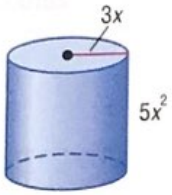
Lesson-by-Lesson Review

7-1 Multiplication Properties of Exponents

Simplify each expression.

11. $x \cdot x^3 \cdot x^5$
12. $(2xy)(-3x^2y^5)$
13. $(-4ab^4)(-5a^5b^2)$
14. $(6x^3y^2)^2$
15. $[(2r^3t)^3]^2$
16. $(-2u^3)(5u)$
17. $(2x^2)^3(x^3)^3$
18. $\frac{1}{2}(2x^3)^3$

19. **GEOMETRY** Use the formula $V = \pi r^2 h$ to find the volume of the cylinder.



Example 1

Simplify $(5x^2y^3)(2x^4y)$.

$(5x^2y^3)(2x^4y)$
$= (5 \cdot 2)(x^2 \cdot x^4)(y^3 \cdot y)$ Commutative Property
$= 10x^6y^4$ Product of Powers

Example 2

Simplify $(3a^2b^4)^3$.

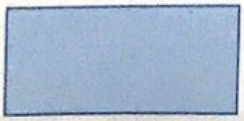
$(3a^2b^4)^3 = 3^3(a^2)^3(b^4)^3$ Power of a Product
$= 27a^6b^{12}$ Simplify.

7-2 Division Properties of Exponents

Simplify each expression. Assume that no denominator equals zero.

20. $\frac{(3x)^0}{2a}$
21. $\left(\frac{3xy^3}{2z}\right)^3$
22. $\frac{12y^{-4}}{3y^{-5}}$
23. $a^{-3}b^0c^6$
24. $\frac{-15x^7y^8z^4}{-45x^3y^5z^3}$
25. $\frac{(3x^{-1})^{-2}}{(3x^2)^{-2}}$
26. $\left(\frac{6xy^{11}z^9}{48x^6yz^{-7}}\right)^0$
27. $\left(\frac{12}{2}\right)\left(\frac{x}{y^5}\right)\left(\frac{y^4}{x^4}\right)$

28. **GEOMETRY** The area of a rectangle is $25x^2y^4$ square feet. The width of the rectangle is $5xy$ feet. What is the length of the rectangle?



Example 3

Simplify $\frac{2k^4m^3}{4k^2m}$. Assume that no denominator equals zero.

$\frac{2k^4m^3}{4k^2m} = \left(\frac{2}{4}\right)\left(\frac{k^4}{k^2}\right)\left(\frac{m^3}{m}\right)$ Group powers with the same base.
$= \left(\frac{1}{2}\right)k^{4-2}m^{3-1}$ Quotient of Powers
$= \frac{k^2m^2}{2}$ Simplify.

Example 4

Simplify $\frac{t^4uv^{-2}}{t^{-3}u^7}$. Assume that no denominator equals zero.

$\frac{t^4uv^{-2}}{t^{-3}u^7} = \left(\frac{t^4}{t^{-3}}\right)\left(\frac{u}{u^7}\right)(v^{-2})$ Group the powers with the same base.
$= (t^{4+3})(u^{1-7})(v^{-2})$ Quotient of Powers
$= t^7u^{-6}v^{-2}$ Simplify.
$= \frac{t^7}{u^6v^2}$ Simplify.

7-3 Rational Exponents

Simplify.

29. $\sqrt[3]{343}$

30. $\sqrt[6]{729}$

31. $625^{\frac{1}{4}}$

32. $\left(\frac{8}{27}\right)^{\frac{1}{3}}$

33. $256^{\frac{3}{4}}$

34. $32^{\frac{2}{5}}$

35. $343^{\frac{4}{3}}$

36. $\left(\frac{4}{49}\right)^{\frac{3}{2}}$

Solve each equation.

37. $6^x = 7776$

38. $4^{4x-1} = 32$

Example 5

Simplify $125^{\frac{2}{3}}$.

$125^{\frac{2}{3}} = \left(\sqrt[3]{125}\right)^2$ — $b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m$

$= \left(\sqrt[3]{5 \cdot 5 \cdot 5}\right)^2$ — $64 = 4^3$

$= 5^2$ or 25 — Simplify.

Example 6

Solve $9^{x-1} = 729$.

$9^{x-1} = 729$ — Original equation

$9^{x-1} = 9^3$ — Rewrite 729 as 9^3.

$x - 1 = 3$ — Power Property of Equality

$x = 4$ — Add 1 to each side.

7-4 Scientific Notation

Express each number in scientific notation.

39. 2,300,000

40. 0.0000543

41. **ASTRONOMY** Earth has a diameter of about 8000 miles. Jupiter has a diameter of about 88,000 miles. Write in scientific notation the ratio of Earth's diameter to Jupiter's diameter.

Example 7

Express 300,000,000 in scientific notation.

Step 1 300,000,000 → 3.00000000

Step 2 The decimal point moved 8 places to the left, so $n = 8$.

Step 3 $300{,}000{,}000 = 3 \times 10^8$

7-5 Exponential Functions

Graph each function. Find the *y*-intercept, and state the domain and range.

42. $y = 2^x$

43. $y = 3^x + 1$

44. $y = 4^x + 2$

45. $y = 2^x - 3$

46. **BIOLOGY** The population of bacteria in a petri dish increases according to the model $p = 550(2.7)^{0.008t}$, where t is the number of hours and $t = 0$ corresponds to 1:00 P.M. Use this model to estimate the number of bacteria in the dish at 5:00 P.M.

Example 8

Graph $y = 3^x + 6$. Find the *y*-intercept, and state the domain and range.

x	$3^x + 6$	y
−3	$3^{-3} + 6$	6.04
−2	$3^{-2} + 6$	6.11
−1	$3^{-1} + 6$	6.33
0	$3^0 + 6$	7
1	$3^1 + 6$	9

The *y*-intercept is (0, 7). The domain is all real numbers, and the range is all real numbers greater than 6.

7-6 Growth and Decay

47. Find the final value of $2500 invested at an interest rate of 2% compounded monthly for 10 years.

48. COMPUTERS Zita's computer is depreciating at a rate of 3% per year. She bought the computer for $1200.

a. Write an equation to represent this situation.

b. What will the computer's value be after 5 years?

Example 9

Find the final value of $2000 invested at an interest r of 3% compounded quarterly for 8 years.

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ Compound interest equati

$= 2000\left(1 + \frac{0.03}{4}\right)^{4(8)}$ $P = 2000$, $r = 0.03$, $n = 4$, and $t = 8$

$\approx \$2540.22$ Use a calculator.

7-7 Geometric Sequences as Exponential Functions

Find the next three terms in each geometric sequence.

49. $-1, 1, -1, 1, \ldots$

50. $3, 9, 27, \ldots$

51. $256, 128, 64, \ldots$

Write the equation for the nth term of each geometric sequence.

52. $-1, 1, -1, 1, \ldots$

53. $3, 9, 27, \ldots$

54. $256, 128, 64, \ldots$

55. SPORTS A basketball is dropped from a height of 20 feet. It bounces to $\frac{1}{2}$ its height after each bounce. Draw a graph to represent the situation.

Example 10

Find the next three terms in the geometric sequence 2, 6, 18, … .

Step 1 Find the common ratio. Each number is 3 times the previous number, so $r = 3$.

Step 2 Multiply each term by the common ratio to find the next three terms.

$18 \times 3 = 54$, $54 \times 3 = 162$, $162 \times 3 = 486$

The next three terms are 54, 162, and 486.

Example 11

Write the equation for the nth term of the geometric sequence $-3, 12, -48, \ldots$.

The common ratio is -4. So $r = -4$.

$a_n = a_1 r^{n-1}$ Formula for the nth term

$a_n = -3(-4)^{n-1}$ $a_1 = -3$ and $r = -4$

7-8 Recursive Formulas

Find the first five terms of each sequence.

56. $a_1 = 11$, $a_n = a_{n-1} - 4$, $n \geq 2$

57. $a_1 = 3$, $a_n = 2a_{n-1} + 6$, $n \geq 2$

Write a recursive formula for each sequence.

58. $2, 7, 12, 17, \ldots$

59. $32, 16, 8, 4, \ldots$

60. $2, 5, 11, 23, \ldots$

Example 12

Write a recursive formula for $3, 1, -1, -3, \ldots$.

Step 1 First subtract each term from the term that follows it.

$1 - 3 = -2$, $-1 - 1 = -2$, $-3 - (-1) = -2$

There is a common difference of -2. The sequence is arithmetic.

Step 2 Use the formula for an arithmetic sequence.

$a_n = a_{n-1} + d$ Recursive formula

$a_n = a_{n-1} + (-2)$ $d = -2$

Step 3 The first term a_1 is 3, and $n \geq 2$.

A recursive formula is $a_1 = 3$, $a_n = a_{n-1} - 2$, $n \geq 2$.