

Solving Inequalities by Addition and Subtraction

Then

Now

Why?

- You solved equations by using addition and subtraction.

- Solve linear inequalities by using addition.

- Solve linear inequalities by using subtraction.

- The data in the table show that the recommended daily allowance of Calories for girls 11–14 years old is less than that of girls between 15–18 years old.

Calories	
Girls 11–14 Years	Girls 15–18
1845	2110

Source: Vital Health Zone

$$1845 < 2110$$

If a 13-year-old girl and a 16-year-old girl each eat 150 more Calories in a day than is suggested, the 16-year-old will still eat more Calories.

$$1845 + 150 \underline{<} 2110 + 150$$

$$1995 < 2260$$



New Vocabulary
inequality
set-builder notation



Common Core State Standards

Content Standards

A.CED.1 Create equations and inequalities in one variable and use them to solve problems.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Mathematical Practices

- Reason abstractly and quantitatively.
- Model with mathematics.

- Solve Inequalities by Addition** An open sentence that contains $<$, $>$, \leq , or \geq is an **inequality**. The example above illustrates the Addition Property of Inequalities.

KeyConcept Addition Property of Inequalities

- Words** If the same number is added to each side of a true inequality, the resulting inequality is also true.
- Symbols** For all numbers a , b , and c , the following are true.
- If $a > b$, then $a + c > b + c$.
 - If $a < b$, then $a + c < b + c$.

This property is also true for \geq and \leq .

Example 1 Solve by Adding

Solve $x - 12 \geq 8$. Check your solution.

$$x - 12 \geq 8$$

Original inequality

$$x - 12 + 12 \geq 8 + 12$$

Add 12 to each side.

$$x \geq 20$$

Simplify.

The solution is the set {all numbers greater than or equal to 20}.

CHECK To check, substitute three different values into the original inequality: 20, a number less than 20, and a number greater than 20.

GuidedPractice

Solve each inequality. Check your solution.

1A. $22 > m - 8$

1B. $d - 14 \geq -19$

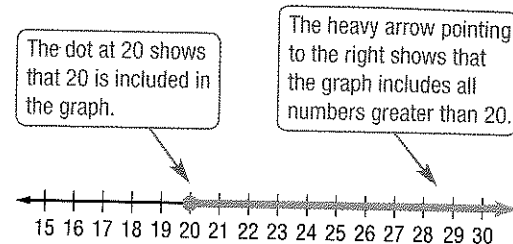


ReadingMath

set-builder notation
 $\{x \mid x \geq 20\}$ is read *the set of all numbers x such that x is greater than or equal to 20.*

A more concise way of writing a solution set is to use **set-builder notation**. In set-builder notation, the solution set in Example 1 is $\{x \mid x \geq 20\}$.

This solution set can be graphed on a number line. Be sure to check if the endpoint of the graph of an inequality should be a circle or a dot. If the endpoint is not included in the graph, use a circle, otherwise use a dot.



2 Solve Inequalities by Subtraction

Subtraction can also be used to solve inequalities.

KeyConcept Subtraction Property of Inequalities

Words	If the same number is subtracted from each side of a true inequality, the resulting inequality is also true.
Symbols	For all numbers a , b , and c , the following are true. <ol style="list-style-type: none"> If $a > b$, then $a - c > b - c$. If $a < b$, then $a - c < b - c$.

This property is also true for \geq and \leq .

Test-TakingTip

Isolating the Variable When solving inequalities, the goal is to isolate the variable on one side of the inequality. This is the same as with solving equations.

Standardized Test Example 2 Solve by Subtracting

Solve $m + 19 > 56$.

- A $\{m \mid m < 41\}$ B $\{m \mid m < 37\}$ C $\{m \mid m > 37\}$ D $\{m \mid m > 41\}$

Read the Test Item

You need to find the solution set for the inequality.

Solve the Test Item

Step 1 Solve the inequality.

$$m + 19 > 56 \quad \text{Original inequality}$$

$$m + 19 - 19 > 56 - 19 \quad \text{Subtract 19 from each side.}$$

$$m > 37 \quad \text{Simplify.}$$

Step 2 Write in set-builder notation: $\{m \mid m > 37\}$. The answer is C.

GuidedPractice

2. Solve $p + 8 \leq 18$.

- F $\{p \mid p \geq 10\}$ G $\{p \mid p \leq 10\}$ H $\{p \mid p \leq 26\}$ J $\{p \mid p \geq 126\}$

StudyTip

Writing Inequalities
 Simplifying the inequality so that the variable is on the left side, as in $a \geq 6$, prepares you to write the solution set in set-builder notation.

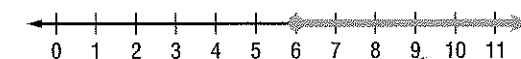
Terms that are constants are not the only terms that can be subtracted. Terms with variables can also be subtracted from each side to solve inequalities.

Example 3 Variables on Each Side

Solve $3a + 6 \leq 4a$. Then graph the solution set on a number line.

$3a + 6 \leq 4a$	Original inequality
$3a - 3a + 6 \leq 4a - 3a$	Subtract $3a$ from each side.
$6 \leq a$	Simplify.

Since $6 \leq a$ is the same as $a \geq 6$, the solution set is $\{a \mid a \geq 6\}$.



GuidedPractice

Solve each inequality. Then graph the solution set on a number line.

3A. $9n - 1 < 10n$

3B. $5h \leq 12 + 4h$

Verbal problems containing phrases like *greater than* or *less than* can be solved by using inequalities. The chart shows some other phrases that indicate inequalities.

ConceptSummary Phrases for Inequalities

$<$	$>$	\leq	\geq
less than fewer than	greater than more than	at most, no more than, less than or equal to	at least, no less than, greater than or equal to

Real-World Example 4 Use an Inequality to Solve a Problem

PETS Felipe needs for the temperature of his leopard gecko's basking spot to be at least 82°F . Currently the basking spot is 62.5°F . How much warmer does the basking spot need to be?

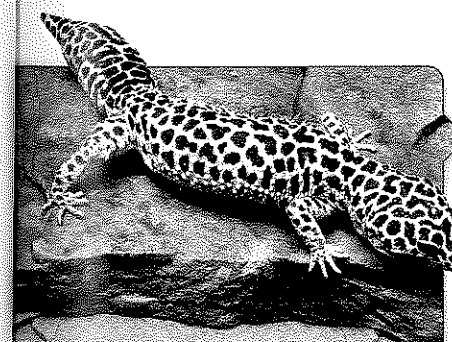
Words	The current temperature needs to be at least 82°F .
Variable	Let t = the number of degrees that the temperature needs to rise.
Inequality	$62.5 + t \geq 82$

$62.5 + t \geq 82$	Original inequality
$62.5 + t - 62.5 \geq 82 - 62.5$	Subtract 62.5 from each side.
$t \geq 19.5$	Simplify.

Felipe needs to raise the temperature of the basking spot 19.5°F or more.

GuidedPractice

4. **SHOPPING** Sanjay has \$65 to spend at the mall. He bought a T-shirt for \$18 and a belt for \$14. If Sanjay wants a pair of jeans, how much can he spend?



Real-WorldLink

Leopard geckos are commonly yellow and white with black spots. They are nocturnal and easy to tame. They do not have toe pads like other geckos, so they do not climb.

Source: Exotic Pets

Check Your Understanding

Step-by-Step Solutions begin on page R13.

Examples 1–3 Solve each inequality. Then graph the solution set on a number line.

1. $x - 3 > 7$
2. $5 \geq 7 + y$
3. $g + 6 < 2$
4. $11 \leq p + 4$
5. $10 > n - 1$
6. $k + 24 > -5$
7. $8r + 6 < 9r$
8. $8n \geq 7n - 3$

Example 4 Define a variable, write an inequality, and solve each problem. Check your solution.

9. Twice a number increased by 4 is at least 10 more than the number.
10. Three more than a number is less than twice the number.
11. **AMUSEMENT** A thrill ride swings passengers back and forth, a little higher each time up to 137 feet. Suppose the height of the swing after 30 seconds is 45 feet. How much higher will the ride swing?

Practice and Problem Solving

Extra Practice is on page R5.

Examples 1–3 Solve each inequality. Then graph the solution set on a number line.

12. $m - 4 < 3$
13. $p - 6 \geq 3$
14. $r - 8 \leq 7$
15. $t - 3 > -8$
16. $b + 2 \geq 4$
17. $13 > 18 + r$
18. $5 + c \leq 1$
19. $-23 \geq q - 30$
20. $11 + m \geq 15$
21. $h - 26 < 4$
22. $8 \leq r - 14$
23. $-7 > 20 + c$
24. $2a \leq -4 + a$
25. $z + 4 \geq 2z$
26. $w - 5 \leq 2w$
27. $3y + 6 \leq 2y$
28. $6x + 5 \geq 7x$
29. $-9 + 2a < 3a$

Example 4 Define a variable, write an inequality, and solve each problem. Check your solution.

30. Twice a number is more than the sum of that number and 9.
31. The sum of twice a number and 5 is at most 3 less than the number.
32. The sum of three times a number and -4 is at least twice the number plus 8.
33. Six times a number decreased by 8 is less than five times the number plus 21.

CCSS MODELING Define a variable, write an inequality, and solve each problem. Then interpret your solution.

34. **FINANCIAL LITERACY** Keisha is babysitting at \$8 per hour to earn money for a car. So far she has saved \$1300. The car that Keisha wants to buy costs at least \$5440. How much money does Keisha still need to earn to buy the car?
35. **TECHNOLOGY** A recent survey found that more than 21 million people between the ages of 12 and 17 use the Internet. Of those, about 16 million said they use the Internet at school. How many teens that are online do not use the Internet at school?
36. **MUSIC** A DJ added 20 more songs to his digital media player, making the total more than 61. How many songs were originally on the player?

37. **TEMPERATURE** The water temperature in a swimming pool increased 4°F this morning. The temperature is now less than 81°F . What was the water temperature this morning?

38. **BASKETBALL** A player's goal was to score at least 150 points this season. So far, she has scored 123 points. If there is one game left, how many points must she score to reach her goal?

39. **SPAS** Samantha received a \$75 gift card for a local day spa for her birthday. She plans to get a haircut and a manicure. How much money will be left on her gift card after her visit?

Service	Cost (\$)
haircut	at least 32
manicure	at least 26

40. **VOLUNTEER** Kono knows that he can only volunteer up to 25 hours per week. If he has volunteered for the times recorded at the right, how much more time can Kono volunteer this week?

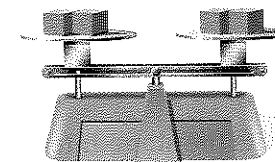
Center	Time (h)
Shelter	3 h 15 min
Kitchen	2 h 20 min

Solve each inequality. Check your solution, and then graph it on a number line.

41. $c + (-1.4) \geq 2.3$
42. $9.1g + 4.5 < 10.1g$
43. $k + \frac{3}{4} > \frac{1}{3}$
44. $\frac{3}{2}p - \frac{2}{3} \leq \frac{4}{9} + \frac{1}{2}p$

45. **MULTIPLE REPRESENTATIONS** In this problem, you will explore multiplication and division in inequalities.

a. **Geometric** Suppose a balance has 12 pounds on the left side and 18 pounds on the right side. Draw a picture to represent this situation.



b. **Numerical** Write an inequality to represent the situation.

c. **Tabular** Create a table showing the result of doubling, tripling, or quadrupling the weight on each side of the balance. Create a second table showing the result of reducing the weight on each side of the balance by a factor of $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$. Include a column in each table for the inequality representing each situation.

d. **Verbal** Describe the effect multiplying or dividing each side of an inequality by the same positive value has on the inequality.

CCSS REASONING If $m + 7 \geq 24$, then complete each inequality.

46. $m \geq ?$
47. $m + ? \geq 27$
48. $m - 5 \geq ?$
49. $m - ? \geq 14$
50. $m - 19 \geq ?$
51. $m + ? \geq 43$

H.O.T. Problems Use Higher-Order Thinking Skills

52. **REASONING** Compare and contrast the graphs of $a < 4$ and $a \leq 4$.
53. **CHALLENGE** Suppose $b > d + \frac{1}{3}$, $c + 1 < a - 4$, and $d + \frac{5}{8} > a + 2$. Order a , b , c , and d from least to greatest.
54. **OPEN ENDED** Write three linear inequalities that are equivalent to $y < -3$.
55. **WRITING IN MATH** Summarize the process of solving and graphing linear inequalities.
56. **WRITING IN MATH** Explain why $x - 2 > 5$ has the same solution set as $x > 7$.



Standardized Test Practice

57. Which equation represents the relationship shown?

- A $y = 7x - 8$
 B $y = 7x + 8$
 C $y = 8x - 7$
 D $y = 8x + 7$

x	y
1	1
2	9
3	17
4	25
5	33
6	41

58. What is the solution set of the inequality $7 + x < 5$?

- F $\{x | x < 2\}$ H $\{x | x < -2\}$
 G $\{x | x > 2\}$ J $\{x | x > -2\}$

59. Francisco has \$3 more than $\frac{1}{4}$ the number of dollars that Kayla has. Which expression represents how much money Francisco has?

- A $3\left(\frac{1}{4}k\right)$ C $3 - \frac{1}{4}k$
 B $\frac{1}{4}k + 3$ D $\frac{1}{4} + 3k$

60. **GRIDDED RESPONSE** The mean score for 10 students on the chemistry final exam was 178. However, the teacher had made a mistake and recorded one student's score as ten points less than the actual score. What should the mean score be?

Spiral Review

Find the inverse of each function. (Lesson 4-7)

61. $f(x) = 7x - 28$

62. $f(x) = \frac{2}{5}x + 12$

63. $f(x) = -\frac{1}{3}x - 8$

64. $f(x) = 12x + 16$

Write the slope-intercept form of an equation for the line that passes through the given point and is perpendicular to the graph of each equation. (Lesson 4-4)

65. $(-2, 0)$, $y = x - 6$

66. $(-3, 1)$, $y = -3x + 7$

67. $(1, -3)$, $y = \frac{1}{2}x + 4$

68. $(-2, 7)$, $2x - 5y = 3$

69. **TRAVEL** On an island cruise in Hawaii, each passenger is given a lei. A crew member hands out 3 red, 3 blue, and 3 green leis in that order. If this pattern is repeated, what color lei will the 50th person receive? (Lesson 3-6)

Find the n th term of each arithmetic sequence described. (Lesson 3-5)

70. $a_1 = 52$, $d = 12$, $n = 102$

71. $-9, -7, -5, -3, \dots$ for $n = 18$

72. $0.5, 1, 1.5, 2, \dots$ for $n = 50$

73. **JOBS** Refer to the time card shown. Write a direct variation equation relating your pay to the hours worked and find your pay if you work 30 hours. (Lesson 3-4)

Weekly Time Card

Day	Hours
FRIDAY	2.0
SATURDAY	3.5
SUNDAY	2.0
TOTAL HOURS	7.5
PAY	\$52.50

Skills Review

Solve each equation.

74. $8y = 56$

75. $4p = -120$

76. $-3a = -21$

77. $2c = \frac{1}{5}$

78. $\frac{r}{2} = 21$

79. $-\frac{3}{4}g = -12$

80. $\frac{2}{5}w = -4$

81. $-6x = \frac{2}{3}$

