Study Guide

KeyConcepts

Systems of Equations (Lessons 6-1 through 6-5)

- A system with a graph of two intersecting lines has one solution and is *consistent and independent*.

- Graphing a system of equations can only provide approximate solutions. For exact solutions, you must use algebraic methods.

- In the substitution method, one equation is solved for a variable and the expression substituted into the second equation to find the value of another variable.

- In the elimination method, one variable is eliminated by adding or subtracting the equations.

- Sometimes multiplying one or both equations by a constant makes it easier to use the elimination method.

- The best method for solving a system of equations depends on the coefficients of the variables.

Systems of Inequalities (Lesson 6-6)

- A system of inequalities is a set of two or more inequalities with the same variables.

- The solution of a system of inequalities is the intersection of the graphs.

FOLDABLES StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.
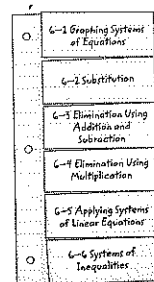
KeyVocabulary

augmented matrix (p. 370)	inconsistent (p. 335)
consistent (p. 335)	independent (p. 335)
dependent (p. 335)	matrix (p. 370)
dimension (p. 370)	substitution (p. 344)
element (p. 370)	system of equations (p. 335)
elimination (p. 350)	system of inequalities (p. 372)

VocabularyCheck

State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

1. If a system has at least one solution, it is said to be <u>consistent</u>.

2. If a consistent system has exactly <u>two</u> solution(s), it is said to be independent.

3. If a consistent system has an infinite number of solutions, it is said to be <u>inconsistent</u>.

4. If a system has no solution, it is said to be <u>inconsistent</u>.

5. <u>Substitution</u> involves substituting an expression from one equation for a variable in the other.

6. In some cases, <u>dividing</u> two equations in a system together will eliminate one of the variables. This process is called elimination.

7. A set of two or more inequalities with the same variables is called a <u>system of equations</u>.

8. When the graphs of the inequalities in a system of inequalities <u>do not intersect</u>, there are no solutions to the system.

Lesson-by-Lesson Review

6-1 Graphing Systems of Equations

Graph each system and determine the number of solutions that it has. If it has one solution, name it.

9. $x - y = 1$
   $x + y = 5$

10. $y = 2x - 4$
    $4x + y = 2$

11. $2x - 3y = -6$
    $y = -3x + 2$

12. $-3x + y = -3$
    $y = x - 3$

13. $x + 2y = 6$
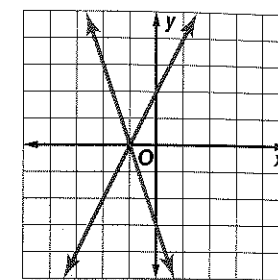 $3x + 6y = 8$

14. $3x + y = 5$
    $6x = 10 - 2y$

15. **MAGIC NUMBERS** Sean is trying to find two numbers with a sum of 14 and a difference of 4. Define two variables, write a system of equations, and solve by graphing.

Example 1

Graph the system and determine the number of solutions it has. If it has one solution, name it.

$y = 2x + 2$
$y = -3x - 3$

The lines appear to intersect at the point $(-1, 0)$. You can check this by substituting -1 for x and 0 for y.

CHECK	$y = 2x + 2$	Original equation
	$0 \stackrel{?}{=} 2(-1) + 2$	Substitution
	$0 \stackrel{?}{=} -2 + 2$	Multiply.
	$0 = 0 ✓$	
	$y = -3x - 3$	Original equation
	$0 \stackrel{?}{=} -3(-1) - 3$	Substitution
	$0 \stackrel{?}{=} 3 - 3$	Multiply.
	$0 = 0 ✓$	

The solution is $(-1, 0)$.

6-2 Substitution

Use substitution to solve each system of equations.

16. $x + y = 3$
    $x = 2y$

17. $x + 3y = -28$
    $y = -5x$

18. $3x + 2y = 16$
    $x = 3y - 2$

19. $x - y = 8$
    $y = -3x$

20. $y = 5x - 3$
    $x + 2y = 27$

21. $x + 3y = 9$
    $x + y = 1$

22. **GEOMETRY** The perimeter of a rectangle is 48 inches. The length is 6 inches greater than the width. Define the variables, and write equations to represent this situation. Solve the system by using substitution.

Example 2

Use substitution to solve the system.

$3x - y = 18$
$y = x - 4$

$3x - y = 18$	First equation
$3x - (x - 4) = 18$	Substitute $x - 4$ for y.
$2x + 4 = 18$	Simplify.
$2x = 14$	Subtract 4 from each side.
$x = 7$	Divide each side by 2.

Use the value of x and either equation to find the value for y.

$y = x - 4$	Second equation
$= 7 - 4$ or 3	Substitute and simplify.

The solution is $(7, 3)$.

6-3 Elimination Using Addition and Subtraction

Use elimination to solve each system of equations.

23. $x + y = 13$
$x - y = 5$

24. $-3x + 4y = 21$
$3x + 3y = 14$

25. $x + 4y = -4$
$x + 10y = -16$

26. $2x + y = -5$
$x - y = 2$

27. $6x + y = 9$
$-6x + 3y = 15$

28. $x - 4y = 2$
$3x + 4y = 38$

29. $2x + 2y = 4$
$2x - 8y = -46$

30. $3x + 2y = 8$
$x + 2y = 2$

31. BASEBALL CARDS Cristiano bought 24 baseball cards for $50. One type cost $1 per card, and the other cost $3 per card. Define the variables, and write equations to find the number of each type of card he bought. Solve by using elimination.

Example 3

Use elimination to solve the system of equations.

$3x - 5y = 11$
$x + 5y = -3$

$$\begin{array}{rl} 3x - 5y &= 11 \\ (+)\quad x + 5y &= -3 \\ \hline 4x &= 8 \qquad \text{The variable } y \text{ is eliminated.} \\ x &= 2 \qquad \text{Divide each side by 4.} \end{array}$$

Now, substitute 2 for x in either equation to find the value of y.

$$\begin{array}{rll} 3x - 5y &= 11 & \text{First equation} \\ 3(2) - 5y &= 11 & \text{Substitute.} \\ 6 - 5y &= 11 & \text{Multiply.} \\ -5y &= 5 & \text{Subtract 6 from each side.} \\ y &= -1 & \text{Divide each side by } -5. \end{array}$$

The solution is $(2, -1)$.

6-4 Elimination Using Multiplication

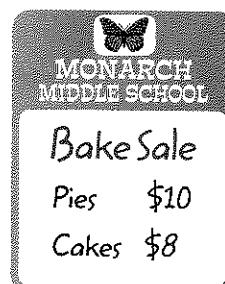**Use elimination to solve each system of equations.**

32. $x + y = 4$
$-2x + 3y = 7$

33. $x - y = -2$
$2x + 4y = 38$

34. $3x + 4y = 1$
$5x + 2y = 11$

35. $-9x + 3y = -3$
$3x - 2y = -4$

36. $8x - 3y = -35$
$3x + 4y = 33$

37. $2x + 9y = 3$
$5x + 4y = 26$

38. $-7x + 3y = 12$
$2x - 8y = -32$

39. $8x - 5y = 18$
$6x + 6y = -6$

40. BAKE SALE On the first day, a total of 40 items were sold for $356. Define the variables, and write a system of equations to find the number of cakes and pies sold. Solve by using elimination.

MONARCH
MIDDLE SCHOOL

Bake Sale

Pies $10

Cakes $8

Example 4

Use elimination to solve the system of equations.

$3x + 6y = 6$
$2x + 3y = 5$

Notice that if you multiply the second equation by -2, the coefficients of the y-terms are additive inverses.

$$\begin{array}{ll} 3x + 6y = 6 & \qquad 3x + 6y = 6 \\ 2x + 3y = 5 \quad \boxed{\text{Multiply by } -2} & \quad (+)\ -4x - 6y = -10 \\ & \qquad \overline{\quad -x \quad\quad = -4} \\ & \qquad\qquad\quad x = 4 \end{array}$$

Now, substitute 4 for x in either equation to find the value of y.

$$\begin{array}{rll} 2x + 3y &= 5 & \text{Second equation} \\ 2(4) + 3y &= 5 & \text{Substitution} \\ 8 + 3y &= 5 & \text{Multiply.} \\ 3y &= -3 & \text{Subtract 8 from both sides.} \\ y &= -1 & \text{Divide each side by 3.} \end{array}$$

The solution is $(4, -1)$.

6-5 Applying Systems of Linear Equations

Determine the best method to solve each system of equations. Then solve the system.

41. $y = x - 8$
$y = -3x$

42. $y = -x$
$y = 2x$

43. $x + 3y = 12$
$x = -6y$

44. $x + y = 10$
$x - y = 18$

45. $3x + 2y = -4$
$5x + 2y = -8$

46. $6x + 5y = 9$
$-2x + 4y = 14$

47. $3x + 4y = 26$
$2x + 3y = 19$

48. $11x - 6y = 3$
$5x - 8y = -25$

49. COINS Tionna has saved dimes and quarters in her piggy bank. Define the variables, and write a system of equations to determine the number of dimes and quarters. Then solve the system using the best method for the situation.

$4.00
25 coins
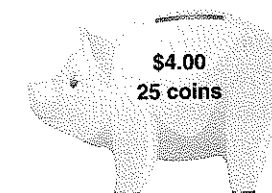
50. FAIR At a county fair, the cost for 4 slices of pizza and 2 orders of French fries is $21.00. The cost of 2 slices of pizza and 3 orders of French fries is $16.50. To find out how much a single slice of pizza and an order of French fries costs, define the variables and write a system of equations to represent the situation. Determine the best method to solve the system of equations. Then solve the system. (Lesson 6-5)

Example 5

Determine the best method to solve the system of equations. Then solve the system.

$3x + 5y = 4$
$4x + y = -6$

The coefficient of y is 1 in the second equation. So solving by substitution is a good method. Solve the second equation for y.

$$\begin{array}{rll} 4x + y &= -6 & \text{Second equation} \\ y &= -6 - 4x & \text{Subtract } 4x \text{ from each side.} \end{array}$$

Substitute $-6 - 4x$ for y in the first equation.

$$\begin{array}{rll} 3x + 5(-6 - 4x) &= 4 & \text{Substitute.} \\ 3x - 30 - 20x &= 4 & \text{Distributive Property} \\ -17x - 30 &= 4 & \text{Simplify.} \\ -17x &= 34 & \text{Add 30 to each side.} \\ x &= -2 & \text{Divide by } -17. \end{array}$$

Last, substitute -2 for x in either equation to find y.

$$\begin{array}{rll} 4x + y &= -6 & \text{Second equation} \\ 4(-2) + y &= -6 & \text{Substitute.} \\ -8 + y &= -6 & \text{Multiply.} \\ y &= 2 & \text{Add 8 to each side.} \end{array}$$

The solution is $(-2, 2)$.

6-6 Systems of Inequalities

Solve each system of inequalities by graphing.

51. $x > 3$
$y < x + 2$

52. $y \leq 5$
$y > x - 4$

53. $y < 3x - 1$
$y \geq -2x + 4$

54. $y \leq -x - 3$
$y \geq 3x - 2$

55. JOBS Kishi makes $7 an hour working at the grocery store and $10 an hour delivering newspapers. She cannot work more than 20 hours per week. Graph two inequalities that Kishi can use to determine how many hours she needs to work at each job if she wants to earn at least $90 per week.

Example 6

Solve the system of inequalities by graphing.

$y < 3x + 1$
$y \geq -2x + 3$

The solution set of the system is the set of ordered pairs in the intersection of the two graphs. This portion is shaded in the graph below.
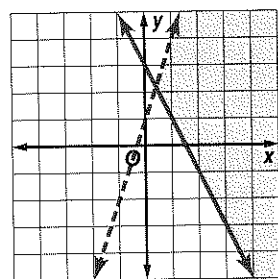

Graph each system and determine the number of solutions that it has. If it has one solution, name it.

1. $y = 2x$
$y = 6 - x$

2. $y = x - 3$
$y = -2x + 9$

3. $x - y = 4$
$x + y = 10$

4. $2x + 3y = 4$
$2x + 3y = -1$

Use substitution to solve each system of equations.

5. $y = x + 8$
$2x + y = -10$

6. $x = -4y - 3$
$3x - 2y = 5$

7. GARDENING Corey has 42 feet of fencing around his garden. The garden is rectangular in shape, and its length is equal to twice the width minus 3 feet. Define the variables, and write a system of equations to find the length and width of the garden. Solve the system by using substitution.
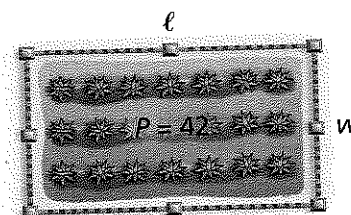
8. MULTIPLE CHOICE Use elimination to solve the system.

$$6x - 4y = 6$$
$$-6x + 3y = 0$$

A $(5, 6)$

B $(-3, -6)$

C $(1, 0)$

D $(4, -8)$

9. SHOPPING Shelly has $175 to shop for jeans and sweaters. Each pair of jeans costs $25, each sweater costs $20, and she buys 8 items. Determine the number of pairs of jeans and sweaters Shelly bought.

Use elimination to solve each system of equations.

10. $x + y = 13$
$x - y = 5$

11. $3x + 7y = 2$
$3x - 4y = 13$

12. $x + y = 8$
$x - 3y = -4$

13. $2x + 6y = 18$
$3x + 2y = 13$

14. MAGAZINES Julie subscribes to a sports magazine and a fashion magazine. She received 24 issues this year. The number of fashion issues is 6 less than twice the number of sports issues. Define the variables, and write a system of equations to find the number of issues of each magazine.

Determine the best method to solve each system of equations. Then solve the system.

15. $y = 3x$
$x + 2y = 21$

16. $x + y = 12$
$y = x - 4$

17. $x + y = 15$
$x - y = 9$

18. $3x + 5y = 7$
$2x - 3y = 11$

19. OFFICE SUPPLIES At a sale, Ricardo bought 24 reams of paper and 4 inkjet cartridges for $320. Britney bought 2 reams of paper and 1 inkjet cartridge for $50. The reams of paper were all the same price and the inkjet cartridges were all the same price. Write a system of equations to represent this situation. Determine the best method to solve the system of equations. Then solve the system.

Solve each system of inequalities by graphing.

20. $x > 2$
$y < 4$

21. $x + y \leq 5$
$y \geq x + 2$

22. $3x - y > 9$
$y > -2x$

23. $y \geq 2x + 3$
$-4x - 3y > 12$