LESSON 6-4

Elimination Using Multiplication

Then

- You used elimination with addition and subtraction to solve systems of equations.

Now

1. Solve systems of equations by using elimination with multiplication.
2. Solve real-world problems involving systems of equations.

Why?

- The table shows the number of cars at Scott's Auto Repair Shop for each type of service.

Item	Repairs	Maintenance
body	3	4
engine	2	2

The manager has allotted 1110 minutes for body work and 570 minutes for engine work. The system $3r + 4m = 1110$ and $2r + 2m = 570$ can be used to find the average time for each service.

Exactostock /SuperStock

Common Core State Standards

Content Standards

A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Mathematical Practices

1 Make sense of problems and persevere in solving them.

1 Elimination Using Multiplication

In the system above, neither variable can be eliminated by adding or subtracting. You can use multiplication to solve.

KeyConcept Solving by Elimination

Step 1 Multiply at least one equation by a constant to get two equations that contain opposite terms.

Step 2 Add the equations, eliminating one variable. Then solve the equation.

Step 3 Substitute the value from Step 2 into one of the equations and solve for the other variable. Write the solution as an ordered pair.

PT

Example 1 Multiply One Equation to Eliminate a Variable

Use elimination to solve the system of equations.

$5x + 6y = -8$
$2x + 3y = -5$

Steps 1 and 2

$$\begin{array}{l} 5x + 6y = -8 \\ 2x + 3y = -5 \end{array} \xrightarrow{\text{Multiply each term by } -2} \begin{array}{rl} & 5x + 6y = -8 \\ (+) & -4x - 6y = 10 \quad \text{Add.} \\ \hline & x = 2 \quad y \text{ is eliminated.} \end{array}$$

Step 3

$2x + 3y = -5$	Second equation
$2(2) + 3y = -5$	Substitution, $x = 2$
$4 + 3y = -5$	Simplify.
$3y = -9$	Subtract 4 from each side and simplify.
$y = -3$	Divide each side by 3 and simplify.

The solution is $(2, -3)$.

GuidedPractice

1A. $6x - 2y = 10$
$3x - 7y = -19$

1B. $9r + q = 13$
$3r + 2q = -4$

Sometimes you have to multiply each equation by a different number in order to solve the system.

Example 2 Multiply Both Equations to Eliminate a Variable

Use elimination to solve the system of equations.

$4x + 2y = 8$
$3x + 3y = 9$

Method 1 Eliminate x.

$4x + 2y = 8$ (Multiply by 3.)
$3x + 3y = 9$ (Multiply by −4.)

$$\begin{array}{rl} 12x + 6y = & 24 \\ (+)\ -12x - 12y = & -36 \\ \hline -6y = & -12 \end{array}$$

Add equations. x is eliminated.

$$\frac{-6y}{-6} = \frac{-12}{-6}$$

Divide each side by −6.

$$y = 2$$

Simplify.

Now substitute 2 for y in either equation to find the value of x.

$3x + 3y = 9$ Second equation
$3x + 3(2) = 9$ Substitute 2 for y.
$3x + 6 = 9$ Simplify.
$3x = 3$ Subtract 6 from each side and simplify.
$\frac{3x}{3} = \frac{3}{3}$ Divide each side by 3.
$x = 1$ The solution is (1, 2).

Method 2 Eliminate y.

$4x + 2y = 8$ (Multiply by 3.)
$3x + 3y = 9$ (Multiply by −2.)

$$\begin{array}{rl} 12x + 6y = & 24 \\ (+)\ -6x - 6y = & -18 \\ \hline 6x = & 6 \end{array}$$

Add equations. y is eliminated.

$$\frac{6x}{6} = \frac{6}{6}$$

Divide each side by 6.

$$x = 1$$

Simplify.

Now substitute 1 for x in either equation to find the value of y.

$3x + 3y = 9$ Second equation
$3(1) + 3y = 9$ Substitute 1 for x.
$3 + 3y = 9$ Simplify.
$3y = 6$ Subtract 3 from each side and simplify.
$\frac{3y}{3} = \frac{6}{3}$ Divide each side by 3.
$y = 2$ Simplify.

The solution is (1, 2), which matches the result obtained with Method 1.

CHECK Substitute 1 for x and 2 for y in the first equation.

$4x + 2y = 8$ Original equation
$4(1) + 2(2) \stackrel{?}{=} 8$ Substitute (1, 2) for (x, y).
$4 + 4 \stackrel{?}{=} 8$ Multiply.
$8 = 8$ ✓ Add.

StudyTip
Choosing a Variable to Eliminate Unless the problem is asking for the value of a specific variable, you may use multiplication to eliminate either variable.

Math HistoryLink
Leonardo Pisano
(1170–1250) Leonardo Pisano is better known by his nickname *Fibonacci*. His book introduced the Hindu-Arabic place-valued decimal system. Systems of linear equations are studied in this work.

GuidedPractice

2A. $5x - 3y = 6$
$2x + 5y = -10$

2B. $6a + 2b = 2$
$4a + 3b = 8$

2 Solve Real-World Problems Sometimes it is necessary to use multiplication before elimination in real-world problem solving too.

Real-World Example 3 Solve a System of Equations

FLIGHT A personal aircraft traveling with the wind flies 520 miles in 4 hours. On the return trip, the airplane takes 5 hours to travel the same distance. Find the speed of the airplane if the air is still.

You are asked to find the speed of the airplane in still air.

Let a = the rate of the airplane if the air is still.
Let w = the rate of the wind.

	r	t	d	$r \cdot t = d$
With the Wind	$a + w$	4	520	$(a + w)4 = 520$
Against the Wind	$a - w$	5	520	$(a - w)5 = 520$

So, our two equations are $4a + 4w = 520$ and $5a - 5w = 520$.

$4a + 4w = 520$ (Multiply by 5.)
$5a - 5w = 520$ (Multiply by 4.)

$$\begin{array}{rl} 20a + 20w = & 2600 \\ (+)\ 20a - 20w = & 2080 \\ \hline 40a = & 4680 \end{array}$$

w is eliminated.

$$\frac{40a}{40} = \frac{4680}{40}$$

Divide each side by 40.

$$a = 117$$

Simplify.

The rate of the airplane in still air is 117 miles per hour.

GuidedPractice

3. CANOEING A canoeist travels 4 miles downstream in 1 hour. The return trip takes the canoeist 1.5 hours. Find the rate of the boat in still water.

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Examples 1–2 **Use elimination to solve each system of equations.**

1. $2x - y = 4$
$7x + 3y = 27$

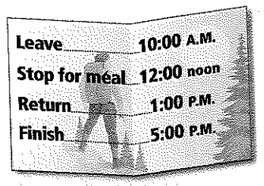
2. $2x + 7y = 1$
$x + 5y = 2$

3 $4x + 2y = -14$
$5x + 3y = -17$

4. $9a - 2b = -8$
$-7a + 3b = 12$

Example 3

5. CCSS SENSE-MAKING A kayaking group with a guide travels 16 miles downstream, stops for a meal, and then travels 16 miles upstream. The speed of the current remains constant throughout the trip. Find the speed of the kayak in still water.



6. PODCASTS Steve subscribed to 10 podcasts for a total of 340 minutes. He used his two favorite tags, Hobbies and Recreation and Soliloquies. Each of the Hobbies and Recreation episodes lasted about 32 minutes. Each Soliloquies episode lasted 42 minutes. To how many of each tag did Steve subscribe?

Practice and Problem Solving

Extra Practice is on page R6.

Examples 1–2 **Use elimination to solve each system of equations.**

7. $x + y = 2$
$-3x + 4y = 15$

8. $x - y = -8$
$7x + 5y = 16$

9. $x + 5y = 17$
$-4x + 3y = 24$

10. $6x + y = -39$
$3x + 2y = -15$

11. $2x + 5y = 11$
$4x + 3y = 1$

12. $3x - 3y = -6$
$-5x + 6y = 12$

13. $3x + 4y = 29$
$6x + 5y = 43$

14. $8x + 3y = 4$
$-7x + 5y = -34$

15. $8x + 3y = -7$
$7x + 2y = -3$

16. $4x + 7y = -80$
$3x + 5y = -58$

17. $12x - 3y = -3$
$6x + y = 1$

18. $-4x + 2y = 0$
$10x + 3y = 8$

Example 3 **19. NUMBER THEORY** Seven times a number plus three times another number equals negative one. The sum of the two numbers is negative three. What are the numbers?

20. FOOTBALL A field goal is 3 points and the extra point after a touchdown is 1 point. In a recent post-season, Adam Vinatieri of the Indianapolis Colts made a total of 21 field goals and extra point kicks for 49 points. Find the number of field goals and extra points that he made.

Use elimination to solve each system of equations.

21. $2.2x + 3y = 15.25$
$4.6x + 2.1y = 18.325$

22. $-0.4x + 0.25y = -2.175$
$2x + y = 7.5$

23. $\frac{1}{4}x + 4y = 2\frac{3}{4}$
$3x + \frac{1}{2}y = 9\frac{1}{4}$

24. $\frac{2}{5}x + 6y = 24\frac{1}{5}$
$3x + \frac{1}{2}y = 3\frac{1}{2}$

25. CCSS MODELING The TOBOR robot saves 120 minutes of a nurse's time n and 180 minutes of support staff time s each day. Another robot that aids stroke patients' limbs is estimated to save 90 minutes of nursing time and 120 minutes of support staff time each day.

a. To be cost effective, TOBOR must save a total of 1500 minutes per day. Write an equation that represents this relationship.

b. To make the stroke assistant cost effective, it must save a total of 1050 minutes per day. Write an equation that represents this relationship.

c. Solve the system of equations, and interpret the solution in the context of the situation.

26. GEOMETRY The graphs of $x + 2y = 6$ and $2x + y = 9$ contain two of the sides of a triangle. A vertex of the triangle is at the intersection of the graphs.

a. What are the coordinates of the vertex?

b. Draw the graph of the two lines. Identify the vertex of the triangle.

c. The line that forms the third side of the triangle is the line $x - y = -3$. Draw this line on the previous graph.

d. Name the other two vertices of the triangle.

27. ENTERTAINMENT At an entertainment center, two groups of people bought batting tokens and miniature golf games, as shown in the table.

Group	Number of Batting Tokens	Number of Miniature Golf Games	Total Cost
A	16	3	$30
B	22	5	$43

a. Define the variables, and write a system of linear equations from this situation.

b. Solve the system of equations, and explain what the solution represents.

28. TESTS Mrs. Henderson discovered that she had accidentally reversed the digits of a test score and did not give a student 36 points. Mrs. Henderson told the student that the sum of the digits was 14 and agreed to give the student his correct score plus extra credit if he could determine his actual score. What was his correct score?

H.O.T. Problems Use Higher-Order Thinking Skills

29. REASONING Explain how you could recognize a system of linear equations with infinitely many solutions.

30. CCSS CRITIQUE Jason and Daniela are solving a system of equations. Is either of them correct? Explain your reasoning.

Jason

$2r + 7t = 11$
$r - 9t = -7$
$2r + 7t = 11$
$(-)\ 2r - 18t = -14$
$25t = 25$
$t = 1$
$2r + 7t = 11$
$2r + 7(1) = 11$
$2r + 7 = 11$
$2r = 4$
$\frac{2r}{2} = \frac{4}{2}$
$r = 2$

The solution is (2, 1).

Daniela

$2r + 7t = 11$
$(-)\ r - 9t = -7$
$r = 18$
$2r + 7t = 11$
$2(18) + 7t = 11$
$36 + 7t = 11$
$7t = -25$
$\frac{7t}{7} = -\frac{25}{7}$
$t = -3.6$

The solution is (18, -3.6).

31. OPEN ENDED Write a system of equations that can be solved by multiplying one equation by -3 and then adding the two equations together.

32. CHALLENGE The solution of the system $4x + 5y = 2$ and $6x - 2y = b$ is $(3, a)$. Find the values of a and b. Discuss the steps that you used.

33. WRITING IN MATH Why is substitution sometimes more helpful than elimination, and vice versa?

Standardized Test Practice

34. What is the solution of this system of equations?

$2x - 3y = -9$
$-x + 3y = 6$

A (3, 3) **C** (−3, 1)
B (−3, 3) **D** (1, −3)

35. A buffet has one price for adults and another for children. The Taylor family has two adults and three children, and their bill was $40.50. The Wong family has three adults and one child. Their bill was $38. Which system of equations could be used to determine the price for an adult and for a child?

F $x + y = 40.50$
$x + y = 38$

G $2x + 3y = 40.50$
$3x + y = 38$

H $2x + 3y = 40.50$
$x + 3y = 38$

J $2x + 2y = 40.50$
$3x + y = 38$

36. SHORT RESPONSE A customer at the paint store has ordered 3 gallons of ivy green paint. Melissa mixes the paint in a ratio of 3 parts blue to one part yellow. How many quarts of blue paint does she use?

37. PROBABILITY The table shows the results of a number cube being rolled. What is the experimental probability of rolling a 3?

Outcome	Frequency
1	4
2	8
3	2
4	0
5	5
6	1

A $\frac{2}{3}$ **B** $\frac{1}{3}$ **C** 0.2 **D** 0.1

Spiral Review

Use elimination to solve each system of equations. (Lesson 6-3)

38. $f + g = -3$
$f - g = 1$

39. $6g + h = -7$
$6g + 3h = -9$

40. $5j + 3k = -9$
$3j + 3k = -3$

41. $2x - 4z = 6$
$x - 4z = -3$

42. $-5c - 3v = 9$
$5c + 2v = -6$

43. $4b - 6n = -36$
$3b - 6n = -36$

44. JOBS Brandy and Adriana work at an after-school child care center. Together they cared for 32 children this week. Brandy cared for 0.6 times as many children as Adriana. How many children did each girl care for? (Lesson 6-2)

Solve each inequality. Then graph the solution set. (Lesson 5-5)

45. $|m - 5| \leq 8$ **46.** $|q + 11| < 5$ **47.** $|2w + 9| > 11$ **48.** $|2r + 1| \geq 9$

Skills Review

Translate each sentence into a formula.

49. The area A of a triangle equals one half times the base b times the height h.

50. The circumference C of a circle equals the product of 2, π, and the radius r.

51. The volume V of a rectangular box is the length ℓ times the width w multiplied by the height h.

52. The volume of a cylinder V is the same as the product of π and the radius r to the second power multiplied by the height h.

53. The area of a circle A equals the product of π and the radius r squared.

54. Acceleration A equals the increase in speed s divided by time t in seconds.