Solving Inequalities by Multiplication and Division

⋅⋅Then	⋅⋅Now	⋅⋅Why?
● You solved equations by using multiplication and division.	● 1 Solve linear inequalities by using multiplication. 2 Solve linear inequalities by using division.	● Terrell received a gift card for $20 of music downloads. If each download costs $0.89, the number of downloads he can purchase can be represented by the inequality $0.89d \leq 20$.
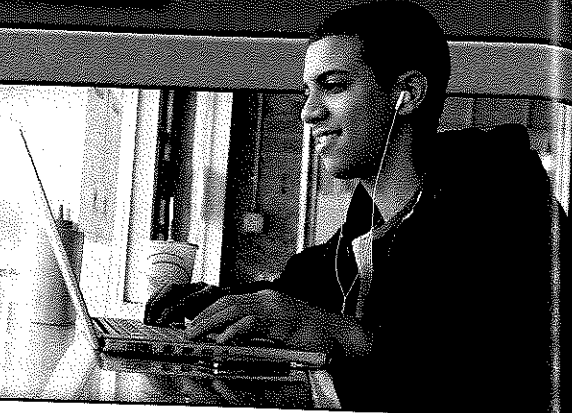
1 Solve Inequalities by Multiplication

If you multiply each side of an inequality by a positive number, then the inequality remains true.

$$4 > 2 \qquad \text{Original inequality}$$
$$4(3) \overset{?}{\_} 2(3) \qquad \text{Multiply each side by 3.}$$
$$12 > 6 \qquad \text{Simplify.}$$

Notice that the direction of the inequality remains the same.

If you multiply each side of an inequality by a negative number, the inequality symbol changes direction.

$$7 < 9 \qquad \text{Original inequality}$$
$$7(-2) \overset{?}{\_} 9(-2) \qquad \text{Multiply each side by } -2.$$
$$-14 > -18 \qquad \text{Simplify.}$$

These examples demonstrate the **Multiplication Property of Inequalities**.

KeyConcept Multiplication Property of Inequalities

Words	Symbols	Examples
If both sides of an inequality that is true are multiplied by a positive number, the resulting inequality is also true.	For any real numbers a and b and any positive real number c, if $a > b$, then $ac > bc$. And, if $a < b$, then $ac < bc$.	$6 > 3.5$ $6(2) > 3.5(2)$ $12 > 7$ and $2.1 < 5$ $2.1(0.5) < 5(0.5)$ $1.05 < 2.5$
If both sides of an inequality that is true are multiplied by a negative number, the direction of the inequality sign is reversed to make the resulting inequality also true.	For any real numbers a and b and any negative real number c, if $a > b$, then $ac < bc$. And, if $a < b$, then $ac > bc$.	$7 > 4.5$ $7(-3) < 4.5(-3)$ $-21 < -13.5$ and $3.1 < 5.2$ $3.1(-4) > 5.2(-4)$ $-12.4 > -20.8$

This property also holds for inequalities involving $\leq$ and $\geq$.

StudyTip

CCSS Sense-Making In Example 1, you could also check the solution by substituting a number greater than 672 and verifying that the resulting inequality is false.
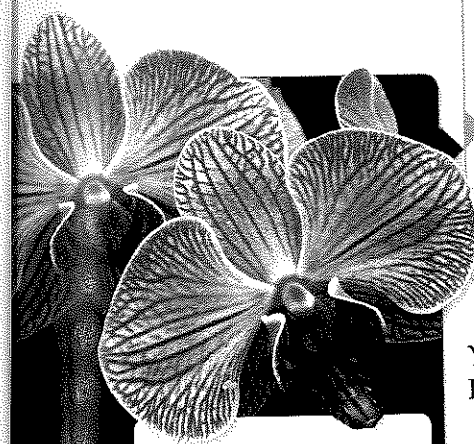
Real-WorldLink

More than 30,000 different orchid species flower in the wild on every continent except Antarctica.

Source: Aloha Orchid Nursery

Real-World Example 1 Write and Solve an Inequality

SURVEYS Of the students surveyed at Madison High School, fewer than eighty-four said they have never purchased an item online. This is about one eighth of those surveyed. How many students were surveyed?

Understand You know the number of students who have never purchased an item online and the portion this is of the number of students surveyed.

Plan Let n = the number of students surveyed. Write an open sentence that represents this situation.

Words	One eighth	times	the number of students surveyed	is less than	84.
Inequality	$\frac{1}{8}$	$\cdot$	n	$<$	84.

Solve Solve for n.

$$\frac{1}{8}n < 84 \qquad \text{Original inequality}$$
$$(8)\frac{1}{8}n < (8)84 \qquad \text{Multiply each side by 8.}$$
$$n < 672 \qquad \text{Simplify.}$$

Check Check the endpoint with 672 and the direction of the inequality with a value less than 672.

$$\frac{1}{8}(672) \overset{?}{=} 84 \qquad \text{Check endpoint.} \qquad \frac{1}{8}(0) \overset{?}{<} 84 \qquad \text{Check direction.}$$
$$84 = 84 \checkmark \qquad\qquad\qquad 0 < 84 \checkmark$$

The solution set is $\{n \mid n < 672\}$, so fewer than 672 students were surveyed.
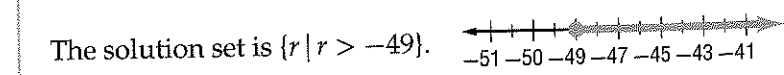
▶ **Guided**Practice

1. **BIOLOGY** Mount Kinabalue in Malaysia has the greatest concentration of wild orchids on Earth. It contains more than 750 species, or about one fourth of all orchid species in Malaysia. How many orchid species are there in Malaysia?

You can also use multiplicative inverses with the Multiplication Property of Inequalities to solve an inequality.

Example 2 Solve by Multiplying

Solve $-\frac{3}{7}r < 21$. Graph the solution on a number line.

$$-\frac{3}{7}r < 21 \qquad \text{Original inequality}$$
$$\left(-\frac{7}{3}\right)\left(-\frac{3}{7}r\right) > \left(-\frac{7}{3}\right)21 \qquad \text{Multiply each side by } -\frac{7}{3}. \text{ Reverse the inequality symbol.}$$
$$r > -49 \qquad \text{Simplify. Check by substituting values.}$$

The solution set is $\{r \mid r > -49\}$.

$$-51\ -50\ -49\ -47\ -45\ -43\ -41$$

▶ **Guided**Practice

Solve each inequality. Check your solution.

2A. $-\frac{n}{6} \leq 8$ **2B.** $-\frac{4}{3}p > -10$ **2C.** $\frac{1}{5}m \geq -3$ **2D.** $\frac{3}{8}t < 5$

Negatives A negative sign in an inequality does not necessarily mean that the direction of the inequality should change. For example, when solving $\frac{x}{6} > -3$, do not change the direction of the inequality.

2 Solve Inequalities by Division

If you divide each side of an inequality by a positive number, then the inequality remains true.

$-10 < -5$	Original inequality
$\frac{-10}{5} \; ? \; \frac{-5}{5}$	Divide each side by -5.
$-2 < -1$	Simplify.

Notice that the direction of the inequality remains the same. If you divide each side of an inequality by a negative number, the inequality symbol changes direction.

$15 < 18$	Original inequality
$\frac{15}{-3} \; ? \; \frac{18}{-3}$	Divide each side by -3.
$-5 > -6$	Simplify.

These examples demonstrate the **Division Property of Inequalities**.

KeyConcept Division Property of Inequalities

Words	Symbols	Examples	
If both sides of a true inequality are divided by a positive number, the resulting inequality is also true.	For any real numbers a and b and any positive real number a, if $a > b$, then $\frac{a}{c} > \frac{b}{c}$. And, if $a < b$, then $\frac{a}{c} < \frac{b}{c}$.	$4.5 > 2.1$ $\frac{4.5}{3} > \frac{2.1}{3}$ and $1.5 > 0.7$	$1.5 < 5$ $\frac{1.5}{0.5} < \frac{5}{0.5}$ $3 < 10$
If both sides of a true inequality are divided by a negative number, the direction of the inequality sign is reversed to make the resulting inequality also true.	For any real numbers a and b, and any negative real number c, if $a > b$, then $\frac{a}{c} < \frac{b}{c}$. And, if $a < b$, then $\frac{a}{c} < \frac{b}{c}$.	$6 > 2.4$ $\frac{6}{-6} < \frac{2.4}{-6}$ and $-1 < -0.4$	$-1.8 < 3.6$ $\frac{-1.8}{-9} < \frac{3.6}{-9}$ $0.2 > -0.4$

This property also holds true for inequalities involving $\leq$ and $\geq$.

Example 3 Divide to Solve an Inequality

Solve each inequality. Graph the solution on a number line.

a. $60t > 8$

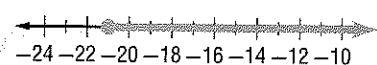|	
$60t > 8$	Original inequality
$\frac{60t}{60} > \frac{8}{60}$	Divide each side by 60.
$t > \frac{2}{15}$	Simplify.

$\left\{ t \mid t > \frac{2}{15} \right\}$

number line: $-6 \; -4 \; -2 \; 0 \; 2 \; 4 \; 6$
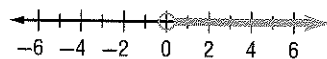
b. $-7d \leq 147$

$-7d \leq 147$	Original inequality
$\frac{-7d}{-7} \geq \frac{147}{-7}$	Divide each side by -7.
$d \geq -21$	Simplify.

$\{d \mid d \geq -21\}$

number line: $-24 \, -22 \, -20 \, -18 \, -16 \, -14 \, -12 \, -10$

GuidedPractice

3A. $8p < 58$

3B. $-42 > 6r$

3C. $-12h > 15$

3D. $-\frac{1}{2}n < 6$

Thomas Harriot (1560–1621) Harriot was a prolific astronomer. He was the first to map the Moon's surface and to see sunspots. Harriot is best known for his work in algebra.

Example 1

1. FUNDRAISING The Jefferson Band Boosters raised more than $5500 from sales of their $15 band DVD. Define a variable, and write an inequality to represent the number of DVDs they sold. Solve the inequality and interpret your solution.

Examples 2–3 Solve each inequality. Graph the solution on a number line.

2. $30 > \frac{1}{2}n$ **3.** $-\frac{3}{4}r \leq -6$ **4.** $-\frac{c}{6} \geq 7$ **5.** $\frac{h}{2} < -5$

6. $9t > 108$ **7.** $-84 < 7v$ **8.** $-28 \leq -6x$ **9.** $40 \geq -5z$

Example 1 Define a variable, write an inequality, and solve each problem. Then interpret your solution.

10. CELL PHONE PLAN Mario purchases a prepaid phone plan for $50 at $0.13 per minute. How many minutes can Mario talk on this plan?

11. FINANCIAL LITERACY Rodrigo needs at least $560 to pay for his spring break expenses, and he is saving $25 from each of his weekly paychecks. How long will it be before he can pay for his trip?

**Examples 2–3** Solve each inequality. Graph the solution on a number line.

12. $\frac{1}{4}m \leq -17$ **13.** $\frac{1}{2}a < 20$ **14.** $-11 > -\frac{c}{11}$

15. $-2 \geq -\frac{d}{34}$ **16.** $-10 \leq \frac{x}{-2}$ **17.** $-72 < \frac{f}{-6}$

18. $\frac{2}{3}h > 14$ **19.** $-\frac{3}{4}j \geq 12$ **20.** $-\frac{1}{6}n \leq -18$

21. $6p \leq 96$ **22.** $4r < 64$ **23.** $32 > -2y$

24. $-26 < 26t$ **25.** $-6v > -72$ **26.** $-33 \geq -3z$

27. $4b \leq -3$ **28.** $-2d < 5$ **29.** $-7f > 5$

30. CHEERLEADING To remain on the cheerleading squad, Lakita must attend at least $\frac{3}{5}$ of the study table sessions offered. She attends 15 sessions. If Lakita met the requirements, what is the maximum number of study table sessions?

31. BRACELETS How many bracelets can Caitlin buy for herself and her friends if she wants to spend no more than $22?   **$4.75**

32. CCSS PRECISION The National Honor Society at Pleasantville High School wants to raise at least $500 for a local charity. Each student earns $0.50 for every quarter of a mile walked in a walk-a-thon. How many miles will the students need to walk?

33. MUSEUM The American history classes are planning a trip to a local museum. Admission is $8 per person. Determine how many people can go for $260.

34. GASOLINE If gasoline costs $3.15 per gallon, how many gallons of gasoline, to the nearest tenth, can Jan buy for $24?

Brand X Pictures/PunchStock

Match each inequality to the graph of its solution.

35. $-\frac{2}{3}h \leq 9$ **36.** $25j \geq 8$ **37.** $3.6p < -4.5$ **38.** $2.3 < -5t$

a.

b.

c.

d.
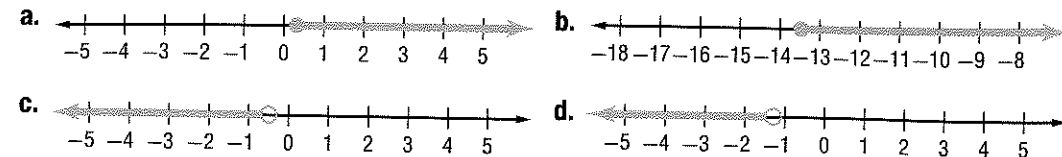
39. CANDY Fewer than 42 employees at a factory stated that they preferred fudge over fruit candy. This is about two thirds of the employees. How many employees are there?

40. TRAVEL A certain travel agency employs more than 275 people at all of its branches. Approximately three fifths of all the people are employed at the west branch. How many people work at the west branch?

41. MULTIPLE REPRESENTATIONS The equation for the volume of a pyramid is $\frac{1}{3}$ the area of the base times the height.

a. Geometric Draw a pyramid with a square base b cm long and a height of h cm.

b. Numerical Suppose the pyramid has a volume of 72 cm^3. Write an equation to find the height.

c. Tabular Create a table showing the value of h when $b = 1, 3, 6, 9,$ and 12.

d. Numerical Write an inequality for the possible lengths of b such that $b < h$. Write an inequality for the possible lengths of h such that $b > h$.
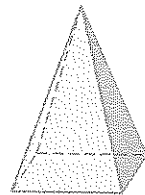
H.O.T. Problems Use Higher-Order Thinking Skills

42. ERROR ANALYSIS Taro and Jamie are solving $6d \geq -84$. Is either of them correct? Explain your reasoning.

Taro	Jamie
$6d \geq -84$	$6d \geq -84$
$\frac{6d}{6} \geq \frac{-84}{6}$	$\frac{6d}{6} \leq \frac{-84}{6}$
$d \geq -14$	$d \leq -14$

43. CHALLENGE Solve each inequality for x. Assume that $a > 0$.

a. $-ax < 5$ **b.** $\frac{1}{a}x \geq 8$ **c.** $-6 \geq ax$

44. STRUCTURE Determine whether $x^2 > 1$ and $x > 1$ are equivalent. Explain.

45. REASONING Explain whether the statement *If $a > b$, then $\frac{1}{a} > \frac{1}{b}$* is *sometimes, always,* or *never* true.

**46. OPEN ENDED** Create a real-world situation to represent the inequality $-\frac{5}{8} \geq x$.

**47. WRITING IN MATH** How are solving linear inequalities and linear equations similar? different?

48. Juan's international calling card costs 9¢ for each minute. Which inequality can be used to find how long he can talk to a friend if he does not want to spend more than \$2.50 on the call?

A $0.09 \geq 2.50m$

B $0.09 \leq 2.50m$

C $0.09m \geq 2.50$

D $0.09m \leq 2.50$

49. SHORT RESPONSE Find the value of x.
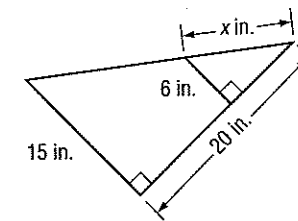
50. What is the greatest rate of decrease of this function?
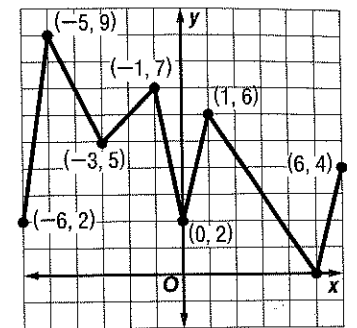
F -5 H -2

G -3 J 1

51. What is the value of x if $4x - 3 = -2x$?

A -2 C $\frac{1}{2}$

B $-\frac{1}{2}$ D 2

Spiral Review

**Solve each inequality. Check your solution, and then graph it on a number line.** (Lesson 5-1)

52. $-8 + 4a < 6a$ **53.** $2y + 11 \geq -24y$ **54.** $7 - 2b > 12b$

Find the inverse of each function. (Lesson 4-7)

55. $f(x) = -6x + 18$ **56.** $f(x) = \frac{3}{7}x + 9$ **57.** $f(x) = 4x - 5$

58. HOME DECOR Pam is having blinds installed at her home. The cost c of installation for any number of blinds b can be described by $c = 25 + 6.5b$. Graph the equation and determine how much it would cost if Pam has 8 blinds installed. (Lesson 3-1)

59. RESCUE A boater radioed for a helicopter to pick up a sick crew member. At that time, the boat and the helicopter were at the positions shown. How long will it take for the helicopter to reach the boat? (Lesson 2-9)
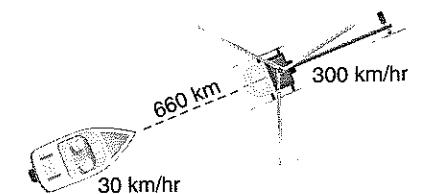
Solve each equation. (Lesson 2-5)

60. $|x + 3| = 10$ **61.** $|2x - 8| = 6$ **62.** $|3x + 1| = -2$

Skills Review

Solve each equation.

63. $4y + 11 = 19$ **64.** $2x - 7 = 9 + 4x$ **65.** $\frac{1}{4} + 2x = 4x - 8$

66. $\frac{1}{3}(6w - 3) = 3w + 12$ **67.** $\frac{7r + 5}{2} = 13$ **68.** $\frac{1}{2}a = \frac{a - 3}{4}$