Solving Compound Inequalities

:·Then	:·Now	:·Why?
● You solved absolute value equations with two cases.	**1** Solve compound inequalities containing the word *and*, and graph their solution set. **2** Solve compound inequalities containing the word *or*, and graph their solution set.	● To ride certain roller coasters, you must be at least 52 inches tall, and your height cannot exceed 72 inches. If *h* represents the height of a rider, we can write two inequalities to represent this.

at least 52 inches   cannot exceed 72 inches

$$h \geq 52 \qquad\qquad h \leq 72$$

The inequalities $h \geq 52$ and $h \leq 72$ can be combined and written without using *and* as $52 \leq h \leq 72$.

1 Inequalities Containing *and* When considered together, two inequalities such as $h \geq 52$ and $h \leq 72$ form a **compound inequality**. A compound inequality containing *and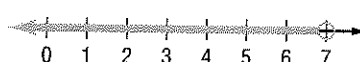* is only true if both inequalities are true. Its graph is where the graphs of the two inequalities overlap. This is called the **intersection** of the two graphs.

The intersection can be found by graphing each inequality and then determining where the graphs intersect.
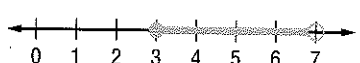
$$x \geq 3$$
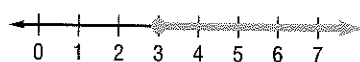
$$x < 7$$

$$x \geq 3 \text{ and } x < 7$$
$$3 \leq x < 7$$

The statement $3 \leq x < 7$ can be read as *x is greater than or equal to 3 and less than 7* or *x is between 3 and 7 including 3.*

Example 1 Solve and Graph an Intersection

Solve $-2 \leq x - 3 < 4$. Then graph the solution set.

First, express $-2 \leq x - 3 < 4$ using *and*. Then solve each inequality.

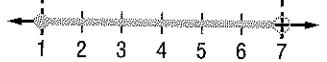|		
$-2 \leq x - 3$	**and**	$x - 3 < 4$	Write the inequalities.
$-2 + 3 \leq x - 3 + 3$		$x - 3 + 3 < 4 + 3$	Add 3 to each side.
$1 \leq x$		$x < 7$	Simplify.

The solution set is $\{x \mid 1 \leq x < 7\}$. Now graph the solution set.

Graph $1 \leq x$ or $x \geq 1$.

Graph $x < 7$.

Find the intersection of the graphs.

GuidedPractice

Solve each compound inequality. Then graph the solution set.

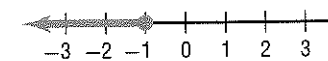**1A.** $y - 3 \geq -11$ and $y - 3 \leq -8$ **1B.** $6 \leq r + 7 < 10$

2 Inequalities Containing *or* Another type of compound inequality contains the word *or*. A compound inequality containing *or* is true if at least one of the inequalities is true. Its graph is the **union** of the graphs of two inequalities.
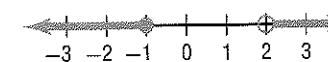
$$x > 2$$
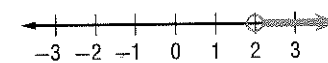
$$x \leq -1$$

$$x > 2 \text{ or } x \leq -1$$

When solving problems involving inequalities, *within* is meant to be inclusive, so use $\geq$ or $\leq$. *Between* is meant to be exclusive, so use $<$ or $>$.

Real-World Example 2 Write and Graph a Compound Inequality

SOUND The human ear can only detect sounds between the frequencies 20 Hertz and 20,000 Hertz. Write and graph a compound inequality that describes the frequency of sounds humans cannot hear.

The problem states that humans can hear the frequencies between 20 Hz and 20,000 Hz. We are asked to find the frequencies humans cannot hear.

Words	The frequency	is at most	20 Hertz	or	The frequency	is at least	20,000 Hertz.
Variable	Let *f* be the frequency.						
Inequality	*f*	$\leq$	20	or	*f*	$\geq$	20,000

Now, graph the solution set.
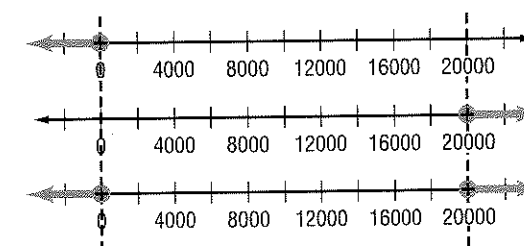
Graph $f < 20$.

Graph $f > 20,000$.

Find the union.

Notice that the graphs do not intersect. Humans cannot hear sounds at a frequency less than 20 Hertz or greater than 20,000 Hertz. The compound inequality is $\{f \mid f < 20 \text{ or } f > 20,000\}$.

GuidedPractice

2. MANUFACTURING A company is manufacturing an action figure that must be at least 11.2 centimeters and at most 11.4 centimeters tall. Write and graph a compound inequality that describes how tall the action figure can be.

Example 3 Solve and Graph a Union

Solve $-2m + 7 \leq 13$ or $5m + 12 > 37$. Then graph the solution set.

$-2m + 7 \leq 13$	**or**	$5m + 12 > 37$	
$-2m + 7 - 7 \leq 13 - 7$	Subtract.	$5m + 12 - 12 > 37 - 12$	
$-2m \leq 6$	Simplify.	$5m > 25$	
$\dfrac{-2m}{-2} \geq \dfrac{6}{-2}$	Divide.	$\dfrac{5m}{5} > \dfrac{25}{5}$	
$m \geq -3$	Simplify.	$m > 5$	
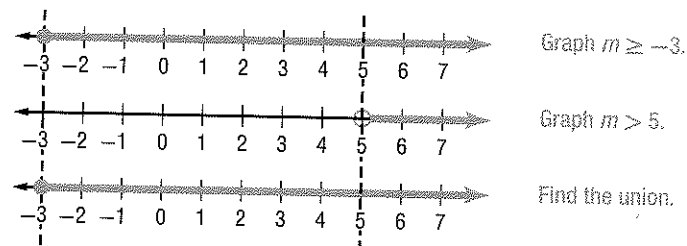
Graph $m \geq -3$.

Graph $m > 5$.

Find the union.

Notice that the graph of $m \geq -3$ contains every point in the graph of $m > 5$. So, the union is the graph of $m \geq -3$. The solution set is $\{m \mid m \geq -3\}$.

Guided Practice

Solve each compound inequality. Then graph the solution set.

3A. $a + 1 < 4$ or $a - 1 \geq 3$

3B. $x \leq 9$ or $2 + 4x < 10$

Check Your Understanding

⬤ = Step-by-Step Solutions begin on page R13.

Examples 1, 3 Solve each compound inequality. Then graph the solution set.

1. $4 \leq p - 8$ and $p - 14 \leq 2$

2. $r + 6 < -8$ or $r - 3 > -10$

3. $4a + 7 \geq 31$ or $a > 5$

4. $2 \leq g + 4 < 7$

Example 2

5. **CCSS SENSE-MAKING** The recommended air pressure for the tires of a mountain bike is at least 35 pounds per square inch (psi), but no more than 80 pounds per square inch. If a bike's tires have 24 pounds per square inch, what is the recommended range of air that should be put into the tires?

Practice and Problem Solving

Extra Practice is on page R5.

Examples 1, 3 Solve each compound inequality. Then graph the solution set.

6. $f - 6 < 5$ and $f - 4 \geq 2$

7. $n + 2 \leq -5$ and $n + 6 \geq -6$

8. $y - 1 \geq 7$ or $y + 3 < -1$

9. $t + 14 \geq 15$ or $t - 9 < -10$

10. $-5 < 3p + 7 \leq 22$

11. $-3 \leq 7c + 4 < 18$

12. $5h - 4 \geq 6$ and $7h + 11 < 32$

13. $22 \geq 4m - 2$ or $5 - 3m \leq -13$

14. $-4a + 13 \geq 29$ and $10 < 6a - 14$

15. $-y + 5 \geq 9$ or $3y + 4 < -5$

Practice and Problem Solving

Extra Practice is on page R5.

Example 2

16. **SPEED** The posted speed limit on an interstate highway is shown. Write an inequality that represents the sign. Graph the inequality.

SPEED LIMIT **70** MINIMUM **40**

17. **NUMBER THEORY** Find all sets of two consecutive positive odd integers with a sum that is at least 8 and less than 24.
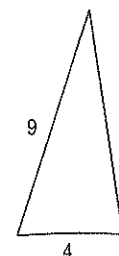
Write a compound inequality for each graph.

18.

19.

20.

21.

22.

23.
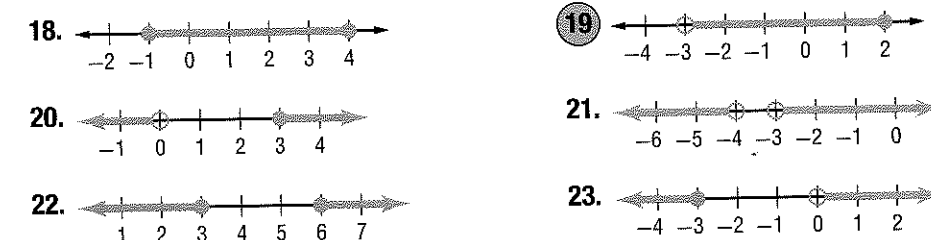
Solve each compound inequality. Then graph the solution set.

24. $3b + 2 < 5b - 6 \leq 2b + 9$

25. $-2a + 3 \geq 6a - 1 > 3a - 10$

26. $10m - 7 < 17m$ or $-6m > 36$

27. $5n - 1 < -16$ or $-3n - 1 < 8$

28. **COUPON** Juanita has a coupon for 10% off any digital camera at a local electronics store. She is looking at digital cameras that range in price from $100 to $250.

  a. How much are the cameras after the coupon is used?

  b. If the tax amount is 6.5%, how much should Juanita expect to spend?

Define a variable, write an inequality, and solve each problem. Then check your solution.

29. Eight less than a number is no more than 14 and no less than 5.

30. The sum of 3 times a number and 4 is between −8 and 10.

31. The product of −5 and a number is greater than 35 or less than 10.

32. One half a number is greater than 0 and less than or equal to 1.

33. **SNAKES** Most snakes live where the temperature ranges from 75°F to 90°F, inclusive. Write an inequality for temperatures where snakes will *not* thrive.

34. **FUNDRAISING** Yumas is selling gift cards to raise money for a class trip. He can earn prizes depending on how many cards he sells. So far, he has sold 34 cards. How many more does he need to sell to earn a prize in category 4?

Cards	Prize
1–15	1
16–30	2
31–45	3
46–60	4
+61	5

35. **TURTLES** Atlantic sea turtle eggs that incubate below 23°C or above 33°C rarely hatch. Write the temperature requirements in two ways: as a pair of simple inequalities, and as a compound inequality.

36. **CCSS STRUCTURE** The *Triangle Inequality Theorem* states that the sum of the measures of any two sides of a triangle is greater than the measure of the third side.

  a. Write and solve three inequalities to express the relationships among the measures of the sides of the triangle shown at the right.

  b. What are four possible lengths for the third side of the triangle?

  c. Write a compound inequality for the possible values of x.

37. HURRICANES The Saffir-Simpson Hurricane Scale rates hurricanes on a scale from 1 to 5 based on their wind speed.

Category	Wind Speed (mph)	Example (year)
1	74–95	Gaston (2004)
2	96–110	Frances (2004)
3	111–130	Ivan (2004)
4	131–155	Charley (2004)
5	> 155	Andrew (1992)

a. Write a compound inequality for the wind speeds of a category 3 and a category 4 hurricane.

b. What is the intersection of the two graphs of the inequalities you found in part a?

38. MULTIPLE REPRESENTATIONS In this problem, you will investigate measurements. The **absolute error** of a measurement is equal to one half the unit of measure. The **relative error** of a measure is the ratio of the absolute error to the expected measure.

a. **Tabular** Copy and complete the table.

Measure	Absolute Error	Relative Error
14.3 cm	$\frac{1}{2}(0.1) = 0.05$ cm	$\frac{\text{absolute error}}{\text{expected measure}} = \frac{0.05 \text{ cm}}{14.3 \text{ cm}}$ ≈ 0.0035 or 0.4%
1.85 cm		
61.2 cm		
237 cm		

b. **Analytical** You measured a length of 12.8 centimeters. Compute the absolute error and then write the range of possible measures.

c. **Logical** To what precision would you have to measure a length in centimeters to have an absolute error of less than 0.05 centimeter?

d. **Analytical** To find the relative error of an area or volume calculation, add the relative errors of each linear measure. If the measures of the sides of a rectangular box are 6.5 centimeters, 7.2 centimeters, and 10.25 centimeters, what is the relative error of the volume of the box?

H.O.T. Problems   Use Higher-Order Thinking Skills

39. ERROR ANALYSIS Chloe and Jonas are solving $3 < 2x - 5 < 7$. Is either of them correct? Explain your reasoning.

| Chloe	Jonas
$3 < 2x - 5 < 7$	$3 < 2x - 5 < 7$
$3 < 2x < 12$	$8 < 2x < 7$
$\frac{3}{2} < x < 6$	$4 < x < \frac{7}{2}$

40. PERSEVERANCE Solve each inequality for x. Assume a is constant and $a > 0$.

a. $-3 < ax + 1 \le 5$

b. $-\frac{1}{a}x + 6 < 1$ or $2 - ax > 8$

41. OPEN ENDED Create an example of a compound inequality containing *or* that has infinitely many solutions.

42. CHALLENGE Determine whether the following statement is *always, sometimes,* or *never* true. Explain. *The graph of a compound inequality that involves an or statement is bounded on the left and right by two values of x.*

43. WRITING IN MATH Give an example of a compound inequality you might encounter at an amusement park. Does the example represent an intersection or a union?

Standardized Test Practice

44. What is the solution set of the inequality $-7 < x + 2 < 4$?

A $\{x \mid -5 < x < 6\}$ C $\{x \mid -9 < x < 2\}$

B $\{x \mid -5 < x < 2\}$ D $\{x \mid -9 < x < 6\}$

45. GEOMETRY What is the surface area of the rectangular solid?

F 249.6 cm^2

G 278.4 cm^2

H 313.6 cm^2

J 371.2 cm^2

8 cm, 5.8 cm, 8 cm
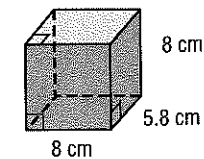
46. GRIDDED RESPONSE What is the next term in the sequence?

$\frac{13}{2}, \frac{18}{5}, \frac{23}{8}, \frac{28}{11}, \frac{33}{14}, \ldots$

47. After paying a $15 membership fee, members of a video club can rent movies for $2. Nonmembers can rent movies for $4. What is the least number of movies which must be rented for it to be less expensive for members?

A 9   C 7

B 8   D 6

Spiral Review

48. BABYSITTING Marilyn earns $150 per month delivering newspapers plus $7 an hour babysitting. If she wants to earn at least $300 this month, how many hours will she have to babysit? (Lesson 5-3)

49. MAGAZINES Carlos has earned more than $260 selling magazine subscriptions. Each subscription was sold for $12. How many did Carlos sell? (Lesson 5-2)

50. PUNCH Raquel is mixing lemon-lime soda and a fruit juice blend that is 45% juice. If she uses 3 quarts of soda, how many quarts of fruit juice must be added to produce punch that is 30% juice? (Lesson 2-9)

Solve each proportion. If necessary, round to the nearest hundredth. (Lesson 2-6)

51. $\frac{14}{x} = \frac{20}{8}$ **52.** $\frac{0.47}{6} = \frac{1.41}{m}$ **53.** $\frac{16}{7} = \frac{9}{b}$

54. $\frac{2+y}{5} = \frac{10}{3}$ **55.** $\frac{8}{9} = \frac{2r-3}{4}$ **56.** $\frac{6-2y}{8} = \frac{2}{18}$

Determine whether each relation is a function. Explain. (Lesson 1-7)

57.

Domain	2	6	10	7
Range	5	0	5	0

58.

Domain	-5	2	-3	2
Range	-10	-7	-5	-3

59. $\{(-4, 11), (-2, 7), (1, 3), (-4, -1)\}$

60. $\{(2, 7), (5, -3), (7, 6), (10, 7)\}$

Evaluate each expression. (Lesson 1-2)

61. $5 + (4 - 2^2)$ **62.** $\frac{3}{8}[8 \div (7 - 4)]$ **63.** $2(4 \cdot 9 - 3) + 5 \cdot \frac{1}{5}$

Skills Review

Solve each equation.

64. $4p - 2 = -6$ **65.** $18 = 5p + 3$ **66.** $9 = 1 + \frac{m}{7}$

67. $1.5a - 8 = 11$ **68.** $20 = -4c - 8$ **69.** $\frac{b+4}{-2} = -17$

70. $\frac{n-3}{8} = 20$ **71.** $6y - 16 = 44$ **72.** $130 = 11k + 9$