pairs in a table of values is the same, or constant, then the ordered pairs, when plotted, will form a straight line.

To determine if a table represents a linear function, you can calculate the rate of change between every consecutive pair of ordered pairs and make sure you obtain the same value every time.

1. Calculate the rate of change between the points represented by the given ordered pairs. Show your work.

x	y
4	13
9	28
11	34
16	47

   a. (4, 13) and (9, 28)

   b. (9, 28) and (11, 34)

   c. (11, 34) and (16, 47)

   d. Will the ordered pairs listed in the table form a straight line when plotted? Explain your reasoning.

2. Determine whether the ordered pairs listed in each table will form a straight line when plotted. Show your work. Explain your reasoning.

a.

x	y
2	7
6	13
8	16
20	34

b.

x	y
1	33
2	40
3	47
4	54
5	61

3. What was different about the table in Question 2, part (b)? How did that affect your calculations?

When the values for the independent variable in a table are consecutive integers, you can examine only the column with the dependent variable and calculate the differences between consecutive values. If the differences are the same each time, then you know that the rate of change is the same each time. The ordered pairs in the table will therefore form a straight line when plotted.

The differences have been calculated for the table shown.

x	y
1	99
2	86
3	73
4	60
5	47

$86 - 99 = -13$

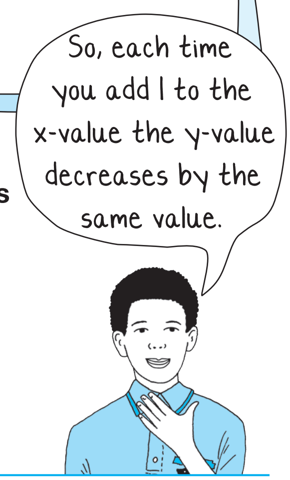
$73 - 86 = -13$

$60 - 73 = -13$

$47 - 60 = -13$

The differences between consecutive values for the dependent variable are the same each time. So, the rate of change is the same each time as well. The ordered pairs in this table will therefore form a straight line when plotted.

In this process, you are calculating *first differences*. **First differences** are the values determined by subtracting consecutive y-values in a table when the x-values are consecutive integers.



4. Determine whether the ordered pairs in each table will form straight lines when plotted. Show your work and explain your reasoning.

a.

x	y
1	25
2	34
3	45
4	52
5	61

b.

x	y
1	12
2	8
3	4
4	0
5	−4

c.

x	y
1	1
2	4
3	9
4	16
5	25

d.

x	y
1	15
2	18
3	21
4	24
5	27

Be prepared to share your solutions and methods.

3

3.3 To Put It in Context

Determining Rate of Change from a Context

Learning Goals

In this lesson, you will:

- Determine the rate of change from a context.
- Create a graph, a table, and an equation from a context.
- Connect the rate of change represented in a context to the rate of change in other representations.
- Generate the values of two coordinate pairs from information given in context.

3

Context is important. The word usually refers to all the events or thoughts surrounding what someone says or writes. When someone takes another person's words "out of context," he or she is usually quoting what the other person said without considering all the events surrounding what that person said.

Can you give some other examples of context? What other ways can people take another person's words or deeds "out of context"?

Problem 1 Soccer Tournament

The Salem Middle School soccer team travels to a tournament. They began their bus trip the evening before the tournament by traveling 210 miles and staying overnight at a hotel. The following morning, they continued their trip by traveling an additional three hours until they reached their destination 180 miles from the hotel. They arrived there in time for their tournament, which began at 11:00 AM.

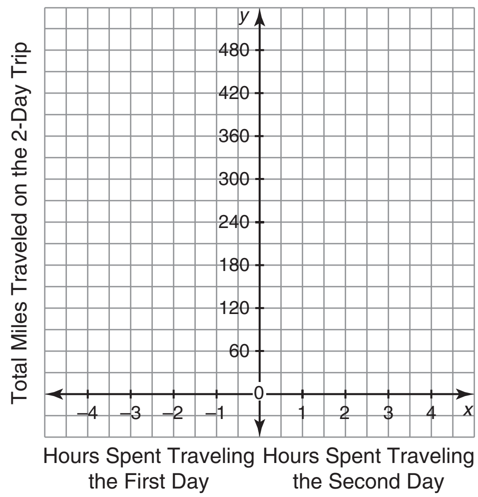
1. What is the rate at which the bus traveled during the second portion of the trip? Show your work.

2. What was the total distance of the trip? Show your work.

3. If the bus traveled the same average rate during both segments of the trip, what is the total number of hours the team traveled on the bus? Show your work.

4. Why do you think the team did not make the entire trip the morning of the tournament?

5. Complete the graph to represent the context.



6. Explain why the graph represents a linear function.

7. Demonstrate the rate of change graphically and by using the formula. Show your work.

8. What is happening in terms of the context in the second quadrant of the graph?

9. Complete the table representing the context.

Travel Time for Second Day (hours)	Total Miles Traveled on the Two-Day Trip

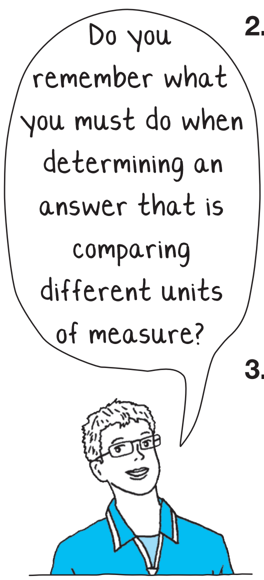
10. The unit rate (miles per hour) is **not** an entry in the table. Calculate the unit rate using the table of values. Show your work.

11. Recalculate the unit rate by using different values from the table. Show your work.

Problem 2 Calculating Rate of Change from a Context

Write the rate described in each context.

1. Bella's Pizza Shop charges $4.50 for a small pizza, $7 for a medium pizza, and $9 for a large pizza. Toppings cost extra depending on the size of the pizza ordered. Bruce ordered a large pizza with three toppings that cost a total of $12.60. What is the unit rate of cost per number of toppings for a large pizza? Show your work.

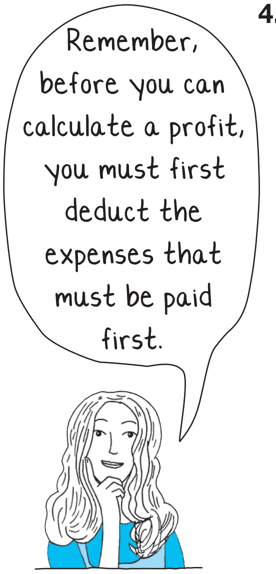


2. A maintenance crew is paving a road. They are able to pave one-eighth of a mile of road during each working shift. A working shift is 7 hours. What is the unit rate of yards of road paved per hour? Show your work.

3. Melanie baked breakfast rolls for a band camp fundraiser. She baked 15 dozen breakfast rolls in 3 hours.

   a. What is the average rate of breakfast rolls baked per minute?

   b. Why do you think that "average" rate is asked instead of "rate"?



4. One hundred twenty teenagers attended the community center's dance. Each ticket costs $5. The community center's expenses for the dance are $140 for the disc jockey (DJ), and $60 for other expenses. What is the profit the center made in dollars for each ticket sold?
Show your work.

3

5. Jonathan goes to bed at 9:30 PM on school nights and wakes up at 6:00 AM. On Fridays and Saturdays, he goes to bed at 11:00 PM and wakes up at 9:00 AM. What is Jonathan's average rate of sleep hours per night? Show your work.

6. Mike had a balance of $81 on his credit card for a department store. He just purchased 3 sweatshirts, and his balance is now $146.85. What is the cost of one sweatshirt? Show your work.

7. One dieter in a weight-loss contest weighed 149 pounds after 8 weeks on his diet. By Week 13, he weighed 134 pounds. What was his average weight loss per week? Show your work.

8. Kathy is working after school to finish assembling the 82 favors needed for the school dance. When she starts at 3:15 PM, she counts the 67 favors that are already assembled. She works until 4:30 PM to finish the job.

   a. How many favors can Kathy assemble in a minute?

   b. How many minutes does it take Kathy to assemble one favor?

   c. Which rate is more meaningful in this situation? Explain your reasoning.

9. Eddie rented a moving van to travel across the country. The odometer registered 34,567 miles after he drove for 4 hours. After 7 hours of driving, the odometer read 34,741 miles.

   a. What was Eddie's driving rate in miles per hour?

b. When Eddie calculated his driving rate, he converted the information to coordinate points and then used the formula $\frac{y_2 - y_1}{x_2 - x_1}$. Examine his work.

The points are (4, 34,567) and (7, 34,741).

Using the formula $\frac{y_2 - y_1}{x_2 - x_1}$, I got $\frac{34{,}741 - 34{,}567}{7 - 4} = \frac{174}{3} = \frac{58}{1}$.

This means that on average I was traveling 58 miles per hour.

How does your process of calculating Eddie's driving rate compare to his work?

3

10. Julie used her gift card for the local coffee shop to buy iced teas for herself and five friends. After she and one friend placed their orders, the balance on Julie's gift card was \$14.85. After all 6 members of the group got their iced teas, she had a balance of \$3.97 on her gift card.

Use Eddie's method from Question 9, part (b) to determine the cost for one glass of iced tea. Show all work.

Be prepared to share your solutions and methods.

3.4 All Together Now!

Determining Rate of Change from an Equation

Learning Goals

In this lesson, you will:

- Determine the rate of change from an equation that has been solved for *y*.
- Create a table, a scenario, and a graph from an equation.
- Connect the rate of change represented in an equation to the rate of change in other representations.
- Determine if an equation has a rate of change that is increasing, decreasing, zero, or undefined.
- Compare rates of change by comparing the coefficients of *x* in different equations.

Key Terms

- slope
- slope-intercept form

3

The Duquesne (pronounced "doo - KANE") Incline in Pittsburgh, Pennsylvania is what is known as a funicular (foo - NICK - you - lur). A funicular is a railway that pulls cars up and down a slope. Funiculars played important roles in many cities' histories. Funiculars were ways people could commute to work from their homes in hillsides to factories along river banks.

The Duquesne Incline, which has a slope of 30°, is one of the most popular tourist attractions in Pittsburgh.

Problem 1 Cut and Sort Linear Relations

1. Carefully cut out the graphs, tables, contexts, and equations on the following pages. Match each equation with its correct graph, table, or context. Explain how you matched the equations with the representations.

2. Compare the graphs.

   a. How are they different? How can you tell this difference by looking at their equations?

   b. Analyze the point where each graph crosses the y-axis. How can you tell this point by looking at the equation for each graph?

   c. What is the rate of change for each graph? How is the rate of change represented in each equation?

3. Analyze the equation for each table.

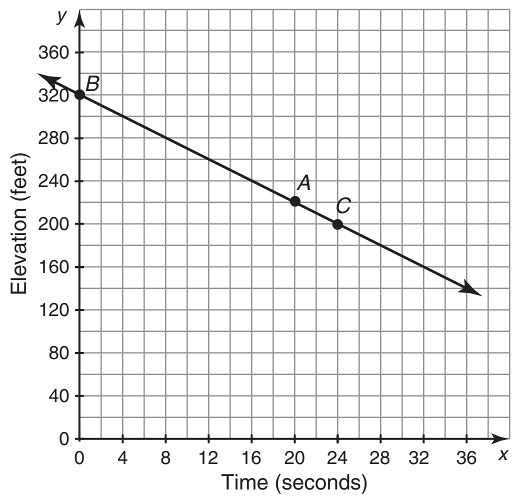
   a. Determine the coefficient of x for each equation using a formula.

   b. How can the number that is added in each equation be determined from the table?

4. Analyze the equation for each context. Explain what each term of the equation means in each context.

3

A



B

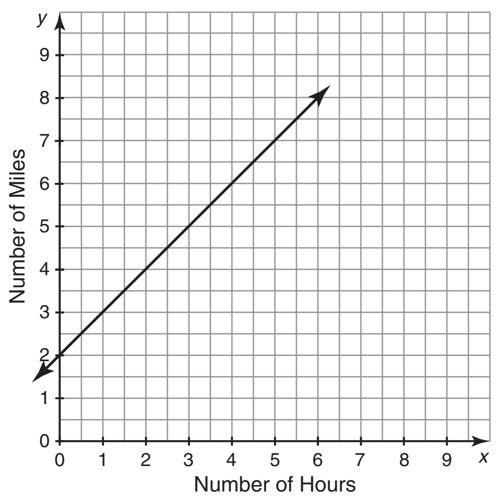
Michele read the first 40 pages of a mystery novel before she fell asleep. The next day, she read one page every two minutes until she finished the book, which was a total of 325 pages.

D

Number of Carnival Ride Tickets	Cost (in dollars)
x	*y*
0	6
4	9
8	12
16	18
32	30

3

C



F

Bella's Pizza Shop charges $4.50 for a small pizza, $7.00 for a medium pizza, and $9.00 for a large pizza. Additional toppings cost extra depending on the size of the pizza ordered. Bruce ordered a large pizza with three toppings that cost a total of $12.60.

E

Number of Games Ron Won Today	Number of Credits on Ron's Player's Card
x	*y*
0	120
12	216
18	264
25	320
40	440

3

$y = 1.2x + 9$	$y = \frac{3}{4}x + 6$
$y = \frac{1}{2}x + 40$	$y = x + 2$
$y = -5x + 320$	$y = 8x + 120$

3

3

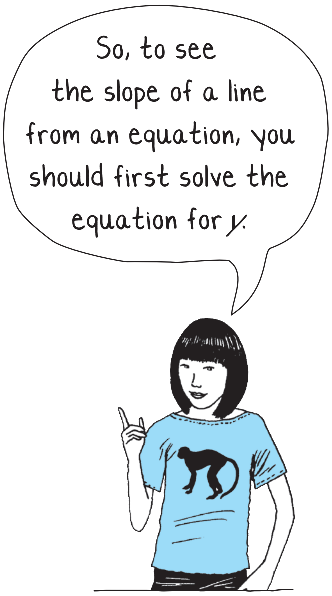
So far, you have determined the rate of change either through formal and informal methods. Now you will learn about a new name for a rate of change. **Slope** is another mathematical term for rate of change. The slope of a line can be calculated in the same ways as rate of change is calculated:

- from a graph using $\frac{\text{rise}}{\text{run}}$ as a measure of the steepness of a line;
- from a table using $\frac{y_2 - y_1}{x_2 - x_1}$ or informally through subtraction of table values;
- from an equation that has been solved for y, as the coefficient of x; and
- from a context using text clues or coordinate point values given in the problem.

All of these methods are also leading us to explore the *slope-intercept form*. The **slope-intercept form** of a linear equation is $y = mx + b$, where m is the slope of the line.

Problem 2 Calculating Rate of Change from an Equation

If a linear equation is solved for y, the coefficient of x represents the rate of change, or slope of the line. Determine the slopes of the lines represented by each equation. Show your work.



1. $y = 3x - 9 + 8x$

2. $15x + 3y = 300$

3. $y = 5(2x - 9)$

4. $8y = -6x + 24$

5. $y = x - 3$

6. $y = 9$

7. $4x - 12y = 48$

8. $x = 10$

Problem 3 Investigating Rate of Change from an Equation

You can also use a graphing calculator to investigate slopes. First you will explore the slope for the equation $y = 1x$.

Step 1: Press $\boxed{\mathbf{Y =}}$. Your cursor should be blinking next to $\mathbf{Y_1} =$. Enter **1*x***. Then, press $\boxed{\mathbf{GRAPH}}$.

You will refer to this basic graph as you make changes to the coefficient of x.

Step 2: Press $\boxed{\mathbf{Y =}}$. Using the arrow keys, move to the left to the **** in front of $\mathbf{Y_1}$.

Step 3: Press $\boxed{\mathbf{ENTER}}$ one time until the \ is darkened. Press $\boxed{\mathbf{GRAPH}}$. Your basic graph should be darkened for easy reference.

1. Press $\boxed{\mathbf{Y =}}$.
   Next to $\mathbf{Y_2}$, enter **4*x***.
   a. What do you think this graph will look like in comparison to the graph of $y = 1x$? Verify your answer by pressing $\boxed{\mathbf{GRAPH}}$.

b. Write an equation of another line that is steeper than both of these lines. Verify your answer by entering the equation next to $\mathbf{Y_3}$ and graphing it.

c. How does increasing the coefficient of x affect the rate of change and the graph of the line?

2. Keep the equation $y_1 = 1x$ on the calculator. Clear all other equations.

a. Write an equation of a line that is less steep than $y_1 = 1x$. Verify your answer by entering the equation next to $\mathbf{Y_2}$ and graphing it.

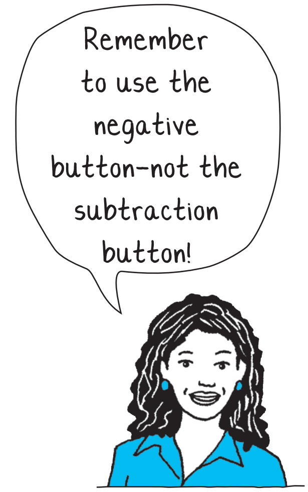
b. Write an equation of a line that is less steep than both of these lines. Verify your answer by entering the equation next to $\mathbf{Y_3}$ and graphing it.

c. How does decreasing the coefficient of x affect the rate of change and the graph of the line?

3. Keep the equation $y_1 = 1x$ on the calculator. Clear all other equations.

Press $\boxed{\mathbf{Y =}}$.

Next to $\mathbf{Y_2}$, enter $\mathbf{-1x}$. Use the (–) sign.



a. What do you think this graph will look like in comparison to the graph of $y = 1x$? Verify your answer by pressing $\boxed{\textbf{GRAPH}}$.

b. Write an equation of another line that is slanted in the same direction as $y = -1x$ but is steeper than that line. Verify your answer by entering the equation next to $\mathbf{Y_3}$ and graphing it.

c. Write an equation of another line that is slanted in the same direction as $y = -1x$ but is less steep than that line. Verify your answer by entering the equation next to $\mathbf{Y_4}$ and graphing it.

d. How does a negative coefficient of x affect the rate of change and the graph of the line?

4. Clear all equations including $y_1 = 1x$ from the calculator.

   a. Enter the equation $y = 1$.
   What do you think this graph will look like? Verify your answer by pressing **GRAPH**.

   b. What is the coefficient of x?

   c. How does a coefficient of 0 affect the rate of change and the graph of the line?

   d. Why do you think it is impossible to graph the equation $x = 1$ on the graphing calculator?

Be prepared to share your solutions and methods.

3

3.5 Where It Crosses

Determining y-Intercepts from Various Representations

Learning Goals

In this lesson, you will:

- Determine the y-intercept of a linear function from a context, a table, a graph, or an equation.
- Write the y-intercept in coordinate form.
- Explain the meaning of the y-intercept when given the context of a linear function.
- Explain how the y-intercept is useful in graphing a linear function.
- Explain what makes a relationship a direct variation.

Key Terms

- y-intercept
- direct variation

3

In professional football, an interception occurs when the ball is thrown by a player on one team and is caught by a player on the opposing team.

As of 2011, the person with the most career interceptions was a man born in Flint, Michigan, in 1942 and played for the Washington Redskins and the Minnesota Vikings.

Can you name him? How many interceptions did he catch?

Problem 1 Connecting Representations

Questions 1 through 5 provide guidance for completing the graphic organizer that follows.

1. Read the context. Represent that information in the form of a graph, a table, and an equation.
2. Revisit each representation. In each box, show how the rate of change is represented.

The rate of change is one important feature of a linear function. Another important feature is the *y-intercept*. The ***y*-intercept** is the *y*-coordinate of the point where a graph crosses the *y*-axis. The *y*-intercept can also be written in the form (0, *y*).

3

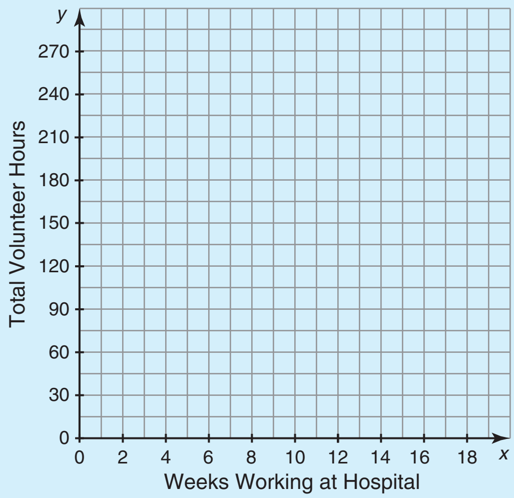
3. Mark the *y*-intercept on the graph. Label the *y*-intercept in coordinate form.
4. What is the meaning of the *y*-intercept in the context?

5. Revisit each representation. Mark where the *y*-intercept is evident in the context, the table, and the equation.

CONTEXT

Eva keeps track of the hours she devotes to volunteering. When she began volunteering on a regular basis at Children's Hospital, she already had 60 volunteer hours at other events. After three weeks of working at Children's Hospital, she had another 36 hours of volunteering.

GRAPH



MULTIPLE REPRESENTATIONS

Number of Weeks Volunteering at the Hospital	Total Hours Volunteering

TABLE

Let y represent:

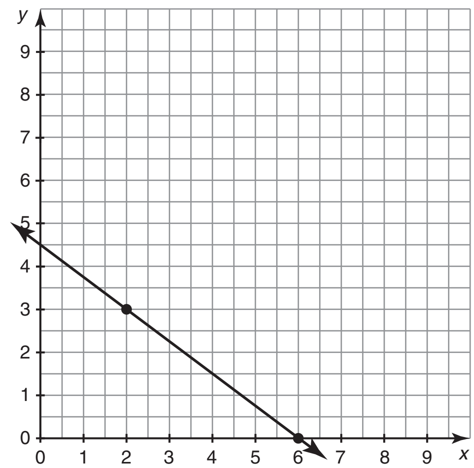
Let x represent:

EQUATION

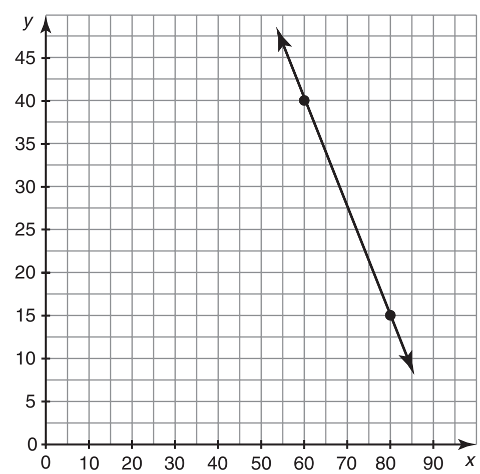
Problem 2 Determining the *y*-Intercept from a Graph

Examine each linear graph and determine the *y*-intercept. Write the *y*-intercept in coordinate form. Show all work.

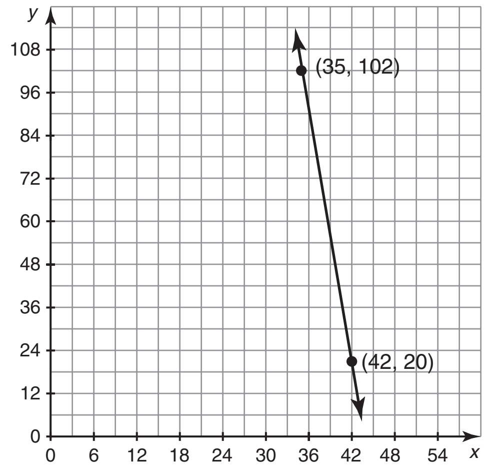
1.



2.



3.



Problem 3 Determining the *y*-Intercept from a Table

Each table represents a linear function. Use the table to identify the *y*-intercept. Write the *y*-intercept in coordinate form. Show all work.

1.

x	*y*
200	14
225	16
250	18
275	20
300	22

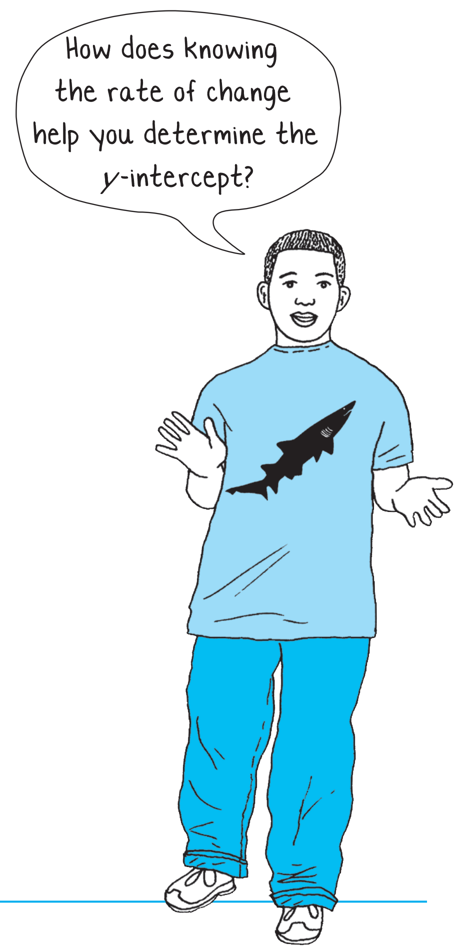
2.

x	y
100	10
105	6
110	2
115	−2
120	−6

3

3.

x	y
16	90
19	91
22	92
25	93
28	94



Problem 4 Determining the *y*-Intercept from a Context

Each context represents a linear function. Read each and determine the *y*-intercept. Write the *y*-intercept in coordinate form. Show all work. Explain what the *y*-intercept represents in the problem situation.

1. Kim spent \$18 to purchase a ride-all-day pass for the amusement park and to play 8 games. After playing a total of 20 games, she realized she'd spent \$24.

2. Mitch saved money he received as gifts to buy a bike. When he added one week's allowance to his savings, he had \$125. After 3 more weeks of saving his allowance, he had \$161 toward the cost of his bike.

3. The cost to ship a package in the mail includes a basic shipping charge plus an additional cost per number of pounds the package weighs. A three-pound package costs \$6.30 to ship. A ten-pound package costs \$14 to ship.

Problem 5 Determining the *y*-Intercept from an Equation

Each equation represents a linear function. Examine each and determine the *y*-intercept. Write the *y*-intercept in coordinate form. Show all work.

1. $4x + 6y = 270$

2. $8x - 4y = 225$

3. $y + 9 = 6(x - 3)$

Problem 6 A Special Case of a Linear Equation

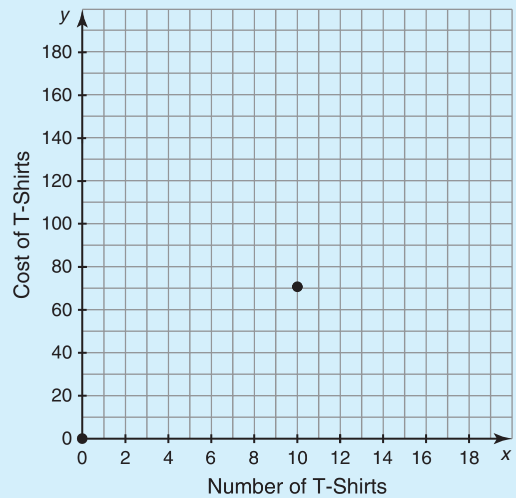
Questions 1 through 3 provide guidance for completing the graphic organizer that follows.

1. Read the context. Represent that information in the form of a graph, a table, and an equation.
2. In each box, show how the rate of change is represented.
3. In each box, show how the *y*-intercept is represented.
4. What is the *y*-intercept? Explain what the *y*-intercept represents in the problem.

CONTEXT

The basketball team won the championship. They are selling special championship T-shirts for a cost of $7 each.

GRAPH



MULTIPLE REPRESENTATIONS

Number of T-shirts	Cost in Dollars

TABLE

Let y represent:

Let x represent:

EQUATION

When the y-intercept is (0, 0), the equation can be written in a simpler form.

It can be written in the form $y = kx$, with $k \neq 0$. No addition or subtraction for the y-intercept value is needed.

When a linear equation is written in this form, the variables x and y show *direct variation*. **Direct variation** is the relationship between two quantities x and y such that the two variables have a constant ratio.

In the example from your graphic organizer, the equation is $y = 7x$. The ratio $\frac{y}{x} = 7$ is true regardless of what coordinate point in the table is used.

5. Why is it not necessary to use the formula $\frac{y_2 - y_1}{x_2 - x_1}$ to determine the rate of change of this relation?

3

6. How can you tell from the graph if an equation shows a direct variation?

Be prepared to share your solutions and methods.

3.6 Slope-Intercept Form

Determining the Rate of Change and *y*-Intercept

Learning Goals

In this lesson, you will:

- Graph lines using the slope and *y*-intercept.
- Calculate the *y*-intercept of a line when given the slope and one point that lies on the line.
- Write equations of lines in slope-intercept form if given two points that lie on the line or the slope and one point that lies on the line.
- Write equations in point-slope form if given the slope and one point that lies on the line.
- Graph lines in standard form by using the intercepts.
- Convert equations from point-slope form and standard form to slope-intercept form.
- Discuss the advantages and disadvantages of point-slope and standard form.

Key Terms

- point-slope form
- standard form

3

A synonym is a word that has the same or almost the same definition of another word. An example of synonyms is "prefer" and "like." In many cases, journalists use synonyms if their writing has many words that repeat within an article or a blog. Sometimes, synonyms can also make an awkwardly written article read more smoothly. Of course, literary critics may sometimes criticize a writer for using too complex synonyms when more common words could easily be used. Can you think of other advantages and disadvantages for using synonyms?

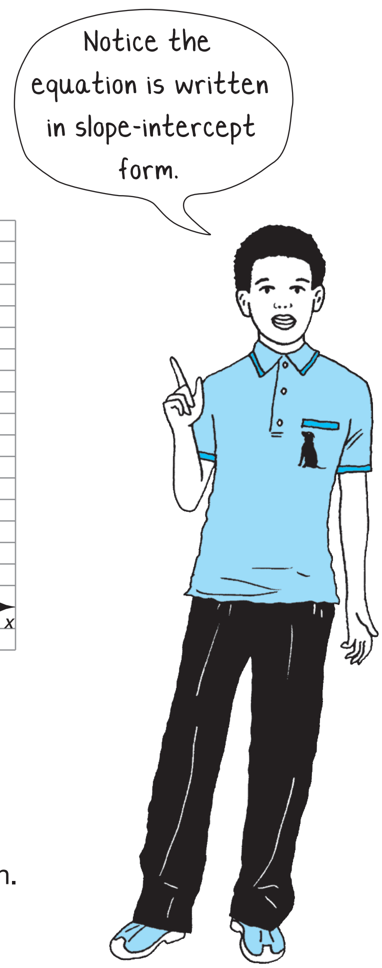
Problem 1 Using Slope-Intercept Form to Graph a Line

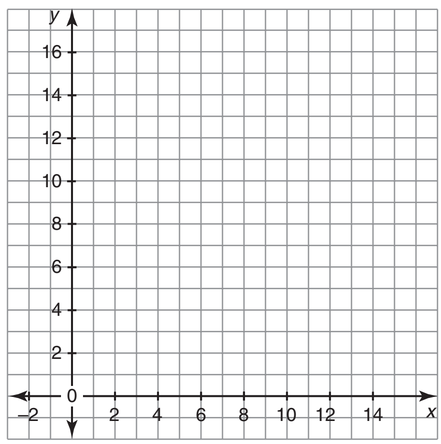
As you learned previously, the slope-intercept form of a linear equation is $y = mx + b$ where m is the slope of the line. However, you did not learn what b represented. In the slope-intercept form, b is the y-intercept of the line. Remember that the slope of the line is the "steepness" of that line.

Douglas is giving away tickets to a concert that he won from a radio station contest. Currently, he has 10 tickets remaining. He gives a pair of tickets to each person who asks for them.

An equation to represent this context is:

y = number of tickets available
x = number of people who request tickets
$y = -2x + 10$





Follow these steps to graph the equation:

Step 1: Write the coordinates for the y-intercept.

Step 2: Plot the y-intercept on the coordinate plane shown.

Step 3: Write the slope as a ratio.

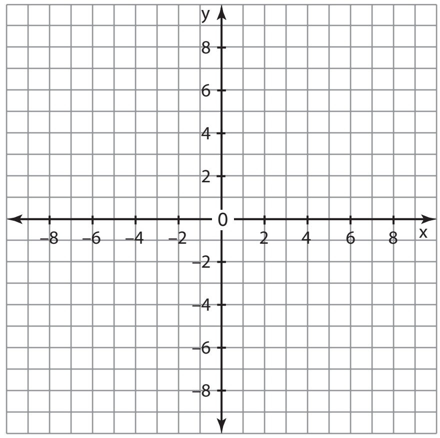
Step 4: Use the slope and count from the y-intercept. To identify another point on the graph, start at the y-intercept and count either down (negative) or up (positive) for the rise. Then, count either left (negative) or right (positive) for the run.

Continue the counting process to plot the next points.

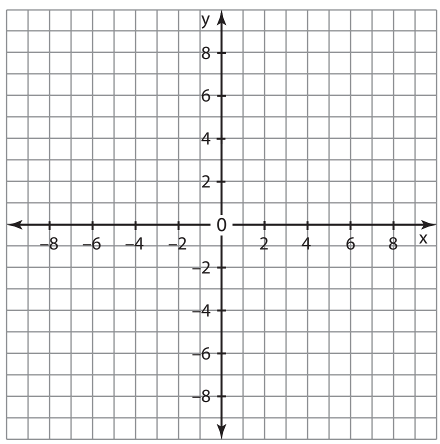
Step 5: Connect the points to make a straight line.

Graph each line. Be careful to take into account the scales on the axes.

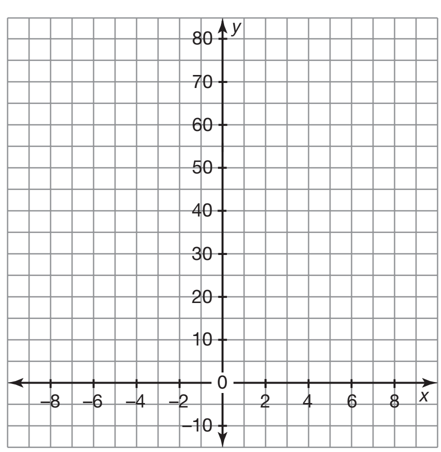
1. $y = \frac{3}{2}x - 1$



2. $y = \frac{-5}{2}x + 3$



3. $y = 10x + 25$



Problem 2 Using Slope-Intercept Form to Calculate the y-Intercept

So far, you have been able to determine the y-intercept of a line when given the linear equation in the slope-intercept form. However, you can determine the y-intercept of a line when given the slope of that line and one point that lies on the line.

Given: $m = -\frac{3}{2}$ and the point (4, 5) that lies on the line.

Step 1: Substitute the values of m, x, and y into the *equation for a line* $y = mx + b$. The x- and y-values are obtained from the point that is given.

$$y = mx + b$$
$$5 = -\frac{3}{2}(4) + b$$

Step 2: Solve the equation for b.

$$5 = -6 + b$$
$$5 + 6 = -6 + b + 6$$
$$11 = b$$

The y-intercept is 11.

Calculate the y-intercept of each line when given the slope and one point that lies on the line.

1. $m = 9$; (2, 11)

2. $m = 2.25$; (16, -52)

3. $m = \frac{-3}{8}$; (50, 7)

Problem 3 Using Slope-Intercept Form to Write Equations of Lines

So far, you have determined the *y*-intercept from the slope-intercept form of a linear equation, and the *y*-intercept from the slope and a point on that lies on the line given. Now, you will write the equation of a line when given two points that lie on the line.

Given: Points (15, −13) and (5, 27) that lie on a line.

Step 1: Calculate the slope using $\frac{y_2 - y_1}{x_2 - x_1}$.

$$\frac{27 - (-13)}{5 - 15} = \frac{40}{-10} = \frac{-4}{1} = -4$$

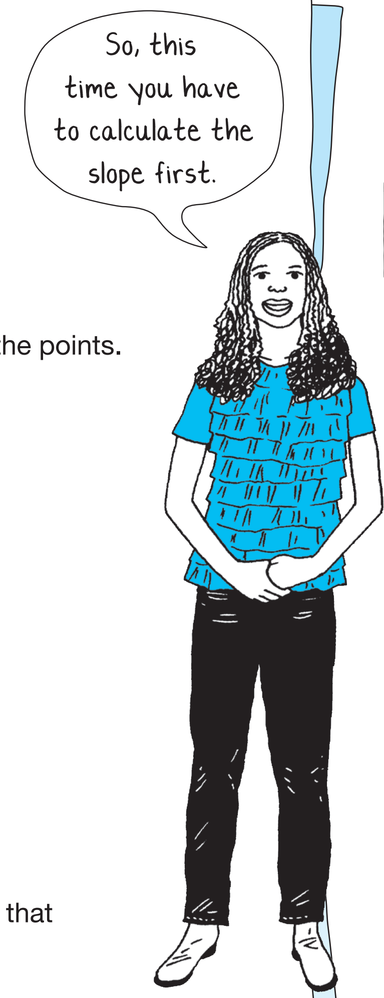
Step 2: Calculate the *y*-intercept by using the slope and one of the points.

$$\begin{aligned} y &= mx + b \\ 27 &= -4(5) + b \\ 27 &= -20 + b \\ 27 + 20 &= -20 + b + 20 \\ 47 &= b \end{aligned}$$

Step 3: Substitute *m* and *b* into the equation $y = mx + b$.

$$\begin{aligned} y &= mx + b \\ y &= -4x + 47 \end{aligned}$$

The equation for a line in which points (15, −13) and (5, 27) lie on that line is $y = -4x + 47$.



3

Write an equation of a line using the given information. Show your work.

1. (7, 15) and (−39, −8)

2. (429, 956) and (249, 836)

3. (6, 19) and (0, −35)

4. The slope is -8. The point (3, 12) lies on the line.

Problem 4 Another Form of a Linear Equation

Let's develop a second form of a linear equation.

Step 1: Begin with the formula for slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Step 2: Rewrite the equation to remove the fraction by multiplying both sides of the equation by $(x_2 - x_1)$.

$$m(x_2 - x_1) = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x_2 - x_1)$$

Step 3: After simplifying, the result is:

$$m(x_2 - x_1) = (y_2 - y_1)$$

Step 4: Remove the subscripts for the second point.

$$m(x - x_1) = (y - y_1)$$

The formula $m(x - x_1) = (y - y_1)$ is the **point-slope form** of a linear equation that passes through the point (x_1, y_1) and has slope m.

Step 5: Finally, substitute the values for m, x, and y into the point-slope form of the equation. The x- and y-values should be substituted in for x_1 and y_1.

1. Write the equation of a line in point-slope form with a slope of -8 and the point (3, 12) that lies on the line.

2. While this equation took little time to write, it is difficult to visualize its graph or even its y-intercept. To determine the y-intercept, manipulate the equation using algebra to write the equation in $y = mx + b$ form. Show all work.

3

3. What is the y-intercept of this line?

4. Write the equation of each line in point-slope form. Then, state the y-intercept of the line. Show all work.

   a. slope $= -5$; (16, 32) lies on the line

   b. $m = \frac{2}{3}$; (9, -18) lies on the line

c. rate of change is -4.5; $(-80, 55)$ lies on the line

5. What are the advantages and disadvantages of using point-slope form?

3

Problem 5 Exploring Standard Form of a Linear Equation

Tickets for the school play cost \$5.00 for students and \$8.00 for adults. On opening night, \$1600 was collected in ticket sales.

This situation can be modeled by the equation $5x + 8y = 1600$. You can define the variables as shown.

x = number of student tickets sold
y = number of adult tickets sold

This equation was not written in slope-intercept form. It was written in *standard form*.

The **standard form** of a linear equation is $Ax + By = C$, where A, B, and C are constants and A and B are not both zero.

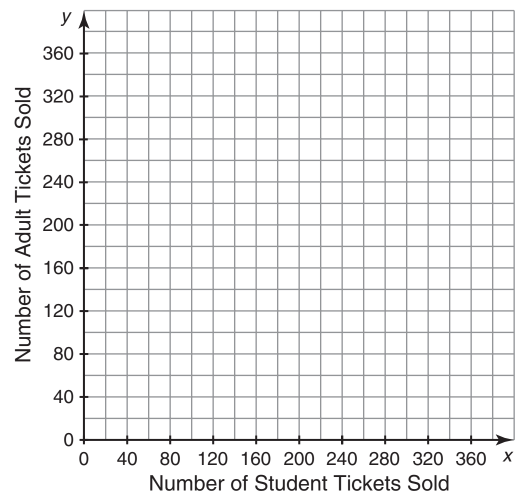
1. Explain what each term of the equation represents in the problem situation.

2. What is the independent variable? What is the dependent variable? Explain your reasoning.

3. Calculate the x-intercept and y-intercept for this equation. Show your work.

4. What are the meanings of the x-intercept and y-intercept?

5. Use the x-intercept and y-intercept to graph the equation of the line.



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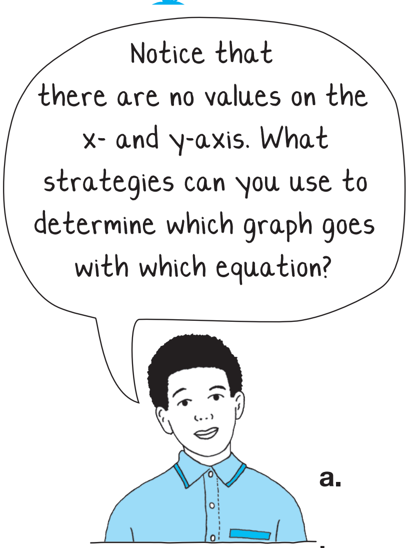
6. What is the slope of this line? Show your work.

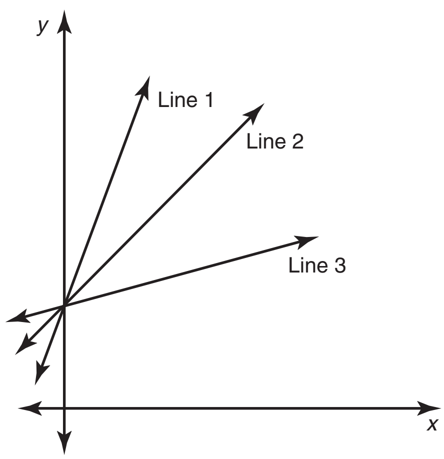
7. What does the slope mean in this problem situation?

8. If 100 student tickets were sold, how many adult tickets were sold? Show your work.

Talk the Talk

1. Match each graph with the correct equation written in standard form. Show your work and explain your reasoning.





a. $3x - 12y = -60$
b. $6x - 2y = -10$
c. $9x - 9y = -45$

a.

b.

c.

2. What are the advantages and disadvantages of using standard form?

Be prepared to share your solutions and methods.

3

Chapter 3 Summary

Key Terms

- rate (3.1)
- rate of change (3.1)
- per (3.1)
- unit rate (3.1)
- rise (3.1)
- run (3.1)
- $\frac{\text{rise}}{\text{run}}$ (3.1)
- first differences (3.2)
- slope (3.4)
- slope-intercept form (3.4)
- *y*-intercept (3.5)
- direct variation (3.5)
- point-slope form (3.6)
- standard form (3.6)

3.1 Using Rise over Run to Calculate the Rate of Change from a Graph

Rate of change is a phrase used when a rate is used to describe a rate of increase (or decrease) in a real-life situation. A unit rate is a rate which has a denominator of 1 unit.

The ratio $\frac{\text{rise}}{\text{run}}$ is a representation of the rate of change shown in a graph.

Example

Choose two points on the graph that can be read without estimation.

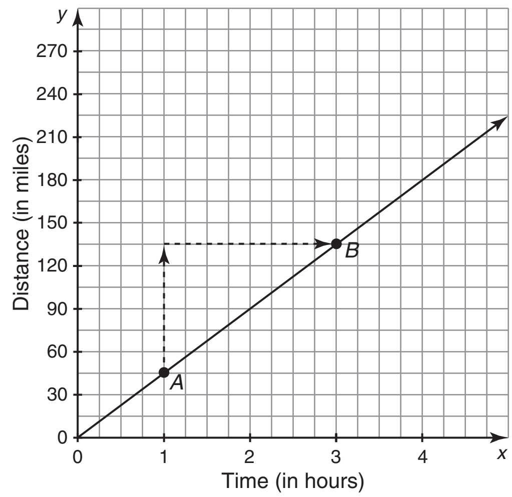
Point *A* (1, 45) and point *B* (3, 135)

The rise is the vertical change from the first point to the second point. The rise is 90 miles.

The run is the horizontal change from the first point to the second point. The run is 2 hours.

$\frac{\text{rise}}{\text{run}} = \frac{90 \text{ miles}}{2 \text{ hours}}$

The unit rate is $\frac{45 \text{ miles}}{1 \text{ hour}}$.



3.2 Determining Rate of Change from a Table

There is a formal mathematical process that can be used to calculate the rate of change of a linear relation from a table of values with at least two coordinate pairs.

The rate of change of a linear relation is $\frac{y_2 - y_1}{x_2 - x_1}$, where the first point is at (x_1, y_1), and the second point is at (x_2, y_2).

Example

Two points are chosen from the table of values and labeled as shown.

$$(x_1, y_1) = (3, 3.75)$$

$$(x_2, y_2) = (5, 6.25)$$

3

Number of Raffle Tickets	Total Cost of Raffle Tickets (in dollars)
3	3.75
5	6.25
7	8.75
9	11.25

Substitute the values into the formula.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{6.25 - 3.75}{5 - 3}$$

$$= \frac{2.5}{2}$$

$$= \frac{1.25}{1}$$

The unit rate is $\frac{\$1.25}{1\ \text{ticket}}$.

3.3 Determining Rate of Change from a Context

A context representing a linear function often provides enough information to determine the rate of change of the function.

Example

Joelle is selling fruit smoothies at a summer festival. During the first two days of the festival, Joelle sells 120 smoothies. By the conclusion of the four-day event, Joelle had sold an additional 88 smoothies.

$$120 \text{ smoothies} + 88 \text{ smoothies} = 208 \text{ smoothies total}$$

$$\frac{208 \text{ smoothies}}{4 \text{ days}} = \frac{52 \text{ smoothies}}{1 \text{ day}}$$

The unit rate is $\frac{52 \text{ smoothies}}{1 \text{ day}}$.

3

3.4 Determining Rate of Change from an Equation

Slope is another mathematical term for rate of change. If a linear equation is solved for y, the coefficient of x represents the rate of change, or slope of the line. An equation in this form, $y = mx + b$, where m is the slope of the line, is said to be in slope-intercept form.

Example

A slope of a line can be determined for the equation $15y - 6x = 30$.

$$\begin{aligned} 15y - 6x &= 30 \\ 15y &= 6x + 30 \\ y &= \frac{6}{15}x + 2 \\ y &= \frac{2}{5}x + 2 \end{aligned}$$

The equation is now in slope-intercept form. The slope of the line, m, is $\frac{2}{5}$.

3.5 Determining the *y*-Intercept of a Linear Equation

The *y*-intercept is the *y*-coordinate of the point where a graph crosses the *y*-axis. The *y*-intercept can also be written as the coordinate pair (0, *y*).

Example

To determine the *y*-intercept from the equation $2x + 6y = 24$, substitute $x = 0$ into the equation and solve for *y*.

$$\begin{aligned} 2x + 6y &= 24 \\ 2(0) + 6y &= 24 \\ 6y &= 24 \\ y &= 4 \end{aligned}$$

The *y*-intercept is (0, 4).

3

3.6 Using Slope-Intercept Form to Graph a Line

The slope-intercept form of a linear equation is $y = mx + b$, where *m* is the slope of the line and *b* is the *y*-intercept of the line. An equation in this form provides all the information necessary to graph the line representing the equation.

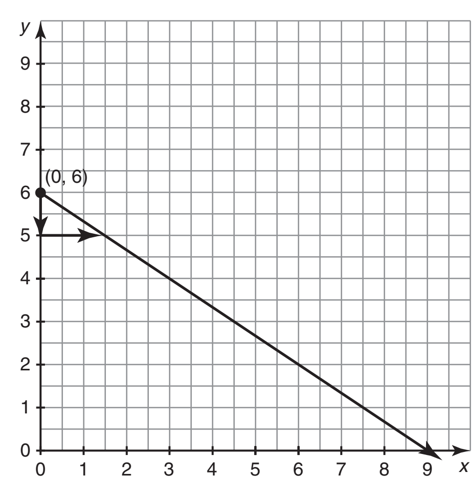
Example

A sketch of a graph can be created for the equation

$y = -\frac{2}{3}x + 6.$

From the equation, it is known that the *y*-intercept is (0, 6). Plot this point on the graph.

From the equation, it is known that the slope is $-\frac{2}{3}$. Use the slope and count from the *y*-intercept to graph another point on the line. Finally, connect the points.



3.6 Using the Point-Slope Form of a Linear Equation

The point-slope form of a linear equation that passes through the point (x_1, y_1) and has slope m is $m(x - x_1) = (y - y_1)$.

Example

An equation can be written for a line with a slope of $-\frac{5}{6}$ which goes through the point $(12, -8)$.

$$m(x - x_1) = (y - y_1)$$

$$-\frac{5}{6}(x - 12) = (y - (-8))$$

$$-\frac{5}{6}x + 10 = y + 8$$

$$-\frac{5}{6}x + 2 = y$$

$$y = -\frac{5}{6}x + 2$$

3

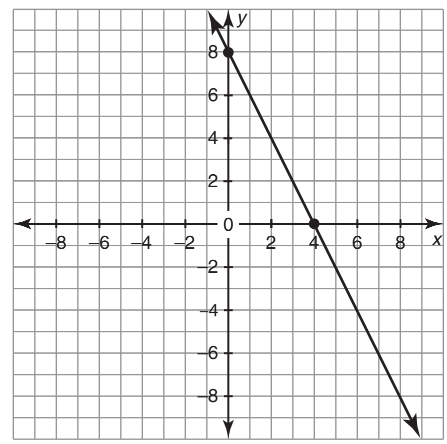
3.6 Using the Standard Form of a Linear Equation

The standard form of a linear equation is $Ax + By = C$, where A, B, and C are constants and A and B are not both zero.

Example

The y-intercept and the x-intercept can be calculated for the linear equation $4x + 2y = 16$ in standard form.

Sketch the graph of the line.



$$\begin{aligned} 4x + 2y &= 16 \\ 4(0) + 2y &= 16 \\ 2y &= 16 \\ y &= 8 \end{aligned}$$

The y-intercept is at (0, 8).

$$\begin{aligned} 4x + 2y &= 16 \\ 4x + 2(0) &= 16 \\ 4x &= 16 \\ x &= 4 \end{aligned}$$

The x-intercept is at (4, 0).