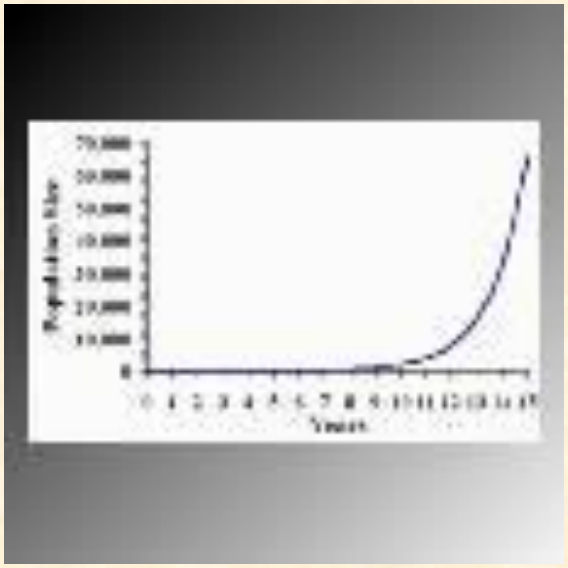
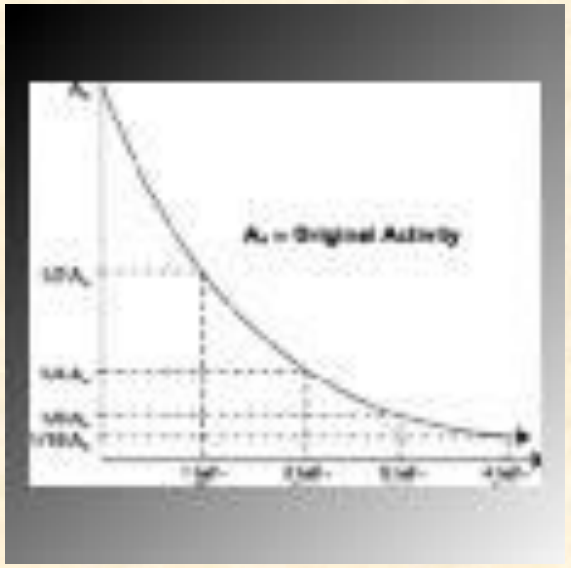
Exponential Functions

Growth



Decay



Graphs of Exponential Functions

Exponential function – A function of the form $y=ab^x$, where b>0 and b≠1.

Step 1 – Make a table of values for the function.

$$y = 3^x$$

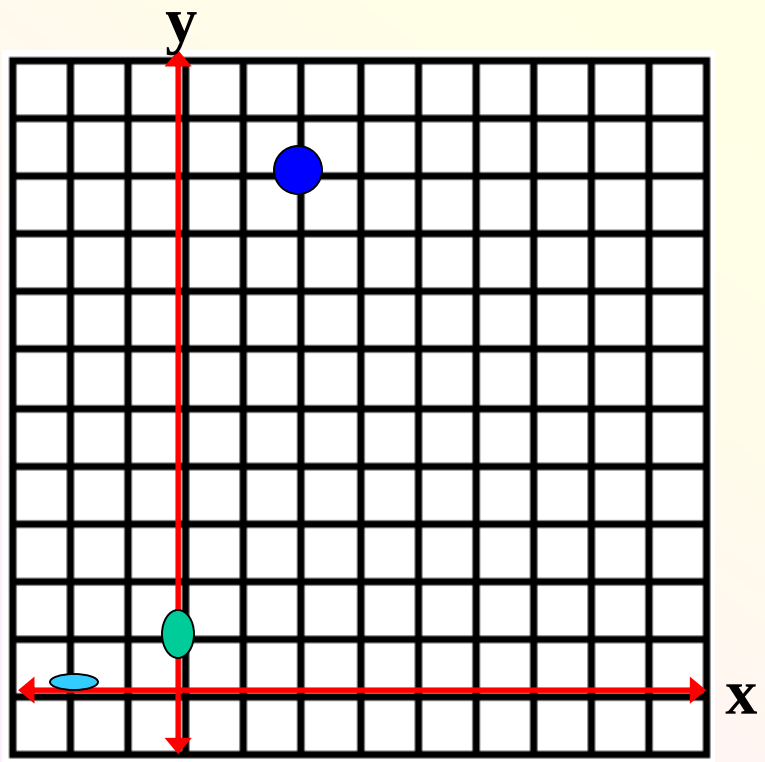
x	$y = 3^x$	y
–2	$3^{-2} = \frac{1}{3^2}$	$\frac{1}{9}$
0	3^0	1
2	3^2	9
3	3^3	27

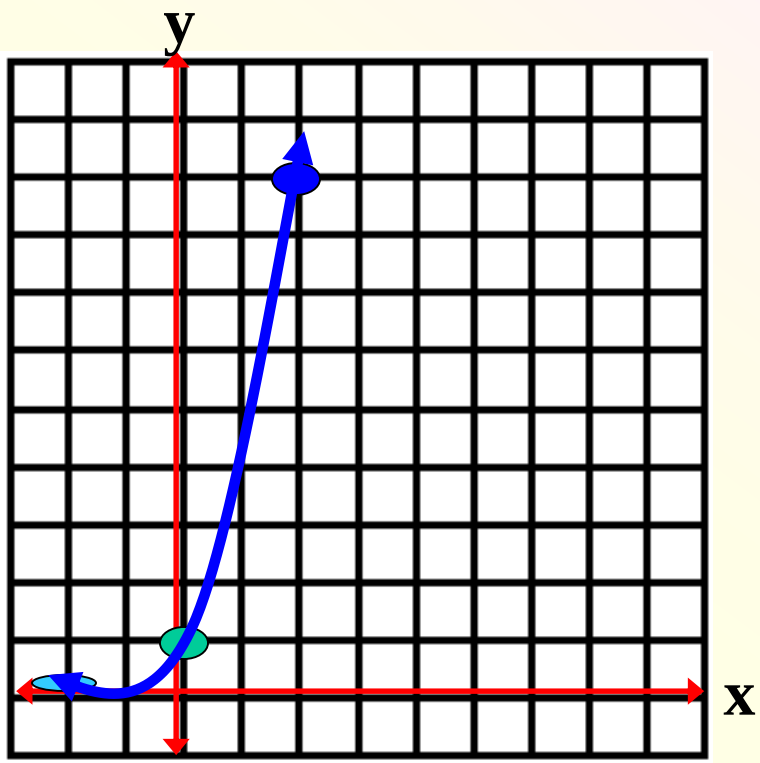
Make the Graph

Now that you have a data table of ordered pairs for the function, you can plot the points on a graph.

(-2, 1/9)	(0,1)	(2,9)

Draw in the curve that fits the plotted points.





Domain and Range

Domain – The collection of all input values of a function. These are usually the "*x*" values.

Range – The collection of all output values of a function. These are usually the "*y*" values.

Describe the domain and range of the function $y = -5^x$.

Domain – The domain of the function is all real numbers since the function is defined for all x-values.

Range – The range of the function is all negative real numbers.

Exponential Growth Functions

If a quantity increases by the same proportion *r* in each unit of time, then the quantity displays exponential growth and can be modeled by the equation

$$y = C(1+r)^t$$

Where

C = initial amount

r = growth rate (percent written as a decimal)

t = time

(1+r) = growth factor where $1 + r > 1$

Example: Compound Interest

You deposit $1500 in an account that pays 2.3% interest compounded yearly,

1) What was the initial principal (**P**) invested?
2) What is the growth rate (**r**)? The growth factor?
3) Using the equation $\mathbf{A = P(1+r)^t}$, how much money would you have after 2 years if you didn't deposit any more money?

1) The initial principal (P) is $1500.
2) The growth rate (r) is 0.023. The growth factor is 1.023.
3) $A = P(1+r)^t$

$A = 1500(1+0.023)^2$

$A = \$1569.79$

Exponential Decay Functions

If a quantity decreases by the same proportion *r* in each unit of time, then the quantity displays exponential decay and can be modeled by the equation

$$y = C(1 - r)^t$$

Where

C = initial amount

r = growth rate (percent written as a decimal)

t = time

(1 - r) = decay factor where $1 - r < 1$

Example: Exponential Decay

You buy a new car for $22,500. The car depreciates at the rate of 7% per year,

1) **What was the initial amount invested?**

2) **What is the decay rate? The decay factor?**

3) **What will the car be worth after the first year? The second year?**

1) The initial investment was $22,500.

2) The decay rate is 0.07. The decay factor is 0.93.

3) $y = C(1-r)^t$ $y = C(1-r)^t$

$y = 22{,}500(1-0.07)^1$ $y = 22{,}500(1-0.07)^2$

$y = \$20{,}925$ $y = \$19460.25$

You Try It

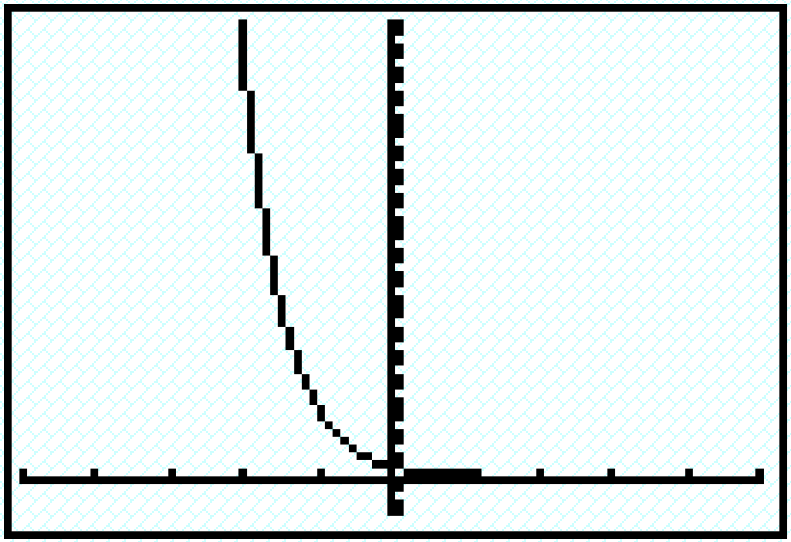
1) Make a table of values for the function $y = \left(\frac{1}{6}\right)^x$ using *x*-values of –2, -1, 0, 1, and 2. Graph the function. Identify the domain and range of the function. Does this function represent exponential growth or exponential decay?

2) Your business had a profit of $25,000 in 1998. If the profit increased by 12% each year, what would your expected profit be in the year 2010? Identify ***C, t, r,*** and *the growth factor***.** Write down the equation you would use and solve.

3) Iodine-131 is a radioactive isotope used in medicine. Its half-life or decay rate of 50% is 8 days. If a patient is given 25mg of iodine-131, how much would be left after 32 days or 4 half-lives. Identify *C*, *t*, *r*, and the *decay factor*. Write down the equation you would use and solve.

Problem 1

x	$y = \left(\frac{1}{6}\right)^x$	y
-2	$\left(\frac{1}{6}\right)^{-2} = 6^2$	36
-1	$\left(\frac{1}{6}\right)^{-1} = 6^1$	6
0	$\left(\frac{1}{6}\right)^{0}$	1
1	$\left(\frac{1}{6}\right)^{1}$	$\frac{1}{6}$
2	$\left(\frac{1}{6}\right)^{2}$	$\frac{1}{36}$



The **domain** of this function is the set of all real numbers.

The **range** of this function is the set of all positive real numbers.

This function represents exponential decay.

Problem 2

C = \$25,000

T = 12

R = 0.12

Growth factor = 1.12

$$y = C(1+r)^{t}$$

$$y = \$25{,}000(1+0.12)^{12}$$

$$y = \$25{,}000(1.12)^{12}$$

$$y = \$97{,}399.40$$

Problem 3

$C = 25$ mg

$T = 4$

$R = 0.5$

Decay factor $= 0.5$

$$y = C(1 - r)^t$$

$$y = 25mg(1 - 0.5)^4$$

$$y = 25mg(0.5)^4$$

$$y = 1.56mg$$