

6-5 Applying Systems of Linear Equations

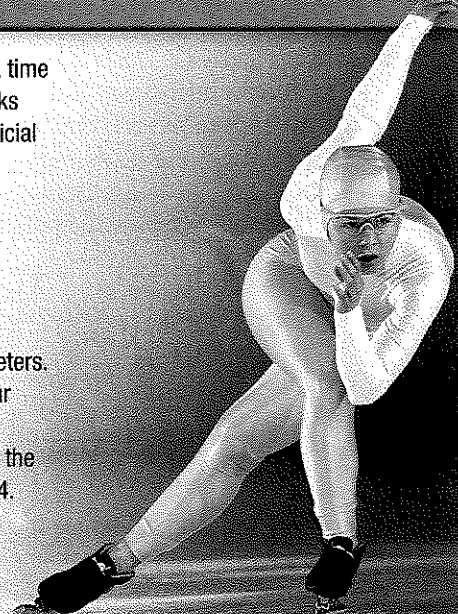
Then **Now** **Why?**

- You solved systems of equations by using substitution and elimination.
- 1** Determine the best method for solving systems of equations.
- 2** Apply systems of equations.
- In speed skating, competitors race two at a time on a double track. Indoor speed skating rinks have two track sizes for race events: an official track and a short track.

Speed Skating Tracks	
official track	x
short track	y

The total length of the two tracks is 511 meters. The official track is 44 meters less than four times the short track. The total length is represented by $x + y = 511$. The length of the official track is represented by $x = 4y - 44$.

You can solve the system of equations to find the length of each track.



Common Core State Standards

Content Standards

A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Mathematical Practices

- 2 Reason abstractly and quantitatively.
- 4 Model with mathematics.

1 Determine the Best Method You have learned five methods for solving systems of linear equations. The table summarizes the methods and the types of systems for which each method works best.

ConceptSummary Solving Systems of Equations

Method	The Best Time to Use
Graphing	To estimate solutions, since graphing usually does not give an exact solution.
Substitution	If one of the variables in either equation has a coefficient of 1 or -1 .
Elimination Using Addition	If one of the variables has opposite coefficients in the two equations.
Elimination Using Subtraction	If one of the variables has the same coefficient in the two equations.
Elimination Using Multiplication	If none of the coefficients are 1 or -1 and neither of the variables can be eliminated by simply adding or subtracting the equations.

Substitution and elimination are algebraic methods for solving systems of equations. An algebraic method is best for an exact solution. Graphing, with or without technology, is a good way to estimate a solution.

A system of equations can be solved using each method. To determine the best approach, analyze the coefficients of each term in each equation.

StudyTip

CCSS Reasoning The system of equations in Example 1 can also be solved by using elimination with multiplication. You can multiply the first equation by 2 and then add to eliminate the x -term.

Example 1 Choose the Best Method

Determine the best method to solve the system of equations. Then solve the system.

$$\begin{aligned} 4x - 4y &= 8 \\ -8x + y &= 19 \end{aligned}$$

Understand To determine the best method to solve the system of equations, look closely at the coefficients of each term.

Plan Neither the coefficients of x nor y are the same or additive inverses, so you cannot add or subtract to eliminate a variable. Since the coefficient of y in the second equation is 1, you can use substitution.

Solve First, solve the second equation for y .

$$-8x + y = 19 \quad \text{Second equation}$$

$$-8x + y + 8x = 19 + 8x \quad \text{Add } 8x \text{ to each side.}$$

$$y = 19 + 8x \quad \text{Simplify.}$$

Next, substitute $19 + 8x$ for y in the first equation.

$$4x - 4y = 8 \quad \text{First equation}$$

$$4x - 4(19 + 8x) = 8 \quad \text{Substitution}$$

$$4x - 76 - 32x = 8 \quad \text{Distributive Property}$$

$$-28x - 76 = 8 \quad \text{Simplify.}$$

$$-28x - 76 + 76 = 8 + 76 \quad \text{Add } 76 \text{ to each side.}$$

$$-28x = 84 \quad \text{Simplify.}$$

$$\frac{-28x}{-28} = \frac{84}{-28} \quad \text{Divide each side by } -28.$$

$$x = -3 \quad \text{Simplify.}$$

Last, substitute -3 for x in the second equation.

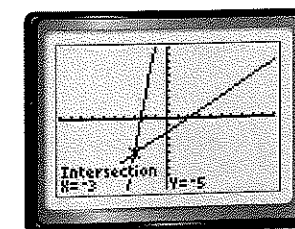
$$-8x + y = 19 \quad \text{Second equation}$$

$$-8(-3) + y = 19 \quad x = -3$$

$$y = -5 \quad \text{Simplify.}$$

The solution of the system of equations is $(-3, -5)$.

Check Use a graphing calculator to check your solution. If your algebraic solution is correct, then the graphs will intersect at $(-3, -5)$.



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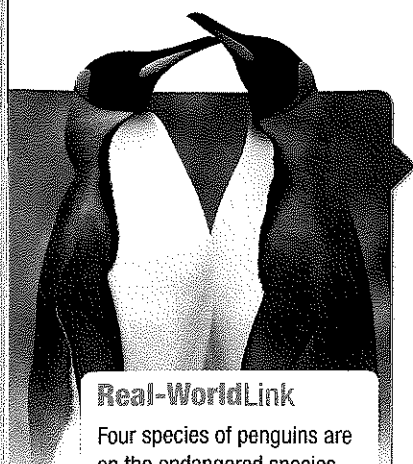
Guided Practice

1A. $\begin{aligned} 5x + 7y &= 2 \\ -2x + 7y &= 9 \end{aligned}$

1C. $\begin{aligned} x - y &= 9 \\ 7x + y &= 7 \end{aligned}$

1B. $\begin{aligned} 3x - 4y &= -10 \\ 5x + 8y &= -2 \end{aligned}$

1D. $\begin{aligned} 5x - y &= 17 \\ 3x + 2y &= 5 \end{aligned}$



Real-WorldLink

Four species of penguins are on the endangered species list. The colonies of penguins that live closest to human inhabitants are the most at risk for extinction.

Source: PBS

2 Apply Systems of Linear Equations When applying systems of linear equations to problems, it is important to analyze each solution in the context of the situation.

Real-World Example 2 Apply Systems of Linear Equations

PENGUINS Of the 17 species of penguins in the world, the largest species is the emperor penguin. One of the smallest is the Galapagos penguin. The total height of the two penguins is 169 centimeters. The emperor penguin is 22 centimeters more than twice the height of the Galapagos penguin. Find the height of each penguin.

The total height of the two species can be represented by $p + g = 169$, where p represents the height of the emperor penguin and g the height of the Galapagos penguin. Next write an equation to represent the height of the emperor penguin.

Words	The emperor penguin is 22 centimeters more than twice the height of the Galapagos penguin.				
Variables	Let p = the height of the emperor penguin and g = the height of the Galapagos penguin.				
Equation	p	$=$	22	$+$	$2g$

First rewrite the second equation.

$$\begin{aligned} p &= 22 + 2g && \text{Second equation} \\ p - 2g &= 22 && \text{Subtract } 2g \text{ from each side.} \end{aligned}$$

You can use elimination by subtraction to solve this system of equations.

$$\begin{aligned} p + g &= 169 && \text{First equation} \\ (-) p - 2g &= 22 && \text{Subtract the second equation.} \\ \hline 3g &= 147 && \text{Eliminate } p. \\ \frac{3g}{3} &= \frac{147}{3} && \text{Divide each side by 3.} \\ g &= 49 && \text{Simplify.} \end{aligned}$$

Next substitute 49 for g in one of the equations.

$$\begin{aligned} p &= 22 + 2g && \text{Second equation} \\ &= 22 + 2(49) && g = 49 \\ &= 120 && \text{Simplify.} \end{aligned}$$

The height of the emperor penguin is 120 centimeters, and the height of the Galapagos penguin is 49 centimeters.

Does the solution make sense in the context of the problem?

Check by verifying the given information. The penguins' heights added together would be $120 + 49$ or 169 centimeters and $22 + 2(49)$ is 120 centimeters.

Guided Practice

2. VOLUNTEERING Jared has volunteered 50 hours and plans to volunteer 3 hours in each coming week. Clementine is a new volunteer who plans to volunteer 5 hours each week. Write and solve a system of equations to find how long it will be before they will have volunteered the same number of hours.

Check Your Understanding

Step-by-Step Solutions begin on page R13.

Example 1 Determine the best method to solve each system of equations. Then solve the system.

- $2x + 3y = -11$
 $-8x - 5y = 9$
- $3x + 4y = 11$
 $2x + y = -1$
- $3x - 4y = -5$
 $-3x + 2y = 3$
- $3x + 7y = 4$
 $5x - 7y = -12$

Example 2 **5. SHOPPING** At a sale, Salazar bought 4 T-shirts and 3 pairs of jeans for \$181. At the same store, Jenna bought 1 T-shirt and 2 pairs of jeans for \$94. The T-shirts were all the same price, and the jeans were all the same price.

- Write a system of equations that can be used to represent this situation.
- Determine the best method to solve the system of equations.
- Solve the system.

Practice and Problem Solving

Extra Practice is on page R6.

Example 1 Determine the best method to solve each system of equations. Then solve the system.

- $-3x + y = -3$
 $4x + 2y = 14$
- $2x + 6y = -8$
 $x - 3y = 8$
- $3x - 4y = -5$
 $-3x - 6y = -5$
- $5x + 8y = 1$
 $-2x + 8y = -6$
- $y + 4x = 3$
 $y = -4x - 1$
- $-5x + 4y = 7$
 $-5x - 3y = -14$

Example 2 **12. FINANCIAL LITERACY** For a Future Teachers of America fundraiser, Denzell sold food as shown in the table. He sold 11 more subs than pizzas and earned a total of \$233. Write and solve a system of equations to represent this situation. Then describe what the solution means.

Item	Selling Price
pizza	\$5.00
sub	\$3.00

- DVDs** Manuela has a total of 40 DVDs of movies and television shows. The number of movies is 4 less than 3 times the number of television shows. Write and solve a system of equations to find the numbers of movies and television shows that she has on DVD.
- CAVES** The Caverns of Sonora have two different tours: the Crystal Palace tour and the Horseshoe Lake tour. The total length of both tours is 3.25 miles. The Crystal Palace tour is a half-mile less than twice the distance of the Horseshoe Lake tour. Determine the length of each tour.
- CCSS MODELING** The *break-even point* is the point at which income equals expenses. Ridgmont High School is paying \$13,200 for the writing and research of their yearbook plus a printing fee of \$25 per book. If they sell the books for \$40 each, how many will they have to sell to break even? Explain.
- PAINBALL** Clara and her friends are planning a trip to a paintball park. Find the cost of lunch and the cost of each paintball. What would be the cost for 400 paintballs and lunch?

**PAINTBALL
IN THE PARK**

- \$25 for 500 paintballs
- \$15 for 200 paintballs

Lunch is included

17. **RECYCLING** Mara and Ling each recycled aluminum cans and newspaper, as shown in the table. Mara earned \$3.77, and Ling earned \$4.65.

Materials	Pounds Recycled	
	Mara	Ling
aluminum cans	9	9
newspaper	26	114

- a. Define variables and write a system of linear equations from this situation.
b. What was the price per pound of aluminum? Determine the reasonableness of your solution.
18. **BOOKS** The library is having a book sale. Hardcover books sell for \$4 each, and paperback books are \$2 each. If Connie spends \$26 for 8 books, how many hardcover books did she buy?
19. **MUSIC** An online music club offers individual songs for one price or entire albums for another. Kendrick pays \$14.90 to download 5 individual songs and 1 album. Geoffrey pays \$21.75 to download 3 individual songs and 2 albums.
- a. How much does the music club charge to download a song?
b. How much does the music club charge to download an entire album?
20. **DRIVING** Malik drove his car for 45 miles at an average speed of r miles per hour. On the return trip, traffic has increased, and Malik's average speed is $\frac{3}{4}r$. The round trip took a total of 1 hour and 45 minutes. Find the average speed for each portion of the trip.

H.O.T. Problems Use Higher-Order Thinking Skills

21. **OPEN ENDED** Formulate a system of equations that represents a situation in your school. Describe the method that you would use to solve the system. Then solve the system and explain what the solution means.
22. **CCSS REASONING** In a system of equations, x represents the time spent riding a bike, and y represents the distance traveled. You determine the solution to be $(-1, 7)$. Use this problem to discuss the importance of analyzing solutions in the context of real-world problems.

23. **CHALLENGE** Solve the following system of equations by using three different methods. Show your work.

$$\begin{aligned} 4x + y &= 13 \\ 6x - y &= 7 \end{aligned}$$

24. **WRITE A QUESTION** A classmate says that elimination is the best way to solve a system of equations. Write a question to challenge his conjecture.

25. **WHICH ONE DOESN'T BELONG?** Which system is different? Explain.

$$\begin{aligned} x - y &= 3 \\ x + \frac{1}{2}y &= 1 \end{aligned}$$

$$\begin{aligned} -x + y &= 0 \\ 5x &= 2y \end{aligned}$$

$$\begin{aligned} y &= x - 4 \\ y &= \frac{2}{x} \end{aligned}$$

$$\begin{aligned} y &= x + 1 \\ y &= 3x \end{aligned}$$

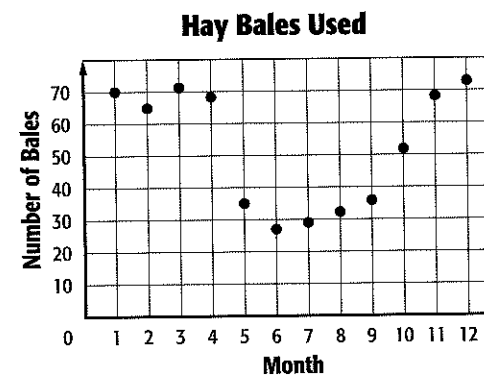
26. **WRITING IN MATH** How do you know what method to use when solving a system of equations?

Standardized Test Practice

27. If $5x + 3y = 12$ and $4x - 5y = 17$, what is y ?

A -1 B 3 C $(-1, 3)$ D $(3, -1)$

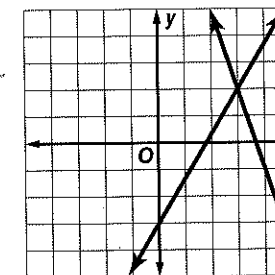
28. **STATISTICS** The scatter plot shows the number of hay bales used on the Bostwick farm during the last year.



- Which is an invalid conclusion?
- F The Bostwicks used less hay in the summer than they did in the winter.
G The Bostwicks used about 629 bales of hay during the year.
H On average, the Bostwicks used about 52 bales each month.
J The Bostwicks used the most hay in February.

29. **SHORT RESPONSE** At noon, Cesar cast a shadow 0.15 foot long. Next to him a streetlight cast a shadow 0.25 foot long. If Cesar is 6 feet tall, how tall is the streetlight?

30. The graph shows the solution to which of the following systems of equations?



- A $y = -3x + 11$
 $3y = 5x - 9$
B $y = 5x - 15$
 $2y = x + 7$
C $y = -3x + 11$
 $2y = 4x - 5$
D $y = 5x - 15$
 $3y = 2x + 18$

Spiral Review

Use elimination to solve each system of equations. (Lesson 6-4)

$$\begin{aligned} 31. \quad x + y &= 3 \\ 3x - 4y &= -12 \end{aligned}$$

$$\begin{aligned} 32. \quad -4x + 2y &= 0 \\ 2x - 3y &= 16 \end{aligned}$$

$$\begin{aligned} 33. \quad 4x + 2y &= 10 \\ 5x - 3y &= 7 \end{aligned}$$

34. **TRAVELING** A youth group is traveling in two vans to visit an aquarium. The number of people in each van and the cost of admission for that van are shown. What are the adult and student prices? (Lesson 6-3)

Van	Number of Adults	Number of Students	Total Cost
A	2	5	\$77
B	2	7	\$95

Graph each inequality. (Lesson 5-6)

$$35. \quad y < 4$$

$$36. \quad x \geq 3$$

$$37. \quad 7x + 12y > 0$$

$$38. \quad y - 3x \leq 4$$

Skills Review

Find each sum or difference.

$$39. \quad (-3.81) + (-8.5)$$

$$40. \quad 12.625 + (-5.23)$$

$$41. \quad 21.65 + (-15.05)$$

$$42. \quad (-4.27) + 1.77$$

$$43. \quad (-78.94) - 14.25$$

$$44. \quad (-97.623) - (-25.14)$$

