

6-3 Elimination Using Addition and Subtraction

Then **Now** **Why?**

- You solved systems of equations by using substitution.
- 1** Solve systems of equations by using elimination with addition.
- 2** Solve systems of equations by using elimination with subtraction.
- In Chicago, Illinois, there are two more months a when the mean high temperature is below 70°F than there are months b when it is above 70°F . The system of equations, $a + b = 12$ and $a - b = 2$, represents this situation.



New Vocabulary
elimination

Common Core State Standards
Content Standards
A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
Mathematical Practices
7 Look for and make use of structure.

1 Elimination Using Addition If you add these equations, the variable b will be eliminated. Using addition or subtraction to solve a system is called **elimination**.

KeyConcept Solving by Elimination

- Step 1** Write the system so like terms with the same or opposite coefficients are aligned.
- Step 2** Add or subtract the equations, eliminating one variable. Then solve the equation.
- Step 3** Substitute the value from Step 2 into one of the equations and solve for the other variable. Write the solution as an ordered pair.

Example 1 Elimination Using Addition

Use elimination to solve the system of equations.

$$\begin{aligned} 4x + 6y &= 32 \\ 3x - 6y &= 3 \end{aligned} \quad \leftarrow \text{Step 1 } 6y \text{ and } -6y \text{ have opposite coefficients.}$$

Step 2 Add the equations.

$$\begin{aligned} 4x + 6y &= 32 \\ (+) 3x - 6y &= 3 \\ \hline 7x &= 35 && \text{The variable } y \text{ is eliminated.} \\ \frac{7x}{7} &= \frac{35}{7} && \text{Divide each side by 7.} \\ x &= 5 && \text{Simplify.} \end{aligned}$$

Step 3 Substitute 5 for x in either equation to find the value of y .

$$\begin{aligned} 4x + 6y &= 32 && \text{First equation} \\ 4(5) + 6y &= 32 && \text{Replace } x \text{ with 5.} \\ 20 + 6y &= 32 && \text{Multiply.} \\ 20 + 6y - 20 &= 32 - 20 && \text{Subtract 20 from each side.} \\ 6y &= 12 && \text{Simplify.} \\ \frac{6y}{6} &= \frac{12}{6} && \text{Divide each side by 6.} \\ y &= 2 && \text{Simplify.} \end{aligned}$$

The solution is $(5, 2)$.



StudyTip

Coefficients When the coefficients of a variable are the same, subtracting the equations will eliminate the variable. When the coefficients are opposites, adding the equations will eliminate the variable.

Problem-SolvingTip

CCSS Perseverance
Checking your answers in both equations of a system helps ensure there are no calculation errors.

GuidedPractice

1A. $-4x + 3y = -3$
 $4x - 5y = 5$

1B. $4y + 3x = 22$
 $3x - 4y = 14$

We can use elimination to find specific numbers that are described as being related to each other.

Example 2 Write and Solve a System of Equations

Negative three times one number plus five times another number is -11 .
Three times the first number plus seven times the other number is -1 .
Find the numbers.

Negative three times one number	plus	five times another number	is	-11 .
$-3x$	+	$5y$	=	-11
Three times the first number	plus	seven times the other number	is	-1 .
$3x$	+	$7y$	=	-1

Steps 1 and 2 Write the equations vertically and add.

$$\begin{aligned} -3x + 5y &= -11 \\ (+) 3x + 7y &= -1 \\ \hline 12y &= -12 && \text{The variable } x \text{ is eliminated.} \\ \frac{12y}{12} &= \frac{-12}{12} && \text{Divide each side by 12.} \\ y &= -1 && \text{Simplify.} \end{aligned}$$

Step 3 Substitute -1 for y in either equation to find the value of x .

$$\begin{aligned} 3x + 7y &= -1 && \text{Second equation} \\ 3x + 7(-1) &= -1 && \text{Replace } y \text{ with } -1. \\ 3x + (-7) &= -1 && \text{Simplify.} \\ 3x + (-7) + 7 &= -1 + 7 && \text{Add 7 to each side.} \\ 3x &= 6 && \text{Simplify.} \\ \frac{3x}{3} &= \frac{6}{3} && \text{Divide each side by 3.} \\ x &= 2 && \text{Simplify.} \end{aligned}$$

The numbers are 2 and -1 .

CHECK	$-3x + 5y = -11$	First equation
	$-3(2) + 5(-1) \stackrel{?}{=} -11$	Substitute 2 for x and -1 for y .
	$-11 = -11 \checkmark$	Simplify.
	$3x + 7y = -1$	Second equation
	$3(2) + 7(-1) \stackrel{?}{=} -1$	Substitute 2 for x and -1 for y .
	$-1 = -1 \checkmark$	Simplify.

GuidedPractice

- 2.** The sum of two numbers is -10 . Negative three times the first number minus the second number equals 2. Find the numbers.

2 Elimination Using Subtraction

Sometimes we can eliminate a variable by subtracting one equation from another.

Standardized Test Example 3

Solve the system of equations.

$$\begin{aligned} 2t + 5r &= 6 \\ 9r + 2t &= 22 \end{aligned}$$

- A $(-7, 15)$ B $(7, \frac{8}{9})$ C $(4, -7)$ D $(4, -\frac{2}{5})$

Read the Test Item

Since both equations contain $2t$, use elimination by subtraction.

Solve the Test Item

Step 1 Subtract the equations.

$$\begin{array}{r} 5r + 2t = 6 \\ (-) 9r + 2t = 22 \\ \hline -4r = -16 \\ r = 4 \end{array}$$

Write the system so like terms are aligned.
The variable t is eliminated.
Simplify.

Step 2 Substitute 4 for r in either equation to find the value of t .

$$\begin{array}{r} 5r + 2t = 6 \\ 5(4) + 2t = 6 \\ 20 + 2t = 6 \\ 20 + 2t - 20 = 6 - 20 \\ 2t = -14 \\ t = -7 \end{array}$$

First equation
 $r = 4$
Simplify.
Subtract 20 from each side.
Simplify.
Simplify.

The solution is $(4, -7)$. The correct answer is C.

Guided Practice

3. Solve the system of equations.
- $$\begin{aligned} 8b + 3c &= 11 \\ 8b + 7c &= 7 \end{aligned}$$
- F $(1.5, -1)$ G $(1.75, -1)$ H $(1.75, 1)$ J $(1.5, 1)$

Real-World Example 4 Write and Solve a System of Equations

JOBS Cheryl and Jackie work at an ice cream shop. Cheryl earns \$8.50 per hour and Jackie earns \$7.50 per hour. During a typical week, Cheryl and Jackie earn \$299.50 together. One week, Jackie doubles her work hours, and the girls earn \$412. How many hours does each girl work during a typical week?

Understand You know how much Cheryl and Jackie each earn per hour and how much they earned together.

Plan Let c = Cheryl's hours and j = Jackie's hours.

Cheryl's pay	plus	Jackie's pay	equals	\$299.50.
$8.50c$	+	$7.50j$	=	299.50
Cheryl's pay	plus	Jackie's pay	equals	\$412.
$8.50c$	+	$7.50(2j)$	=	412

StudyTip

Another Method Instead of subtracting the equations, you could also multiply one equation by -1 and then add the equations.

Solve Subtract the equations to eliminate one of the variables. Then solve for the other variable.

$$\begin{array}{r} 8.50c + 7.50j = 299.50 \\ (-) 8.50c + 7.50(2j) = 412 \\ \hline 8.50c + 7.50j = 299.50 \\ (-) 8.50c + 15j = 412 \\ \hline -7.50j = -112.50 \\ \frac{-7.50j}{-7.50} = \frac{-112.50}{-7.50} \\ j = 15 \end{array}$$

Write the equations vertically.
Simplify.
Subtract. The variable c is eliminated.
Divide each side by -7.50 .
Simplify.

Now substitute 15 for j in either equation to find the value of c .

$$\begin{array}{r} 8.50c + 7.50j = 299.50 \\ 8.50c + 7.50(15) = 299.50 \\ 8.50c + 112.50 = 299.50 \\ 8.50c = 187 \\ c = 22 \end{array}$$

First equation
Substitute 15 for j .
Simplify.
Subtract 112.50 from each side.
Divide each side by 8.50.

Check Substitute both values into the other equation to see if the equation holds true. If $c = 22$ and $j = 15$, then $8.50(22) + 15(15)$ or 412.

Cheryl works 22 hours, while Jackie works 15 hours during a typical week.

Guided Practice

4. **PARTIES** Tamera and Adelina are throwing a birthday party for their friend. Tamera invited 5 fewer friends than Adelina. Together they invited 47 guests. How many guests did each girl invite?

Check Your Understanding

Step-by-Step Solutions begin on page R13.

Examples 1, 3 Use elimination to solve each system of equations.

- $5m - p = 7$
 $7m - p = 11$
- $8x + 5y = 38$
 $-8x + 2y = 4$
- $7f + 3g = -6$
 $7f - 2g = -31$
- $6a - 3b = 27$
 $2a - 3b = 11$

Example 2

Example 4

5. **CCSS REASONING** The sum of two numbers is 24. Five times the first number minus the second number is 12. What are the two numbers?

6. **RECYCLING** The recycling and reuse industry employs approximately 1,025,000 more workers than the waste management industry. Together they provide 1,275,000 jobs. How many jobs does each industry provide?

Real-WorldLink

The five most dangerous jobs for teens are: delivery and other driving jobs, working alone in cash-based businesses, traveling youth crews, cooking, and construction.

Source: National Consumers League

Practice and Problem Solving

Extra Practice is on page R6.

Examples 1, 3 Use elimination to solve each system of equations.

7. $-v + w = 7$
 $v + w = 1$
8. $y + z = 4$
 $y - z = 8$
9. $-4x + 5y = 17$
 $4x + 6y = -6$
10. $5m - 2p = 24$
 $3m + 2p = 24$
11. $a + 4b = -4$
 $a + 10b = -16$
12. $6r - 6t = 6$
 $3r - 6t = 15$
13. $6c - 9d = 111$
 $5c - 9d = 103$
14. $11f + 14g = 13$
 $11f + 10g = 25$
15. $9x + 6y = 78$
 $3x - 6y = -30$
16. $3j + 4k = 23.5$
 $8j - 4k = 4$
17. $-3x - 8y = -24$
 $3x - 5y = 4.5$
18. $6x - 2y = 1$
 $10x - 2y = 5$

Example 2

19. The sum of two numbers is 22, and their difference is 12. What are the numbers?
20. Find the two numbers with a sum of 41 and a difference of 9.
21. Three times a number minus another number is -3 . The sum of the numbers is 11. Find the numbers.

Example 4

22. A number minus twice another number is 4. Three times the first number plus two times the second number is 12. What are the numbers?
23. **TOURS** The Blackwells and Joneses are going to Hershey's Really Big 3D Show in Pennsylvania. Find the adult price and the children's price of the show.

Family	Number of Adults	Number of Children	Total Cost
Blackwell	2	5	\$31.65
Jones	2	3	\$23.75

Use elimination to solve each system of equations.

24. $4(x + 2y) = 8$
 $4x + 4y = 12$
25. $3x - 5y = 11$
 $5(x + y) = 5$
26. $4x + 3y = 6$
 $3x + 3y = 7$
27. $6x - 7y = -26$
 $6x + 5y = 10$
28. $\frac{1}{2}x + \frac{2}{3}y = 2\frac{3}{4}$
 $\frac{1}{4}x - \frac{2}{3}y = 6\frac{1}{4}$
29. $\frac{3}{5}x + \frac{1}{2}y = 8\frac{1}{3}$
 $-\frac{3}{5}x + \frac{3}{4}y = 8\frac{1}{3}$

30. **CCSS SENSE-MAKING** The total height of an office building b and the granite statue that stands on top of it g is 326.6 feet. The difference in heights between the building and the statue is 295.4 feet.

- a. How tall is the statue?
- b. How tall is the building?

31. **BIKE RACING** Professional Mountain Bike Racing currently has 66 teams. The number of non-U.S. teams is 30 more than the number of U.S. teams.

- a. Let x represent the number of non-U.S. teams and y represent the number of U.S. teams. Write a system of equations that represents the number of U.S. teams and non-U.S. teams.
- b. Use elimination to find the solution of the system of equations.
- c. Interpret the solution in the context of the situation.
- d. Graph the system of equations to check your solution.

32. **SHOPPING** Let x represent the number of years since 2004 and y represent the number of catalogs.

Catalogs	Number in 2004	Growth Rate (number per year)
online	7440	1293
print	3805	-1364

Source: MediaPost Publications

- a. Write a system of equations to represent this situation.
- b. Use elimination to find the solution to the system of equations.
- c. Analyze the solution in terms of the situation. Determine the reasonableness of the solution.

33. **MULTIPLE REPRESENTATIONS** Collect 9 pennies and 9 paper clips. For this game, you may use a maximum of 9 objects to create a certain required number of points. Each paper clip is worth 1 point and each penny is worth 3 points. Let p represent a penny and c represent a paper clip.

$$9 \text{ points} = \text{2 pennies} + \text{3 paper clips} = 2p + 3c$$

- a. **Concrete** You must have exactly 15 points using at least one of each piece. Compare your pattern to other students.
- b. **Analytical** Write and solve a system of equations to find the number of paper clips and pennies used.
- c. **Tabular** Make a table showing the number of paper clips used and the total number of points when the number of pennies is 0, 1, 2, 3, 4, or 5.
- d. **Verbal** Does the result in the table match the results in part b? Explain.

H.O.T. Problems Use Higher-Order Thinking Skills

34. **REASONING** Describe the solution of a system of equations if after you added two equations the result was $0 = 0$.
35. **REASONING** What is the solution of a system of equations if the sum of the equations is $0 = 2$?
36. **OPEN ENDED** Create a system of equations that can be solved by using addition to eliminate one variable. Formulate a general rule for creating such systems.
37. **CCSS STRUCTURE** The solution of a system of equations is $(-3, 2)$. One equation in the system is $x + 4y = 5$. Find a second equation for the system. Explain how you derived this equation.
38. **CHALLENGE** If a number is multiplied by 7, the result is 182. The sum of that number's two digits is 8. Define the variables and write the system of equations that you would use to find the number. Then solve the system and find the number.
39. **WRITING IN MATH** Describe when it would be most beneficial to use elimination to solve a system of equations.

Standardized Test Practice

40. SHORT RESPONSE Martina is on a train traveling at a speed of 188 mph between two cities 1128 miles apart. If the train has been traveling for an hour, how many more hours is her train ride?

41. GEOMETRY Ms. Miller wants to tile her rectangular kitchen floor. She knows the dimensions of the floor. Which formula should she use to find the area?

- A $A = lw$ C $P = 2\ell + 2w$
B $V = Bh$ D $c^2 = a^2 + b^2$

42. If the pattern continues, what is the 8th number in the sequence?

$$2, 3, \frac{9}{2}, \frac{27}{4}, \frac{81}{8}, \dots$$

- F $\frac{2187}{64}$ G $\frac{2245}{64}$ H $\frac{2281}{64}$ J $\frac{2445}{64}$

43. What is the solution of this system of equations?

$$\begin{aligned} x + 4y &= 1 \\ 2x - 3y &= -9 \end{aligned}$$

- A (2, -8) C no solution
B (-3, 1) D infinitely many solutions

Spiral Review

Use substitution to solve each system of equations. If the system does not have exactly one solution, state whether it has no solution or infinitely many solutions. (Lesson 6-2)

44. $y = 6x$
 $2x + 3y = 40$

45. $x = 3y$
 $2x + 3y = 45$

46. $x = 5y + 6$
 $x = 3y - 2$

47. $y = 3x + 2$
 $y = 4x - 1$

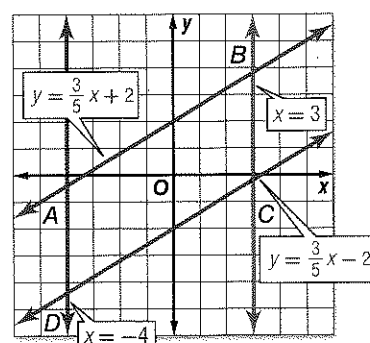
48. $3c = 4d + 2$
 $c = d - 1$

49. $z = v + 4$
 $2z - v = 6$

50. FINANCIAL LITERACY Gregorio and Javier each want to buy a bicycle. Gregorio has already saved \$35 and plans to save \$10 per week. Javier has \$26 and plans to save \$13 per week. (Lesson 6-1)

- a. In how many weeks will Gregorio and Javier have saved the same amount of money?
b. How much will each person have saved at that time?

51. GEOMETRY A *parallelogram* is a quadrilateral in which opposite sides are parallel. Determine whether ABCD is parallelogram. Explain your reasoning. (Lesson 4-4)



Solve each equation. Check your solution. (Lesson 2-2)

52. $6u = -48$

53. $75 = -15p$

54. $\frac{2}{3}a = 8$

55. $-\frac{3}{4}d = 15$

Skills Review

Simplify each expression. If not possible, write *simplified*.

56. $6q - 3 + 7q + 1$

57. $7w^2 - 9w + 4w^2$

58. $10(2 + r) + 3r$

59. $5y - 7(y + 5)$

