SMART START

Simplify each expression.

10. $8^2 \cdot 8^5$

11. $4x^2 \cdot 6x^4$

12. $(z^4)^2$

13. $\dfrac{12^6}{12^4}$

14. $(2a^3 b)^5$

15. $\dfrac{24y^9}{6y^4}$

16. $\dfrac{10^0}{10^5}$

17. $p^4 \cdot p \cdot p^3$

1 Rational Exponents You know that an exponent represents the number of times that the base is used as a factor. But how do you evaluate an expression with an exponent that is not an integer like the one above? Let's investigate **rational exponents** by assuming that they behave like integer exponents.

$\left(b^{\frac{1}{2}}\right)^2 = b^{\frac{1}{2}} \cdot b^{\frac{1}{2}}$	Write as a multiplication expression.
$= b^{\frac{1}{2} + \frac{1}{2}}$	Product of Powers
$= b^1$ or b	Simplify.

Thus, $b^{\frac{1}{2}}$ is a number with a square equal to b. So $b^{\frac{1}{2}} = \sqrt{b}$.

rational exponent (p. 406) For any positive real number b and any integers m and $n > 1$, $b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m$ or $\sqrt[n]{b^m}$. $\frac{m}{n}$ is a rational exponent.

KeyConcept $b^{\frac{1}{2}}$

Words	For any nonnegative real number b, $b^{\frac{1}{2}} = \sqrt{b}$
Examples	$16^{\frac{1}{2}} = \sqrt{16}$ or 4 $\qquad 38^{\frac{1}{2}} = \sqrt{38}$

Example 1 Radical and Exponential Forms

Write each expression in radical form, or write each radical in exponential form.

a. $25^{\frac{1}{2}}$

$25^{\frac{1}{2}} = \sqrt{25}$ Definition of $b^{\frac{1}{2}}$

$= 5$ Simplify.

b. $\sqrt{18}$

$\sqrt{18} = 18^{\frac{1}{2}}$ Definition of $b^{\frac{1}{2}}$

c. $5x^{\frac{1}{2}}$

$5x^{\frac{1}{2}} = 5\sqrt{x}$ Definition of $b^{\frac{1}{2}}$

d. $\sqrt{8p}$

$\sqrt{8p} = (8p)^{\frac{1}{2}}$ Definition of $b^{\frac{1}{2}}$

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1A. $a^{\frac{1}{2}}$ **1B.** $\sqrt{22}$ **1C.** $(7w)^{\frac{1}{2}}$ **1D.** $2\sqrt{x}$

KeyConcept *n*th Root

Words	For any real numbers a and b and any positive integer n, if $a^n = b$, then a is an nth root of b.
Symbols	If $a^n = b$, then $\sqrt[n]{b} = a$.
Example	Because $2^4 = 16$, 2 is a fourth root of 16; $\sqrt[4]{16} = 2$.

Since $3^2 = 9$ and $(-3)^2 = 9$, both 3 and -3 are square roots of 9. Similarly, since $2^4 = 16$ and $(-2)^4 = 16$, both 2 and -2 are fourth roots of 16. The positive roots are called *principal roots*. Radical symbols indicate principal roots, so $\sqrt[4]{16} = 2$.

Example 2 *n*th roots

Simplify.

a. $\sqrt[3]{27}$

$$\sqrt[3]{27} = \sqrt[3]{3 \cdot 3 \cdot 3}$$
$$= 3$$

b. $\sqrt[5]{32}$

$$\sqrt[5]{32} = \sqrt[5]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$$
$$= 2$$

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2A. $\sqrt[3]{64}$

2B. $\sqrt[4]{10{,}000}$

Like square roots, nth roots can be represented by rational exponents.

$$\left(b^{\frac{1}{n}}\right)^n = \underbrace{b^{\frac{1}{n}} \cdot b^{\frac{1}{n}} \cdot \ldots \cdot b^{\frac{1}{n}}}_{n \text{ factors}} \qquad \text{Write as a multiplication expression.}$$

$$= b^{\frac{1}{n} + \frac{1}{n} + \ldots + \frac{1}{n}} \qquad \text{Product of Powers}$$

$$= b^1 \text{ or } b \qquad \text{Simplify.}$$

Thus, $b^{\frac{1}{n}}$ is a number with an nth power equal to b. So $b^{\frac{1}{n}} = \sqrt[n]{b}$.

KeyConcept $b^{\frac{1}{n}}$

Words For any positive real number b and any integer $n > 1$, $b^{\frac{1}{n}} = \sqrt[n]{b}$

Example $8^{\frac{1}{3}} = \sqrt[3]{8} = \sqrt[3]{2 \cdot 2 \cdot 2}$ or 2

Example 3 Evaluate $b^{\frac{1}{n}}$ Expressions

Simplify.

a. $125^{\frac{1}{3}}$

$$125^{\frac{1}{3}} = \sqrt[3]{125} \qquad b^{\frac{1}{n}} = \sqrt[n]{b}$$

$$= \sqrt[3]{5 \cdot 5 \cdot 5} \qquad 125 = 5^3$$

$$= 5 \qquad \text{Simplify.}$$

b. $1296^{\frac{1}{4}}$

$$1296^{\frac{1}{4}} = \sqrt[4]{1296} \qquad b^{\frac{1}{n}} = \sqrt[n]{b}$$

$$= \sqrt[4]{6 \cdot 6 \cdot 6 \cdot 6} \qquad 1296 = 6^4$$

$$= 6 \qquad \text{Simplify.}$$

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3A. $27^{\frac{1}{3}}$

3B. $256^{\frac{1}{4}}$

The Power of a Power Property allows us to extend the definition of $b^{\frac{1}{n}}$ to $b^{\frac{m}{n}}$.

$b^{\frac{m}{n}} = \left(b^{\frac{1}{n}}\right)^m$ Power of a Power

$= \left(\sqrt[n]{b}\right)^m$ or $\sqrt[n]{b^m}$ $b^{\frac{1}{n}} = \sqrt[n]{b}$

KeyConcept $b^{\frac{m}{n}}$

Words For any positive real number b and any integers m and $n > 1$,

$$b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m \text{ or } \sqrt[n]{b^m}$$

Example $8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = 2^2$ or 4

Example 4 Evaluate $b^{\frac{m}{n}}$ Expressions

Simplify.

a. $64^{\frac{2}{3}}$

$64^{\frac{2}{3}} = \left(\sqrt[3]{64}\right)^2$ $b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m$

$= \left(\sqrt[3]{4 \cdot 4 \cdot 4}\right)^2$ $64 = 4^3$

$= 4^2$ or 16 Simplify.

b. $36^{\frac{3}{2}}$

$36^{\frac{3}{2}} = \left(\sqrt[2]{36}\right)^3$ $b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m$

$= 6^3$ $\sqrt{36} = 6$

$= 216$ Simplify.

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4A. $27^{\frac{2}{3}}$

4B. $256^{\frac{5}{4}}$

2 Solve Exponential Equations

In an **exponential equation**, variables occur as exponents. The Power Property of Equality and the other properties of exponents can be used to solve exponential equations.

KeyConcept Power Property of Equality

Words For any real number $b > 0$ and $b \neq 1$, $b^x = b^y$ if and only if $x = y$.

Examples If $5^x = 5^3$, then $x = 3$. If $n = \frac{1}{2}$, then $4^n = 4^{\frac{1}{2}}$.

Example 5 Solve Exponential Equations

Solve each equation.

a. $6^x = 216$

$6^x = 216$ Original equation

$6^x = 6^3$ Rewrite 216 as 6^3.

$x = 3$ Property of Equality

CHECK $6^x = 216$

$6^3 \stackrel{?}{=} 216$

$216 = 216$ ✓

b. $25^{x-1} = 5$

$25^{x-1} = 5$ Original equation

$(5^2)^{x-1} = 5$ Rewrite 25 as 5^2

$5^{2x-2} = 5^1$ Power of a Power, Distributive Property

$2x - 2 = 1$ Power Property of Equality

$2x = 3$ Add 2 to each side.

$x = \frac{3}{2}$ Divide each side by 2.

CHECK $25^{x-1} = 5$

$25^{\frac{3}{2}-1} \stackrel{?}{=} 5$

$25^{\frac{1}{2}} = 5$ ✓

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5A. $5^x = 125$

5B. $12^{2x+3} = 144$