

## SLIDING RIGHT, LEFT, UP, DOWR, AMD DIAGONALLY Translations Using Geometric Figures

## Learning Goals

In this lesson, you will:

- Translate geometric figures horizontally.
- Translate geometric figures vertically.
- Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of translations.


## Key Terms

- transformation
- translation
- image
- pre-image

T.o begin this chapter, cut out the figures shown on this page. You will have a trapezoid, two triangles, and a parallelogram. You will be using these figures in several lessons. What do you know about these shapes?



## Problem 1 Sliding to the Right, Left, Up, and Down



Let's explore different ways to move, or transform, figures across a coordinate plane.
A transformation is the mapping, or movement, of all the points of a figure in a plane according to a common operation.

1. Look at the parallelogram shown on the coordinate plane.


a. Place your parallelogram on the original figure on the coordinate plane shown and slide it 5 units to the left. Trace your parallelogram on the coordinate plane, and label it Figure 1.
b. Place your parallelogram on the original figure on the coordinate plane shown and slide it 5 units down. Trace your parallelogram on the coordinate plane, and label it Figure 2.
c. Place your parallelogram on Figure 1 on the coordinate plane and slide it 5 units down. Trace your parallelogram on the coordinate plane, and label it Figure 3.
d. Describe how all of the parallelograms you traced on the coordinate plane compare with each other.

2. Recall that two geometric figures are considered congruent when they are the same size and the same shape.
a. Did sliding the parallelogram either up or down on the coordinate plane change the size or shape of the parallelogram?
b. Are Figure 1, Figure 2, and Figure 3 all congruent to the original parallelogram shown on the coordinate plane? Explain your reasoning.


When you were sliding the parallelogram to the different places, you were performing translations of the parallelogram. A translation is a transformation that "slides" each point of a figure the same distance and direction. Sliding a figure left or right is a horizontal translation, and sliding it up or down is a vertical translation. The new figure created from the translation is called the image. The original figure is called the pre-image.
3. Look at the triangle shown on the coordinate plane.

a. List the ordered pairs for the vertices of $\triangle A B C$.

b. Place your triangle on $\triangle A B C$, and translate it -6 units vertically. Trace the new triangle, and label the vertices $A^{\prime}, B^{\prime}$, and $C^{\prime}$ in $\triangle A^{\prime} B^{\prime} C^{\prime}$ so the vertices correspond to the vertices $A, B$, and $C$ in $\triangle A B C$.
c. List the ordered pairs for the vertices of $\triangle A^{\prime} B^{\prime} C^{\prime}$.
d. Place your triangle on $\triangle A B C$, and translate it -6 units horizontally. Trace the new triangle, and label the vertices $A^{\prime \prime}, B^{\prime \prime}$, and $C^{\prime \prime}$ in $\triangle A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}$ so the vertices correspond to the vertices $A, B$, and $C$ in $\triangle A B C$.
e. List the ordered pairs for the vertices of $\triangle A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}$.
f. Compare the ordered pairs in $\triangle A B C$ and $\triangle A^{\prime} B^{\prime} C^{\prime}$. How are the values in the ordered pairs affected by the translation?
g. Compare the ordered pairs in $\triangle A B C$ and $\triangle A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}$. How are the values in the ordered pairs affected by the translation?
h. If you were to translate $\triangle A B C 10$ units vertically to form $\triangle D E F$, what would be the ordered pairs of the corresponding vertices?
i. If you were to translate $\triangle A B C 10$ units horizontally to form $\triangle G H J$, what would be the ordered pairs of the corresponding vertices?

4. Recall that two geometric figures are considered congruent when they are the same size and the same shape.
a. Did sliding the triangle either up or down on the coordinate plane change the size or shape of the triangle?
b. Are both of the triangles you drew congruent to the triangle shown on the coordinate plane? Explain your reasoning.
5. Look at the triangle shown on the coordinate plane.

a. List the ordered pairs for the vertices of $\triangle A B C$.
b. Place your triangle on $\triangle A B C$, and translate it -5 units vertically. Trace the new triangle, and label the vertices $A^{\prime}, B^{\prime}$, and $C^{\prime}$ in $\triangle A^{\prime} B^{\prime} C^{\prime}$ so the vertices correspond to the vertices $A, B$, and $C$ in $\triangle A B C$.
c. List the ordered pairs for the vertices of $\triangle A^{\prime} B^{\prime} C^{\prime}$.
d. Place your triangle on $\triangle A B C$, and translate it -5 units horizontally. Trace the new triangle, and label the vertices $A^{\prime \prime}, B^{\prime \prime}$, and $C^{\prime \prime}$ in $\triangle A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}$ so the vertices correspond to the vertices $A, B$, and $C$ in $\triangle A B C$.
e. List the ordered pairs for the vertices of $\triangle A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}$.
f. Compare the ordered pairs in $\triangle A B C$ and $\triangle A^{\prime} B^{\prime} C^{\prime}$. How are the values in the ordered pairs affected by the translation?
g. Compare the ordered pairs in $\triangle A B C$ and $\triangle A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}$. How are the values in the ordered pairs affected by the translation?
h. If you were to translate $\triangle A B C 10$ units vertically to form $\triangle D E F$, what would be the ordered pairs of the corresponding vertices?
i. If you were to translate $\triangle A B C 10$ units horizontally to form $\triangle G H J$, what would be the ordered pairs of the corresponding vertices?
6. Are both triangles congruent to the original triangle shown on the coordinate plane? Explain your reasoning.


## Problem 2 Translating a Trapezoid

1. Look at the trapezoid shown on the coordinate plane.

a. List the ordered pairs for the vertices of trapezoid $A B C D$.
b. Place your trapezoid on trapezoid $A B C D$, and translate it -5 units vertically. Trace the new trapezoid, and label the vertices $A^{\prime}, B^{\prime}, C^{\prime}$, and $D^{\prime}$ in trapezoid $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$ so the vertices correspond to the vertices $A, B, C$, and $D$ in trapezoid $A B C D$.
c. List the ordered pairs for the vertices of trapezoid $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$.
d. Place your trapezoid on trapezoid $A B C D$, and translate it -5 units horizontally. Trace the new trapezoid, and label the vertices $A^{\prime \prime}, B^{\prime \prime}, C^{\prime \prime}$, and $D^{\prime \prime}$ in trapezoid $A^{\prime \prime} B^{\prime \prime} C^{\prime \prime} D^{\prime \prime}$ so the vertices correspond to the vertices $A, B, C$, and $D$ in trapezoid $A B C D$.
e. List the ordered pairs for the vertices of trapezoid $A^{\prime \prime} B^{\prime \prime} C^{\prime \prime} D^{\prime \prime}$.

f. Compare the ordered pairs in trapezoid $A B C D$ and trapezoid $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$. How are the values in the ordered pairs affected by the translation?
g. Compare the ordered pairs in trapezoid $A B C D$ and trapezoid $A^{\prime \prime} B^{\prime \prime} C^{\prime \prime} D^{\prime \prime}$. How are the values in the ordered pairs affected by the translation?
h. If you were to translate trapezoid $A B C D 10$ units vertically to form trapezoid $D E F G$, what would be the ordered pairs of the corresponding vertices?
i. If you were to translate trapezoid $A B C D 10$ units horizontally to form trapezoid HJKM, what would be the ordered pairs of the corresponding vertices?
2. Recall that two geometric figures are considered congruent when they are the same size and the same shape.
a. Did sliding the trapezoid either up or down on the coordinate plane change the size or shape of the trapezoid?
b. Are both trapezoids congruent to the original trapezoid shown on the coordinate plane? Explain your reasoning.

## Talk the Talk



1. Are all images, or new figures that result from a translation, always congruent to the original figure? Explain your reasoning.
2. For any real number $c$ or $d$, describe how the ordered pair $(x, y)$ of any original figure will change when translated:
a. horizontally $c$ units. How do you know if the image translated to the left or to the right?
b. vertically $d$ units. How do you know if the image translated up or down?

Be prepared to share your solutions and methods.


## Sliding lines Translations of Linear Functions

## Learning Goals

In this lesson, you will:

- Translate linear functions horizontally and vertically.
- Use multiple representations such as tables, graphs, and equations to represent linear functions and the translations of linear functions.
ook at the lines below each row of black and white squares. Are these lines straight? Grab a ruler or other straightedge to test.


This very famous optical illusion is called the Zöllner illusion, named after its discoverer, Johann Karl Friedrich Zöllner, who first wrote about it in 1860.

## Problem 1 Translating Linear Functions Up or Down

In the previous lesson, geometric figures were translated vertically (up or down) and horizontally (left or right). In this lesson, you will use that knowledge to translate linear functions both vertically and horizontally.

1. Consider the equation $y=x$. Complete the table of values.

| $x$ | $y$ |
| :---: | :---: |
| -3 |  |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |


2. Use the table of values and the coordinate plane provided to graph the equation $y=x$.

3. Did you connect the points on the graph of the equation? Why or why not?
4. In the previous lesson, a geometric figure was translated down 4 units.
a. How did that affect the value of the $x$-coordinate of each vertex?
b. How did that affect the value of the $y$-coordinate of each vertex?
5. Use your experience of translating a geometric figure to translate the graph of $y=x$ down 4 units. Draw the new line on the coordinate plane in Question 2 and then complete the table of values.

| $x$ | $y$ |
| :---: | :---: |
| -3 |  |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |


6. Compare the graph of $y=x$ to the graph of $y=x$ translated down 4 units.
a. What do you notice?
b. Write an equation in the form $y=$ to represent the translation.

7. Translate the graph of $y=x$ up 4 units. Draw the new line on the coordinate plane in Question 2 and then complete the table of values.

| $x$ | $y$ |
| :---: | :---: |
| -3 |  |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |

8. Compare the graph of $y=x$ to the graph of $y=x$ translated up 4 units.
a. What do you notice?
b. Write an equation in the form $y=$ to represent the translation.
c. Write an equation in the form $x=$ to represent the translation.

9. Label each equation on the coordinate plane in slope-intercept form. What do you notice? What is similar about each line? What is different?

## Problem 2 Translating Linear Functions Left or Right

1. In the previous lesson, a geometric figure was translated to the left 4 units.
a. How did that affect the value of the $x$-coordinate of each vertex?
b. How did that affect the value of the $y$-coordinate of each vertex?

7.2 Translations of Linear Functions

- 399

2. Graph the equation $y=x$ on the coordinate plane.

3. Use your experience of translating a geometric figure to translate the graph of $y=x$ to the left 4 units. Draw the new line on the coordinate plane and then complete the table of values.

4. Compare the graph of $y=x$ to the graph of $y=x$ translated to the left 4 units.
a. What do you notice?
b. Write an equation in the form $y=$ to represent the translation.
c. Write an equation in the form $x=$ to represent the translation.
5. Translate the graph of $y=x$ to the right 4 units. Draw the new line on the coordinate plane in Question 2 and then complete the table of values.

6. Compare the graph of $y=x$ to the graph of $y=x$ translated to the right 4 units.
a. What do you notice?
b. Write an equation in the form $y=$ to represent the translation.
c. Write an equation in the form $x=$ to represent the translation.
7. Label each equation on the coordinate plane in slope-intercept form. What do you notice? What is similar about each line? What is different?

## Problem 3 Making Connections



1. Organize the equations you determined for the graph of each translation performed on the linear equation $y=x$ in the previous problem by completing the last two columns of the table shown.

| Original <br> Equation | Translation <br> Performed | Equation of <br> Translation in the <br> Form of $y=$ | Equation of <br> Translation in the <br> Form of $x=$ |
| :---: | :---: | :---: | :---: |
| $y=x$ | Down 4 Units | $y=$ | $x=$ |
| $y=x$ | Up 4 Units | $y=$ | $x=$ |
| $y=x$ | Left 4 Units | $y=$ | $x=$ |
| $y=x$ | Right 4 Units | $y=$ | $x=$ |

2. Which translations of the linear equation $y=x$ resulted in the same graph?
3. Kieran says that whenever a linear equation written in slope-intercept form shows a plus sign, it is a translation right or up, and when it shows a minus sign it is a translation left or down, because positive always means up and right on the coordinate grid, and negative always means left and down. Is Kieran correct? Justify your answer.

4. Each graph shown is a result of a translation performed on the equation $y=x$. Describe the translation. Then write an equation in slope-intercept form.
a.

b.

5. Each graph shown is a result of a translation performed on the equation $y=-x$. Describe the translation.
a.

b.

6. Each equation shown is a result of a translation performed on the equation $y=x$. Describe the translation.
a. $y=x+12.5$
b. $y=x-15.25$
7. Each equation shown is a result of a translation performed on the equation $y=-x$. Describe the translation.
a. $y=-x-1.2$
b. $y=-x+3.8$

## Talk the Talk

1. The equation shown is a result of a translation performed on the equation $y=x$.

For any real number $h$, describe the possible translations.
$y=x+h$
2. The equation shown is a result of a translation performed on the equation $y=-x$. For any real number $h$, describe the possible translations.

$$
y=-x+h
$$

3. If a function is translated horizontally or vertically, is the resulting line still a function?

# 7.3 <br> <br> ROUAD AAD ROUHD <br> <br> ROUAD AAD ROUHD WE GO! WE GO! Rotations of Geometric Figures Rotations of Geometric Figures on the Coordinate Plane 

 on the Coordinate Plane}

## Learning Goal

In this lesson, you will:

- Rotate geometric figures on the coordinate plane.


## Key Terms

- rotation
- angle of rotation
- point of rotation

$C$entrifuges are devices that spin material around a center point. Centrifuges are used in biology and chemistry, often to separate materials in a gas or liquid.

Tubes are inserted into the device and, as it spins, heavier material is pushed to the bottom of the tubes while lighter material tends to rise to the top.

Human centrifuges are used to test pilots and astronauts. Can you think of other devices that work like centrifuges?

## Problem 1 What Is a Rotation?

You have considered what happens to shapes when you slide them up, down, left, or right. Let's explore what happens when you rotate a geometric figure.


1. Look at the triangles shown in the coordinate plane.


2. Place your triangle on $\triangle A B C$. Without moving vertex $A$, "transform" the triangle into $\triangle A B^{\prime} C^{\prime}$.
3. Describe how you transformed the triangle.
4. Katie says that she can use translations to move triangle $A B C$ to triangle $A B^{\prime} C^{\prime}$. Is she correct? Explain your reasoning.

A rotation is a transformation that turns a figure about a fixed point for a given angle, called the angle of rotation, and a given direction. The angle of rotation is the amount of rotation about a fixed point, or point of rotation. Rotation can be clockwise or counterclockwise.

The point of rotation can be a point on the figure.


Or, it can be a point not on the figure.

The point of rotation stays
fixed.


It can also be a point in the figure.

5. Use your triangle to rotate $\triangle A B C$ in Question 1 by placing your triangle on the figure, putting a pin in it at vertex $C$, and then rotating your triangle first to the left and then to the right.
6. Using $\overline{A C}$ as one side of the angle, measure and draw $\angle A C A^{\prime}$ to be $120^{\circ}$. Then, rotate your triangle clockwise to produce $\triangle C A^{\prime} B^{\prime}$. Label your rotation in the coordinate plane.


7. Use your triangle to rotate $\triangle A B C$ by placing your triangle on the figure, putting a pin in it at any point on side $\overline{A C}$, and then rotating your triangle first clockwise and then counterclockwise. Trace one rotation you performed on the coordinate plane as $\triangle A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}$.

8. Recall that two geometric figures are considered congruent when they are the same size and the same shape.
a. Did rotating the triangle on the coordinate plane in any of the previous questions change the size or shape of the triangle?
b. Is the image of the triangle that resulted from the rotation congruent to the triangle shown on the coordinate plane? Explain your reasoning.

## Problem 2 Rotating a Parallelogram



1. Use your parallelogram to rotate parallelogram $A B C D$ by placing your parallelogram on the figure, putting a pin in it at any point in the interior of the parallelogram, and then rotating your parallelogram first clockwise and then counterclockwise. Trace one rotation you performed on the coordinate plane. Place a point at your center of rotation.


2. Recall that two geometric figures are considered congruent when they are the same size and the same shape.
a. Did rotating the parallelogram on the coordinate plane change the size or shape of the parallelogram?
b. Is the image of the parallelogram that resulted from the rotation congruent to the parallelogram shown on the coordinate plane? Explain your reasoning.

## Problem 3 Rotating a Trapezoid



1. Use your trapezoid to rotate trapezoid $A B C D$ around point $P$ by placing your trapezoid on the figure. Fold a piece of tape in half and tape it to both sides of the trapezoid, making sure that the tape covers point $P$. Put a pin in at point $P$, and rotate your parallelogram first clockwise and then counterclockwise. Trace one rotation you performed on the coordinate plane.
2. Recall that two geometric figures are considered congruent when they are the same size and the same shape.
a. Did rotating the trapezoid on the coordinate plane change the size or shape of the trapezoid?
b. Is the image of the trapezoid congruent to the trapezoid shown on the coordinate plane? Explain your reasoning.

## Talk the Talk



1. Are all images, or new figures that result from a rotation, always congruent to the original figure? Explain your reasoning.
2. Describe the point of rotation in each.
a.

b.

c.


- Chapter 7 Translations, Reflections, and Rotations


# NIRROR, NIRROR Reflections of Geometric Figures on the Coordinate Plane 

Learning Goals
In this lesson, you will:

- Reflect geometric figures across the axes on the coordinate plane.
- Reflect geometric figures across lines on the coordinate plane.

Key Terms

- reflection
- reflection line

T
he astronauts aboard the Apollo Moon missions in 1969 through the 1970s did more than just play golf and take pictures. They also set up equipment on the Moon to help scientists measure the distance from the Moon to the Earth.

This equipment contained sets of mirrors, called retroreflectors. Scientists on Earth can now shoot laser beams at these mirrors and calculate the distance to the Moon by observing how long it takes the laser beam to "bounce back."

## Problem 1 Reflections Across the Axes

In this lesson you will explore what happens to geometric figures that are reflected across different lines.


1. Look at the two triangles shown in the coordinate plane.


a. Describe the positions of the two triangles on this coordinate plane. Do you think the two triangles are congruent?
b. Place a mirror on the $y$-axis facing to the left. Describe what you see when you look at the triangle in the mirror.
c. Place a mirror on the $y$-axis facing to the right. Describe what you see when you look at the triangle in the mirror.

Figures that are mirror images of each other are called reflections. A reflection is a transformation that "flips" a figure across a reflection line. A reflection line is a line that acts as a mirror so that corresponding points are the same distance from the mirror. In this coordinate plane, either triangle is a reflection of the other.
d. What do you think is the reflection line in the diagram shown?
e. Draw the reflection of each of the triangles across the $x$-axis.


2. Reflect parallelogram $A B C D$, using the $y$-axis as the reflection line, to form parallelogram $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$.


a. Connect each vertex of the original parallelogram to the corresponding vertex of the image with line segments $\overline{A A}^{\prime}, \overline{B B}^{\prime}, \overline{C C}^{\prime}$, and $\overline{D D}^{\prime}$.
b. Describe the relationship between the $y$-axis and each of the segments you drew.

c. List the ordered pairs for the vertices of parallelogram $A B C D$ and parallelogram $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$.
d. What do you notice about the ordered pairs of the vertices of the original figure and its reflection across the $y$-axis?

3. Reflect parallelogram $A B C D$ across the $x$-axis by using the $x$-axis as a perpendicular bisector.

a. List the ordered pairs for the vertices of the original parallelogram and the reflected image.
b. What do you notice about the ordered pairs of the vertices of the original figure and its reflection across the $x$-axis?
4. A triangle has vertices at $A(-4,3), B(1,5), C(2,-2)$.
a. If this triangle is reflected across the $x$-axis, what would the ordered pairs of the reflection's vertices be?
b. If this triangle is reflected across the $y$-axis, what would the ordered pairs of the reflection's vertices be?

## Problem 2 Reflections Across Horizontal and Vertical Lines



1. Reflect the triangle across the line $x=-1$.


2. Reflect the triangle across the line $y=2$.

3. Recall that geometric figures are considered congruent when they are the same size and the same shape.
a. Did reflecting the triangle on the coordinate plane change the size or shape of the figure?
b. Is the image of the reflection of the triangle congruent to the original figure shown on the coordinate plane? Explain your reasoning.

## Talk the Talk



1. Are all images, or new figures that result from a reflection, always congruent to the original figure? Explain.
2. Describe the line of reflection in each.
a.

b.


Be prepared to share your solutions and methods.

## Chapter 7 Summary

## Key Terms

- transformation (7.1)
- translation (7.1)
$>$ image (7.1)
- pre-image (7.1)
- rotation (7.3)
- angle of rotation (7.3)
- point of rotation (7.3)
- reflection (7.4)
- reflection line (7.4)


### 7.1 Translating Geometric Figures

A translation is a transformation that "slides" each point of a figure the same distance and direction. Sliding a figure left or right is a horizontal translation and sliding it up or down is a vertical translation. The new figure created from a translation is called the image.

## Example

$\triangle A B C$ with coordinates $A(-2,2), B(0,5)$, and $C(1,1)$ is translated six units horizontally and -4 units vertically.


## 7.2 <br> Translating Linear Functions

You learned that a translation is a transformation that "slides" each point of a geometric figure the same distance and direction. That knowledge can also be applied to linear functions on a coordinate plane.

## Example

Complete the table of values using the function $y=x$.

| $x$ | $y$ |
| :---: | :---: |
| -2 | -2 |
| -1 | -1 |
| 1 | 1 |
| 2 | 2 |

Graph the function using the table of values.


Translate the graph of $y=x$ up 5 units and complete the table of values.

| $x$ | $y$ |
| :---: | :---: |
| -2 | 3 |
| -1 | 4 |
| 1 | 6 |
| 2 | 7 |

Write an equation in the form $y=$ to represent the translation.

$$
y=x+5
$$

## 7.3 <br> Rotating Geometric Figures on a Coordinate Plane

A rotation is a transformation that turns a figure about a fixed point for a given angle and a given direction. The given angle is called the angle of rotation. The angle of rotation is the amount of rotation about a fixed point. The point around which the figure is rotated is called the point of rotation. Rotations can be either clockwise or counterclockwise.

## Example

To rotate $\triangle X Y Z 45^{\circ}$ clockwise around point $Z$, use a protractor to draw a $45^{\circ}$ angle as shown, with point $Z$ as the vertex. Next, rotate the figure clockwise around point $Z$ until the side corresponding to $\overline{Y Z}$ has been rotated $45^{\circ}$. The image is labeled as $\triangle X^{\prime} Y^{\prime} Z$.


## $7 . \Delta 3$ Reflecting Geometric Figures on the Coordinate Plane

A reflection is a transformation that "flips" a figure across a reflection line. A reflection line is a line that acts as a mirror such that corresponding points in the figure and its image are the same distance from the line.

When a figure is reflected across the $x$-axis, the $y$-values of the points on the image have the opposite sign of the $y$-values of the corresponding points on the original figure while the $x$-values remain the same. When a figure is reflected across the $y$-axis, the $x$-values of the points on the image have the opposite sign of the $x$-values of the corresponding points on the original figure while the $y$-values remain the same.

## Example

A square with vertices $P(-1,5), Q(2,8), R(5,5)$, and $S(2,2)$ is reflected across the $x$-axis.
To determine the vertices of the image, change the sign of the $y$-coordinates of the figure's vertices to find the $y$-coordinates of the image's vertices. The $x$-coordinates remain the same. The vertices of the image are $P^{\prime}(-1,-5), Q^{\prime}(2,-8), R^{\prime}(5,-5)$, and $S^{\prime}(2,-2)$.


