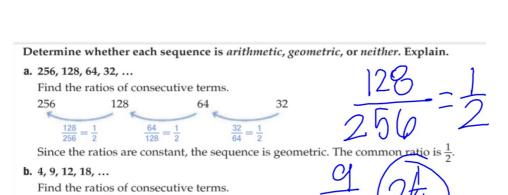**1** **Recognize Geometric Sequences** The first person generates 5 emails. If each of these people sends the email to 5 more people, 25 emails are generated. If each of the 25 people sends 5 emails, 125 emails are generated. The sequence of emails generated, 1, 5, 25, 125, … is an example of a **geometric sequence**.

In a geometric sequence, the first term is nonzero and each term after the first is found by multiplying the previous term by a nonzero constant $r$ called the **common ratio**. The common ratio can be found by dividing any term by its previous term.

5, 10, 20, 40

$$\frac{10}{5} = 2$$

**Determine whether each sequence is *arithmetic*, *geometric*, or *neither*. Explain.**

**a. 256, 128, 64, 32, …**

Find the ratios of consecutive terms.

256        128              64              32

$\frac{128}{256} = \frac{1}{2}$        $\frac{64}{128} = \frac{1}{2}$        $\frac{32}{64} = \frac{1}{2}$

Since the ratios are constant, the sequence is geometric. The common ratio is $\frac{1}{2}$.

$$\frac{128}{256} = \frac{1}{2}$$

**b. 4, 9, 12, 18, …**

Find the ratios of consecutive terms.

4              9              12              18

$\frac{9}{4} = 2\frac{1}{4}$        $\frac{12}{9} = 1\frac{1}{3}$        $\frac{18}{12} = 1\frac{1}{2}$

The ratios are not constant, so the sequence is not geometric.

$$\frac{9}{4} \quad \boxed{2\frac{1}{4}}$$

Find the differences of consecutive terms.

4              9              12              18

$9 - 4 = 5$        $12 - 9 = 3$        $18 - 12 = 6$

There is no common difference, so the sequence is not arithmetic.
Thus, the sequence is neither geometric nor arithmetic.

**1A–1C. See margin.**

**1A.** 1, 3, 9, 27, ...    **1B.** −20, −15, −10, −5, ...    **1C.** 2, 8, 14, 22, ...

geometric
common ratio
3

arithmetic
common difference
5

Neither
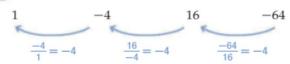no common
difference
& no common
ratio

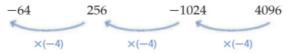**Example 2** Find Terms of Geometric Sequences

Find the next three terms in each geometric sequence.

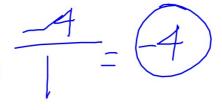a. $1, -4, 16, -64, \ldots$

**Step 1** Find the common ratio.

$$1 \qquad -4 \qquad 16 \qquad -64$$

$$\frac{-4}{1} = -4 \qquad \frac{16}{-4} = -4 \qquad \frac{-64}{16} = -4$$

**Step 2** Multiply each term by the common ratio to find the next three terms.

$$-64 \qquad 256 \qquad -1024 \qquad 4096$$

$$\times(-4) \qquad \times(-4) \qquad \times(-4)$$

The next three terms are 256, $-1024$, and 4096.

① find common ratio $\dfrac{-4}{1} = \boxed{-4}$

$\dfrac{-64}{16} = -4$

**b.** $9, 3, 1, \frac{1}{3} \dots$

**Step 1** Find the common ratio.

$$9 \qquad\qquad 3 \qquad\qquad 1 \qquad\qquad \frac{1}{3}$$

$$\frac{3}{9} = \frac{1}{3} \qquad\qquad \frac{1}{3} = \frac{1}{3} \qquad\qquad \frac{\frac{1}{3}}{3} = \frac{1}{3}$$

The value of $r$ is $\frac{1}{3}$.

**Step 2** Multiply each term by the common ratio to find the next three terms.

$$\frac{1}{3} \qquad\qquad \frac{1}{9} \qquad\qquad \frac{1}{27} \qquad\qquad \frac{1}{81}$$

$$\times\frac{1}{3} \qquad\qquad \times\frac{1}{3} \qquad\qquad \times\frac{1}{3}$$

The next three terms are $\frac{1}{9}, \frac{1}{27}$, and $\frac{1}{81}$.

$$\frac{3}{9} = \frac{1}{3}$$

**2A.** −3, 15, −75, 375, …

**2B.** 24, 36, 54, 81, …

$$\frac{15}{-3} = -5$$

−1875,

9375,

−46875

$$\frac{36}{24} = 1.5 \left(\frac{3}{2}\right)$$

121.5, 182.25,

273.375

practice: page 441
2-8 (even)
and
14-24 (even)