14 VOLUME


Disco balls are spheres that reflect light in all different directions. They were really popular in dance clubs throughout the 1960s, 1970s, and 1980s.

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14.1 DRUM ROLL, PLEASE!
Volume of a Cylinder

Learning Goals

In this lesson, you will:

- Explore the volume of a cylinder using unit cubes.
- Estimate the volume of a right circular cylinder.
- Write a formula for the volume of a cylinder.
- Use a formula to determine the volume of a right circular cylinder.
- Use appropriate units of measure when computing the volume of a right circular cylinder.

Key Terms

- cylinder
- right circular cylinder
- radius of a cylinder
- height of a cylinder
- circumference
- pi

There's an old saying that states, "You can tune a car, but you can't tune a fish." But, can you tune a drum? In fact, you can! Certain drums not only can be used to play rhythms, but some also produce a tone. For example, timpani (sometimes called kettle drums) can be tuned to the key of a song. However, the snare drum is different. Snare drums are the type of drum you most likely hear when there is a drum roll. Most drummers do not like the ring that comes from a snare drum.

To "tune" a drum, musicians alter the tension of the drum heads. This tightening and loosening affect the tones that the drum produces. Also, sometimes adding tape to certain sections of the heads can alter the tone of the drums as well.

What effect does tightening the drum head have on the tone? What effect does loosening the drum head have?

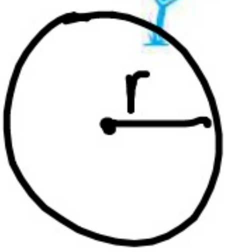
Problem 1 Getting to Know Cylinders!

A **cylinder** is a three-dimensional object with two parallel, congruent circular bases.

A **right circular cylinder** is a cylinder in which the bases are circles and they are aligned one directly above the other.

All cylinders in this chapter are right circular cylinders.

1. Sketch what you think is an example of a cylinder.



2. Does your sketch show circular bases? Does your sketch show bases that are congruent and parallel?

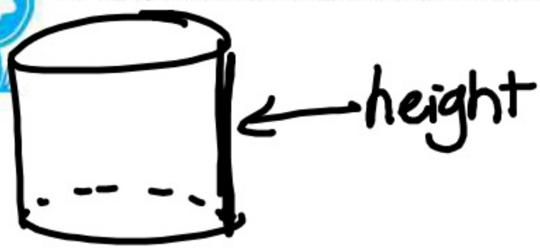
3. Compare your sketch with your classmates' sketches. Did everyone sketch the same cylinder? Explain how the sketches are the same or different.

4. The radius of a cylinder is the radius of one of its bases. Use your sketch to explain what is meant by "radius of the base."

radius of one of the bases
Distance from center of circle to point on circle

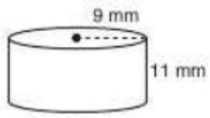
The **radius of a cylinder** is the distance from the center of the base to any point on the base. The radius of a cylinder is the same on both bases. The **height of a cylinder** is the length of a line segment drawn from one base to the other base. This line segment is perpendicular to the other base.

5. Use your sketch to explain what is meant by the "height of a cylinder."



Problem 2 Characteristics of a Cylinder

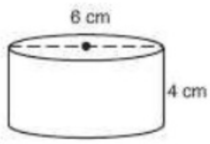
1. Identify the radius, diameter, and height of each cylinder.



radius: 9mm height: 11 mm
diameter: 18mm

a. How did you determine the diameter of the cylinder?

double the radius



radius: 3 cm height: 4cm
diameter: 6 cm

b. How did you determine the radius of the cylinder?

Divide diameter by 2

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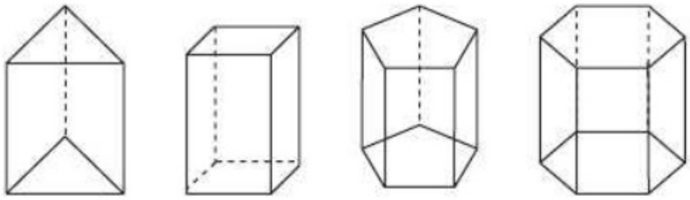
Cylinders and prisms are closely related.



Triangular Prism Rectangular Prism Pentagonal Prism Hexagonal Prism

2. Analyze the prisms shown. What pattern do you see as the number of sides of the base increase?

As the number of sides increases, the prism looks more like a cylinder.

3. Does a cylinder have a front, back, left, right, top, or bottom side? Explain your reasoning.

It has a top and bottom

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4. Imagine a paper label on a can of rice soup. The label wraps all the way around the can as shown.

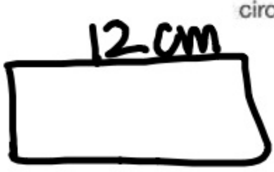
a. If the label is carefully peeled off the can and flattened out, what do you think the shape of the label will be?

The label would be rectangular

b. How is the length of the label related to the circular base of the can?

The length of the label would be the length around the can.

The distance around a circle or a cylinder is known as the **circumference**. The circumference is calculated by the following formula: $C = \pi(d)$ where C represents the circumference, d represents the diameter, and π stands for *pi*. **Pi** is the ratio of the circumference of a circle to its diameter. Generally, π is approximately 3.14, or $\frac{22}{7}$.



c. If the length of the rectangular label is 12 centimeters, what is the circumference of the base of the can?

12cm

$C = \pi(d)$
$12 = \pi(d)$
$6 = \pi(d)$

d. If the length of the rectangular label is 12 centimeters, what is the radius of the base of the can?

$6/\pi$

e. How is the width of the label related to the height of the can?

width is the height of the can

f. If the width of the label is 8 centimeters, what is the height of the can?

8 cm

g. Describe the relationship between the area of the label and the product of the circumference of the base times the height of the can.

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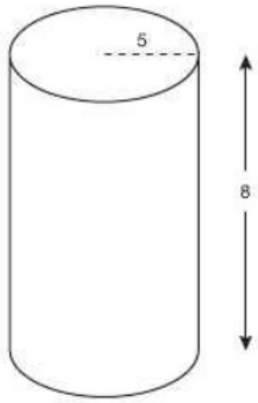
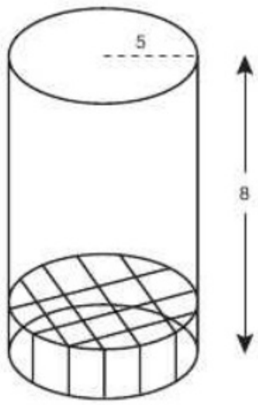
Problem 3 The Volume of a Cylinder

Consider the cylinder shown. The radius of the circular base is 5 units and the height of the cylinder is 8 units.



Suppose the bottom of the cylinder is filled with unit cubes so it looks like this.



As you can see, due to the circular nature of the cylinder, some of the unit cubes had to be broken off to fit inside.

1. The cubes on the bottom layer represent only a part of the total volume of the cylinder. Why?

Cause it is not filled up to the top

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2. What would you need to do to calculate the total volume of the cylinder?

Fill to the top with unit cubes

Since it would be tricky counting all of the broken cubes, it makes more sense to calculate the number of unit cubes in the bottom layer using a formula.

3. What formula could be used to determine the number of unit cubes in the bottom layer?

Area of base (circle)
$A = \pi r^2$

4. What is the radius of the base of the cylinder?

radius: 5

5. What is the formula for the area of a circle?

6. Determine the number of unit cubes in the bottom layer of the cylinder by using the area of a circle formula.

$A = \pi r^2$
$A = (3.14)(5)^2$
$3.14(25)$

$78.5\ u^2$

7. If you fill the cylinder with layers of unit cubes, will each layer contain the same number of unit cubes? Why or why not?

Yes it is the same Circle

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8. How many layers of unit cubes are needed to fill the cylinder? Explain your reasoning.

height: 8

9. Calculate the total number of unit cubes needed to fill the cylinder.

$78.5(8)$

$628u^3$

10. Write a formula for the volume of a cylinder, where V represents the volume of the cylinder, B represents the area of the base (area of the first layer), and h represents the height of the cylinder.

$V = Bh$

11. Write a second formula for the volume of a cylinder, where V represents the volume of the cylinder, r represents the radius of the cylinder, and h represents the height of the cylinder.

$V = \pi r^2 h$

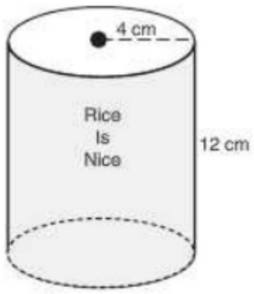
Problem 4 Comparing Prototypes

The director of the marketing department at the Rice Is Nice Company sent a memo to her product development team requesting that the volume of the new cylinder prototype equal 602.88 cm^3.

Two members of the marketing team disagreed about the dimensions of the cylinder prototype.

The formula both team members used to calculate the volume of their cylinder prototype was $V = \pi r^2 h$, where V is the volume of the cylinder, r is the radius of the cylinder, and h is the height of the cylinder. However, they each designed a different cylinder. Each member insists that their co-worker made a math error because both cylinders can't satisfy the director's memo and have different dimensions. They need your help to settle this disagreement!

Cassandra designed the cylinder shown.



1. Cassandra claims the volume of her prototype is 602.88 cubic centimeters. Calculate the volume using the measurements shown on Cassandra's prototype. Use 3.14 for π.

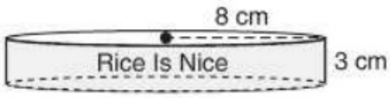
$V = \pi r^2 h$

$V = 3.14(4)^2 12$

$= 3.14(16)(12)$

602.88 cm^3

Robert designed the cylinder prototype shown.



2. Robert claims the volume of his prototype is 602.88 cubic centimeters. Calculate the volume using the measurements shown on Robert's prototype. Use 3.14 for π.

$V = \pi r^2 h$

$V = 3.14(8)^2 3$

$3.14(64)3$

$602.88 cm^3$

3. What would you say to Cassandra and Robert to settle their disagreement?

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Problem 5 The Doubling Effect

1. Juan and Sandy are two other members of the product development team. They are both discussing the effect that doubling the radius of the base has on the volume of a can of rice.

   Juan insists that if the radius of a cylinder doubles, the volume will double. Sandy thinks the volume will be more than double. Who is correct? To help you determine who is correct, consider using an example of a cylinder that has a radius of 4 centimeters and a height of 10 centimeters. Use 3.14 for π. Explain your reasoning.

2. Sandy wanted to make sure Juan understood the effect doubling the length of the radius of a cylinder has on the volume of a cylinder. She created a table, hoping Juan would see a pattern. Complete the table, and identify any patterns that you notice.

Cylinder Radius (cm) r	Cylinder Area of Base (cm) $A = \pi r^2$	Cylinder Height (cm) h	Cylinder Volume (cm^3) $V = \pi r^2 h$
1	$\pi(1)^2 = 3.14$ 1π	1	1π 3.14
2		1	
4		1	
8		1	

3. Juan does not give up easily. He is now trying to convince Sandy about the effect doubling the height has on the volume of a can of rice.

Juan insists that if the height of a cylinder doubles, the volume will double. Sandy says the volume again will be more than double. Who is correct? Once again, consider a cylinder that has a radius of 4 centimeters and a height of 10 centimeters. Use 3.14 for π. Explain your reasoning.

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Problem 6 Solving Problems

1. A circular swimming pool has a diameter of 30 feet and a height of 5 feet. What is the volume of the pool? Use 3.14 for π.

2. How many milliliters of liquid are needed to fill a cylindrical can with a radius of 3 centimeters and a height of 4.2 centimeters? Note: One milliliter is equivalent to one cubic centimeter of liquid.

3. Many newspapers are made from 100% wood. The wood used to make this paper can come from pine trees, which are typically about 60 feet (or 720 inches) tall and have diameters of about 1 foot (12 inches). But only about half of the volume of the tree is turned into paper.

   Suppose it takes about 0.5 cubic inches of wood to make one sheet of paper. About how many sheets can be made from a typical pine tree? Show your work, and explain your reasoning.

Be prepared to share your solutions and methods.

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14.2 PILING ON!

Volume of a Cone

Learning Goals

In this lesson, you will:

- Explore the volume of a cone using a cylinder and birdseed.
- Write a formula for the volume of a cone.
- Use a formula to determine the volume of a cone.
- Use appropriate units of measure when calculating the volume of a cone.

Key Terms

- cone
- height of a cone

Geometry has helped to solve environmental problems. Don't believe it? Here's an example. See if you can get the answer.

One problem with right cylinder-shaped cups is that they stand up on their own when you set them on a table (or on the ground in some cases). This is good in some ways—you can set your drink down and come back to it later. But it also has meant that people set these cups down and forget about them, which creates a severe trash and pollution problem.

Can you think of a paper cup design that would help to prevent this problem? Does your school use cylinder-shaped paper cups? What might you suggest they change them to?

Problem 1 Getting to Know Cones

A **cone** is a three-dimensional figure with a circular or elliptical base and one vertex.

All of the cones associated with this chapter have a circular base and a vertex that is located directly above the center point of the base of the cone.

1. What do you think "elliptical base" means?

2. Sketch what you think is an example of a cone.

3. Does your sketch show a circular or elliptical base? Does your sketch show one vertex?

4. Compare your sketch with your classmates' sketches. Did everyone sketch the same cone? Explain how the sketches are the same or different.

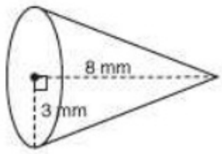
5. How does the radius of a cone compare to the radius of a cylinder?

The **height of a cone** is the length of a line segment drawn from the vertex to the base of the cone. The line segment is perpendicular to the base.

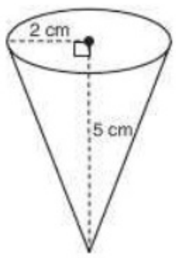
6. Use your sketch to explain what is meant by the "height of a cone"?

Problem 2 Characteristics of a Cone

1. Identify the radius, diameter, and height of each cone.

   a. 

   How did you determine the diameter of the cone?

   b. 

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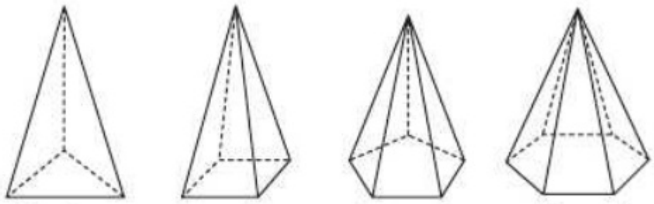
Cones and pyramids are closely related.



2. Analyze each pyramid. What pattern do you notice as the number of sides of the base increase?

3. What solid would a pyramid resemble if the polygonal bases had a million sides?

4. Does a cone have a front, back, left, right, top, or bottom side? Explain your reasoning.

Think about a paper snow cone holder. The paper is in the shape of an open cone. The paper wraps all the way around the side of the snow cone.

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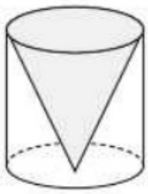
5. Suppose the paper holder was opened up and flattened out.

a. Sketch the shape of the paper snow cone holder.

b. Describe the shape of the flattened open cone.

Problem 3 Volume of a Cone

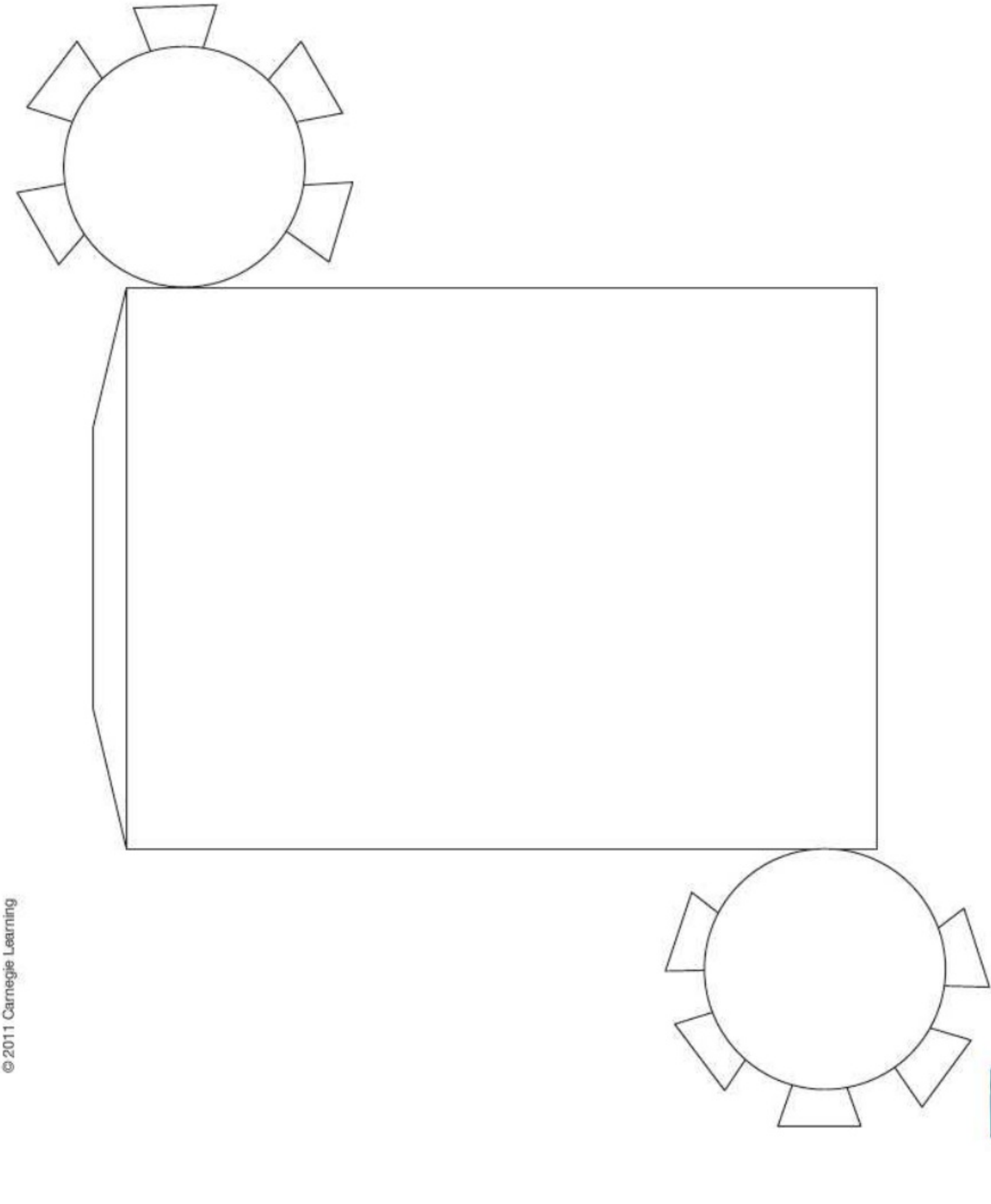
In the figure shown, a cone is placed inside a cylinder.



To explore the volume of a cone, you need to remove the base of the cone and a base of the cylinder.

1. Use the nets provided to create a model of a cylinder and a cone. Don't forget to remove the base of the cone and a base of the cylinder so both solids are open.

   This activity works best if the models are created out of oak tag paper or poster board to provide more support. You can use these nets as a template to copy onto a paper with more support.



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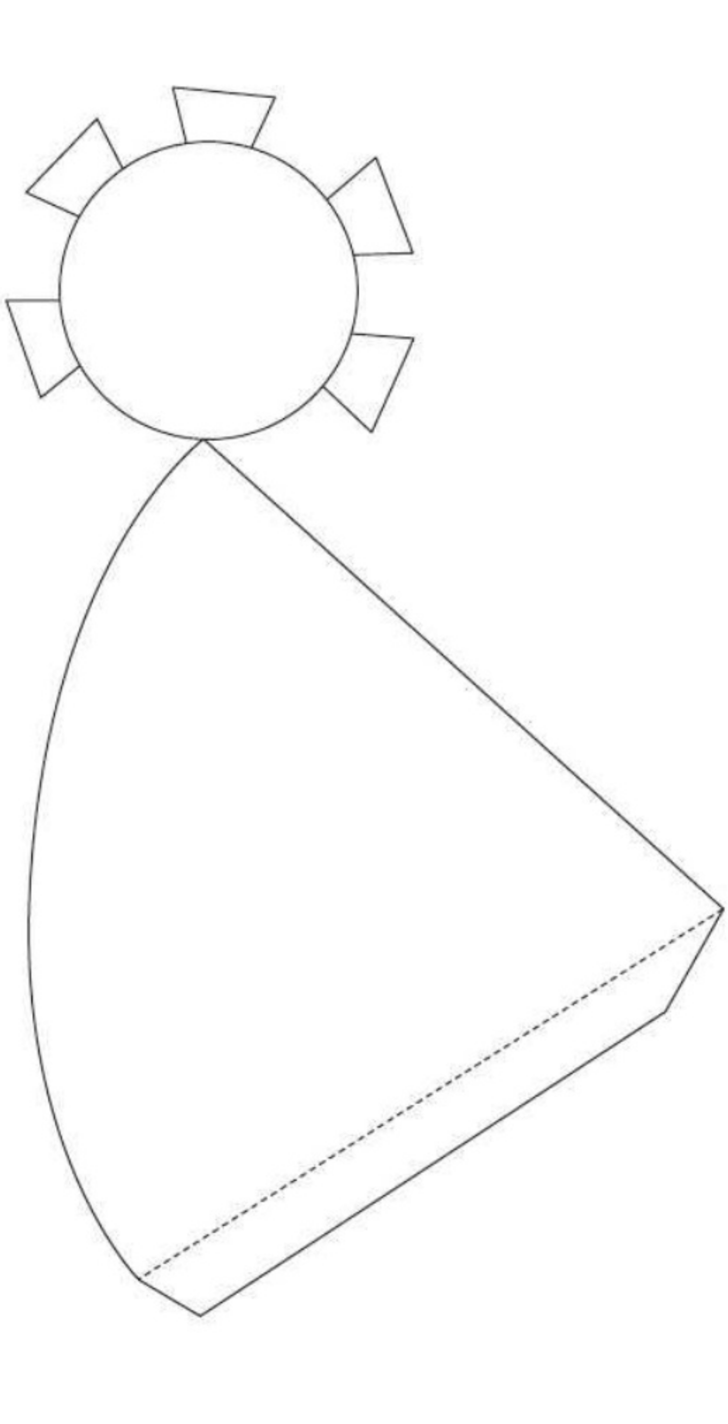

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2. Fill the cone with birdseed, and then pour the birdseed into the cylinder. Continue refilling the cone until you fill the cylinder with birdseed. How many cones of birdseed did it take you to fill the cylinder?

3. Compare the amount of birdseed that you used to fill the cone and to fill the cylinder. In other words, compare the volume of the cone to the volume of the cylinder. What fraction best describes this ratio?

4. *Estimate* the maximum number of unit cubes that would fit inside your model of a cylinder.

5. Using the *estimate* of the volume of the cylinder, *estimate* the volume of the cone.

6. Explain how you estimated the volume of the cone.

7. Use a centimeter ruler to measure the length of the radius and height of the cylinder. Then, *calculate* the actual volume of the cylinder. Use 3.14 for π.

8. How does the *estimation* of the volume of the cylinder compare to the *calculation* of the volume of the cylinder?

9. Using the actual volume of the cylinder, *calculate* the actual volume of the cone.

Problem 4 Volume Formula of a Cone

Recall that the formula for calculating the volume of a cylinder is $V = \pi r^2(h)$, where r represents the radius and h represents the height.

Use the same formula, but adjust it to apply it to a cone using the fraction discussed in the previous problem.

1. What fraction will you use to accurately represent the volume of a cone?

2. What does the variable V represent?

3. What does the variable r represent?

4. What does the variable h represent?

5. Write the formula for the volume of a cone. Define all variables used in the formula.

6. Write a second formula for the volume of a cone, where V represents the volume of a cone, B represents the area of the base of a cone, and h represents the height of the cone.

7. If you know that a cylinder and a cone share the same base and height, how would you determine the volume of the cone?

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Be prepared to share your solutions and methods.

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14.3 ALL BUBBLY
Volume of a Sphere

Learning Goals

In this lesson, you will:

- Explore the volume of a sphere.
- Write a formula for the volume of a sphere.
- Use a formula to determine the volume of a sphere.

Key Terms

- sphere
- center of a sphere
- radius of a sphere
- diameter of a sphere
- antipodes
- great circle
- hemisphere

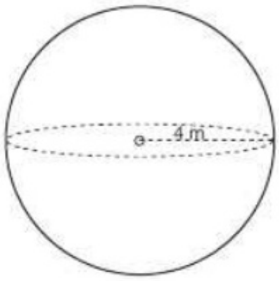
Why don't we ever see planets shaped like cylinders or cones? Everywhere we look in space, planets—and even stars like our sun—are shaped like spheres. Why is that?

The answer is gravity. For a large body in space, gravity pulls every point on the surface equally toward its center. Over time, gravity molds the body into the only possible shape that could form from such a process—a sphere!

Of course, not every object we observe in space is a sphere. Asteroids, comets, and even very small moons often have weird, rough shapes. These objects are too small—and their gravities too weak. For them, the sphere-making process never begins.

Problem 1 Getting to Know Spheres

A **sphere** is defined as the set of all points in three dimensions that are equidistant from a given point called the **center**. Like a circle, a sphere has radii and diameters. A segment drawn from the center of the sphere to a point on the sphere is called a **radius**. A segment drawn between two points on the sphere that passes through the center is a **diameter**. The endpoints of a diameter are the **antipodes**. The length of a diameter is twice the length of a radius. A **great circle** is the circumference of the sphere at the sphere's widest part.



List all of the things you know to be true about this sphere.

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Problem 2 Volume of a Sphere

The figure shown is a **hemisphere.** A hemisphere is a half-sphere.

The formula for the volume of a hemisphere is $\frac{2}{3}\pi r^3$.

1. Use the definition above to write a formula for the volume of a sphere. Explain your reasoning.

2. Earth is not a perfect sphere because of a bulge at the Equator. Earth has two different diameters: equatorial and polar. The equatorial diameter is 25 miles wider than the polar diameter. Use 7926 miles as the approximate diameter of Earth.

   a. Calculate the approximate radius of Earth.

   b. Calculate the approximate volume of Earth.

3. The circumference of an NBA basketball ranges from 29.5 to 30 inches.

   a. Calculate the approximate radius of a basketball with a circumference of 30 inches.

   b. Calculate the approximate volume of a basketball with a circumference of 30 inches.

4. The volume of a Major League baseball is 12.77 cubic inches.

   a. Calculate the approximate radius of a Major League baseball.

b. Calculate the approximate circumference of a Major League baseball.

5. Built in the 1950s by the Stamp Collecting Club at Boy's Town, the World's Largest Ball of Postage Stamps is very impressive. The solid ball has a diameter of 32 inches. It weighs 600 pounds and consists of 4,655,000 postage stamps.

Calculate the volume of the world's largest ball of postage stamps. Use 3.14 for pi.

6. The world's largest ball of paint resides in Alexandria, Indiana. The ball began as an ordinary baseball. People began coating the ball with layers of paint. Imagine this baseball with over 21,140 coats of paint on it! The baseball began weighing approximately 5 ounces and now weighs more than 2700 pounds! Painting this baseball has gone on for more than 32 years and people are still painting it today.

   The amount of paint on this ball could be used to paint a 4-inch-wide strip for over 68 miles. One gallon of paint will paint the ball about 12 times.

   When the baseball had 20,500 coats of paint on it, the circumference along the diameter of the ball was approximately 132.9162 inches. Each layer is approximately 0.001037 in. thick.

   Calculate the volume of the world's largest paint ball. Use 3.14 for pi.

7. The world's largest disco ball hangs from a fixed point and is powered by a 5-ton hydraulic rotator. It weighs nearly 1.5 tons with a volume of approximately 67 cubic meters. Approximately 8000 100-square-centimeter mirror tiles and over 10,000 rivets were used in its creation.

   Calculate the radius of the world's largest disco ball. Use 3.14 for pi.

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8. For over seven years, John Bain has spent his life creating the World's Largest Rubber Band Ball. It is solid to the core with rubber bands. Each rubber band was individually stretched around the ball, creating a giant rubber band ball. The weight of the ball is over 3,120 pounds, the circumference is 15.1 feet, the cost of the materials was approximately $25,000, and the number of rubber bands was 850,000.

   Calculate the volume of the world's largest rubber band ball. Use 3.14 for pi.

9. The world's largest ball of twine is in Darwin, Minnesota. It weighs 17,400 pounds and was created by Francis A. Johnson. He began this pursuit in March 1950. He spent four hours a day, every day wrapping the ball. At some point, the ball had to be lifted with a crane to continue proper wrapping. It took Francis 39 years to complete. Upon completion, it was moved to a circular open air shed on his front lawn for all to view.

   If the volume of the world's largest ball of twine is 7234.56 cubic feet, determine the diameter. Use 3.14 for pi.

10. In 2002, Guinness World Records certified the world's largest ball of transparent tape. It was built over the span of one month, and was created from 238 rolls (about 12 miles in length) of clear packing tape. The tape ball is about 2 feet high, 75 inches in diameter, and weighs 80 pounds.

Calculate the volume of the world's largest transparent tape ball. Use 3.14 for pi.

11. Young people often attempt to break world records. Jessica is no exception. Today her math class studied the volume of a sphere and she had a great idea. After working out the math, Jessica told her best friend Molly that she could stuff 63 inflated regulation-size basketballs into her school locker. Her rectangular locker is 6 feet high, 20 inches wide, and 20 inches deep. The radius of one basketball is 4.76 inches. Molly also did the math and said only 14 basketballs would fit.

How did Molly and Jessica compute their answers? Is either Molly or Jessica correct?

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Be prepared to share you solutions and methods.

14.4 PRACTICE MAKES PERFECT

Volume Problems

Learning Goals

In this lesson, you will:

- Use the volume of a cylinder formula to solve problems.
- Use the volume of a cone formula to solve problems.
- Use the volume of a sphere formula to solve problems.

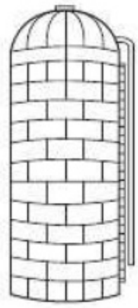
How can you tell when a word problem is asking you about volume and not about something else—like area or surface area? What about clue words?

Take the words *fill* and *cover*, for example. Which one might refer better to area, and which one might refer better to volume? What other strategies can you use to tell the difference?

In this lesson, you will solve only volume problems. Can you identify strategies you can use to figure out when a problem is asking you about volume?

Problem 1 The Silo

A silo is used to store grain that farm animals eat during the winter months. The top of the silo is a hemisphere with a radius of 8 ft. The cylindrical body of the silo shares the same radius as the hemisphere and has a height of 40 ft.



The truck hauling grain to the silo has a rectangular container attached to the back that is 8 feet in length, 5 feet in width, and 4 feet in height. Determine the number of truckloads of grain required to fill an empty silo.

Use 3.14 for pi.

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Problem 2 Frozen Yogurt Cone

The frozen yogurt cone is 12 cm in height and has a diameter of 6 cm. A scoop of frozen yogurt is placed on the wide end of the cone. The scoop is a sphere with a diameter of 6 cm.

If the scoop of frozen yogurt melts into the cone, will the cone overflow? Explain your reasoning.

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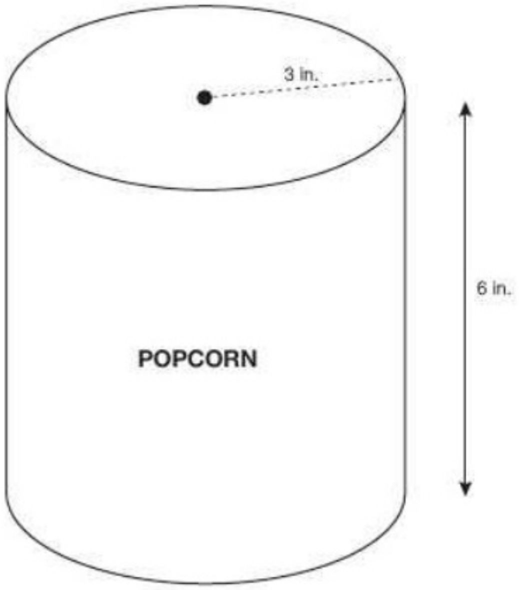
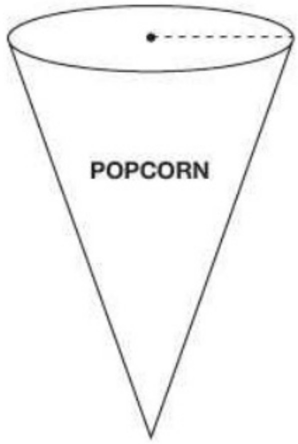
Problem 3 Containers of Popcorn

Consider the cylindrical tub of popcorn shown.



Consider the conical container of popcorn shown.



Assume the conical container and the cylindrical tub of popcorn are full and they hold the same amount of popcorn.

1. Calculate the volume of the cylindrical tub.

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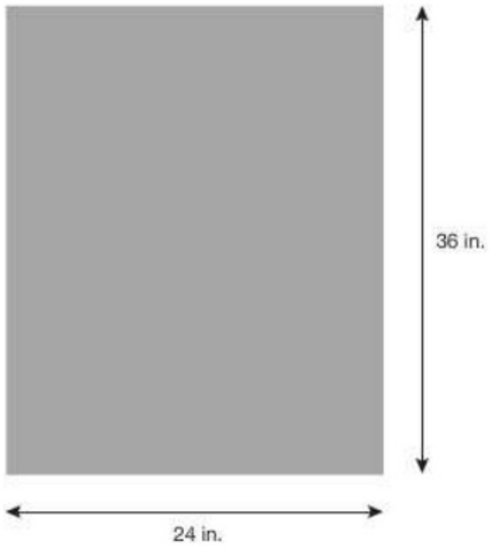
2. Calculate a possible radius and height of the conical container.

3. If the radius of the conical container is the same as the radius of the cylindrical tub, what is the height of the cone?

Problem 4 Poster Board



A standard piece of poster board is 24 inches by 36 inches. A cylindrical shape can be made by taping two ends of the poster board together.

Explain which way the poster board should be taped to result in the cylinder having the greatest volume. Justify your conclusion.

Problem 5 The Doubling Effect

1. If the radius of a cone doubles and the height remains the same, does the volume of the cone double? Explain your reasoning.

2. If the height of a cone doubles and the radius remains the same, does the volume of the cone double? Explain your reasoning.

Be prepared to share you solutions and methods.

Chapter 14 Summary

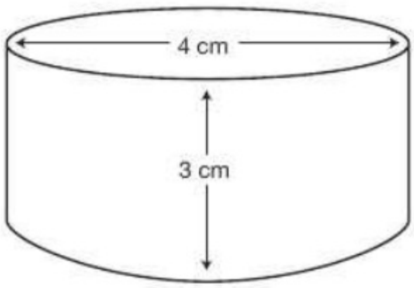
Key Terms

- cylinder (14.1)
- right circular cylinder (14.1)
- radius of a cylinder (14.1)
- height of a cylinder (14.1)
- circumference (14.1)
- pi (14.1)
- cone (14.2)
- height of a cone (14.2)
- sphere (14.3)
- center of a sphere (14.3)
- radius of a sphere (14.3)
- diameter of a sphere (14.3)
- antipodes (14.3)
- great circle (14.3)
- hemisphere (14.3)

14.1 Calculating the Volume of Right Circular Cylinders

A cylinder is a three-dimensional object with two parallel, congruent, circular bases. A right circular cylinder is a cylinder in which the bases are aligned one directly above the other. The two circular portions of the cylinder represent the bases. The area of the base can be determined by using the formula $A = \pi r^2$, where A represents the area of the circle, and r represents the radius of the circle. The volume of a cylinder can be calculated by multiplying the area of the cylinder's base times the height of the cylinder.

Example



The diameter of the base is 4 cm. Therefore, the radius of the base is 2 cm.
The area of the base is $\pi \cdot 2^2 = 3.14(4) = 12.56$ cm².
The volume of the cylinder is $12.56 \cdot 3 = 37.68$ cm³.

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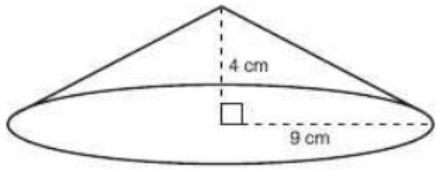
14.2 Calculating the Volume of Cones

A cone is a three-dimensional figure with a circular or elliptical base and one vertex. All of the cones associated with this chapter have a circular base and a vertex that is located directly above the center point of the base of the cone. To calculate the volume of a cone, use the formula $V = \frac{1}{3} \times B \times h$, where V represents the volume of the cone, B represents the area of the base, and h represents the height.

Example



The area of the base is $\pi \cdot 9^2 \approx 3.14(81) \approx 254.34$ cm^2.
The volume of the cone is $\frac{1}{3}(254.34 \cdot 4) = 339.12$ cm^3.

14.3 Calculating the Volume of Spheres

A sphere is the set of all points in three dimensions that are equidistant from a center point. To calculate the volume of a sphere, use the formula $V = \frac{4}{3}\pi r^3$, where V represents the volume of the sphere, and r represents the radius of the sphere.

Example

The radius of a tennis ball is 3.35 cm. Use the radius to find the volume of the tennis ball.

$$\begin{aligned} V &= \frac{4}{3}\pi(3.35^3) \\ &\approx \frac{4}{3}(3.14)(37.60) \\ &\approx 157.42 \end{aligned}$$

The volume of a tennis ball is about 157.42 cm^3.

14.4 Using the Volume of a Sphere to Solve Problems

When the volume of a sphere is known, the radius, diameter, or circumference of the sphere can be calculated using the formula for the volume of a sphere.

Example

The volume of The Sphere sculpture by Fritz Koenig that used to stand between the World Trade Center towers in New York City is 1766.25 ft^3. The formula for the volume of a sphere can be used to find the circumference of the sculpture.

$$\begin{aligned}
V &= \frac{4}{3}\pi r^3 \\
1766.25 &= \frac{4}{3}\pi r^3 \\
\frac{3}{4}(1766.25) &= \frac{3}{4}\left(\frac{4}{3}\right)\pi r^3 \\
1324.6875 &= \pi r^3 \\
\frac{1324.6875}{\pi} &= \frac{\pi r^3}{\pi} \\
421.875 &\approx r^3 \\
\sqrt[3]{421.875} &\approx \sqrt[3]{r^3} \\
7.5 &\approx r
\end{aligned}$$

$$\begin{aligned}
C &= 2\pi r \\
&= 2\pi(7.5) \\
&\approx 47.1
\end{aligned}$$

So, the circumference of the sphere is approximately 47.1 ft.

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