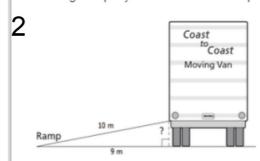
One leg of a right triangle is 14 cm long, and the hypotenuse is 26 cm long. What is the approximate length of the other leg?

- 12 cm
- 18 cm
- 22 cm
- 24 cm

A moving company needs to build a ramp for its moving van, as shown in the diagram.



What must be the height of the ramp, to the nearest tenth of a meter?

- 1.0 m
- 4.4 m
- 13.5 m
- 19.0 m

# **Example 1** Square of a Sum

Find  $(3x + 5)^2$ .

$$(a + b)^2 = a^2 + 2ab + b^2$$

Square of a sum

$$(3x + 5)^2 = (3x)^2 + 2(3x)(5) + 5^2$$
  $a = 3x, b = 5$ 

$$a = 3x$$
,  $b = 5$ 

$$= 9x^2 + 30x + 25$$

 $=9x^2 + 30x + 25$  Simplify. Use FOIL to check your solution.

#### **Guided**Practice

Find each product.

**1A.** 
$$(8c + 3d)^2$$

**1B.** 
$$(3x + 4y)^2$$



# **Example 2** Square of a Difference

Find  $(2x - 5y)^2$ .

$$(a-b)^2 = a^2 - 2ab + b^2$$
 Square of a difference  $(2x-5y)^2 = (2x)^2 - 2(2x)(5y) + (5y)^2$   $a = 2x$  and  $b = 5y$  Simplify.

# **Guided**Practice

Find each product.

**2A.** 
$$(6p-1)^2$$

**2B.** 
$$(a-2b)^2$$



# **Example 4** Product of a Sum and a Difference

Find 
$$(2x^2 + 3)(2x^2 - 3)$$
.

$$(a+b)(a-b) = a^2 - b^2$$
 Product of a sum and a difference

$$(2x^2 + 3)(2x^2 - 3) = (2x^2)^2 - (3)^2$$
  $a = 2x^2$  and  $b = 3$ 

$$=4x^4-9$$
 Simplify.

### **Guided**Practice

Find each product.

**4A.** 
$$(3n+2)(3n-2)$$

**4B.** 
$$(4c - 7d)(4c + 7d)$$

**Use the Distributive Property to Factor** You have used the Distributive Property to multiply a monomial by a polynomial. You can work backward to express a polynomial as the product of a monomial factor and a polynomial factor.

$$1.6w^2 + 6w = 1.6w(w) + 6(w)$$
  
=  $w(1.6w + 6)$ 

So, w(1.6w + 6) is the *factored form* of  $1.6w^2 + 6w$ . **Factoring** a polynomial involves finding the *completely* factored form.



# **Example 1** Use the Distributive Property



Use the Distributive Property to factor each polynomial.

a. 
$$27y^2 + 18y$$

Find the GCF of each term.

$$27y^2 = 3 \cdot 3 \cdot 3 \cdot y \cdot y$$
$$18y = 2 \cdot 3 \cdot 3 \cdot y$$

Factor each term.

Circle common factors.

$$GCF = 3 \cdot 3 \cdot y \text{ or } 9y$$

Write each term as the product of the GCF and the remaining factors. Use the Distributive Property to factor out the GCF.

$$27y^{2} + 18y = 9y(3y) + 9y(2)$$
$$= 9y(3y + 2)$$

Rewrite each term using the GCF. **Distributive Property** 

**b.**  $-4a^2b - 8ab^2 + 2ab$ 

$$-4a^{2}b = -1 \cdot 2 \cdot 2 \cdot a \cdot a \cdot b$$
  
$$-8ab^{2} = -1 \cdot 2 \cdot 2 \cdot 2 \cdot a \cdot b \cdot b$$

Factor each term.

$$-8ab^2 = -1 \cdot 2 \cdot 2 \cdot 2 \cdot a \cdot b \cdot b$$

Circle common factors.

$$2ab = 2 \cdot a \cdot b$$

$$GCF = 2 \cdot a \cdot b \text{ or } 2ab$$

$$-4a^2b - 8ab^2 + 2ab = 2ab(-2a) - 2ab(4b) + 2ab(1)$$
 Rewrite each term using the GCF.  
=  $2ab(-2a - 4b + 1)$  Distributive Property

**Distributive Property** 

#### **Guided**Practice

**1A.** 
$$15w - 3v$$

**1B.** 
$$7u^2t^2 + 21ut^2 - ut$$

Using the Distributive Property to factor polynomials with four or more terms is called **factoring by grouping** because terms are put into groups and then factored. The Distributive Property is then applied to a common binomial factor.

# KeyConcept Factoring by Grouping

Words A polynomial can be factored by grouping only if all of the following conditions exist.

- There are four or more terms.
- Terms have common factors that can be grouped together.
- There are two common factors that are identical or additive inverses of each other.

Symbols

$$ax + bx + ay + by = (ax + bx) + (ay + by)$$
  
=  $x(a + b) + y(a + b)$   
=  $(x + y)(a + b)$ 

# **Example 2** Factor by Grouping



Factor 4qr + 8r + 3q + 6.

$$4qr + 8r + 3q + 6$$
 Original expression
$$= (4qr + 8r) + (3q + 6)$$
 Group terms with common factors.
$$= 4r(q + 2) + 3(q + 2)$$
 Factor the GCF from each group.

Notice that (q + 2) is common in both groups, so it becomes the GCF.

$$=(4r+3)(q+2)$$

**Distributive Property** 

### GuidedPractice

Factor each polynomial.

**2A.** 
$$rn + 5n - r - 5$$

**2B.** 
$$3np + 15p - 4n - 20$$



## **Example 3** Factor by Grouping with Additive Inverses

Factor 2mk - 12m + 42 - 7k.

$$2mk - 12m + 42 - 7k$$

$$= (2mk - 12m) + (42 - 7k)$$

$$= 2m(k-6) + 7(6-k)$$

$$= 2m(k-6) + 7[(-1)(k-6)] \qquad 6-k = -1(k-6)$$

$$= 2m(k-6) - 7(k-6)$$

$$=(2m-7)(k-6)$$

Group terms with common factors.

Factor the GCF from each group.

$$6 - k = -1(k - 6)$$

**Associative Property** 

**Distributive Property** 

#### GuidedPractice

Factor each polynomial.

**3A.** 
$$c - 2cd + 8d - 4$$

**3B.** 
$$3p - 2p^2 - 18p + 27$$

**2 Solve Equations by Factoring** Some equations can be solved by factoring. Consider the following.

3(0) = 0

0(2-2)=0

-312(0) = 0

0(0.25) = 0

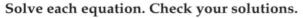
Notice that in each case, at least one of the factors is 0. These examples are demonstrations of the **Zero Product Property**.

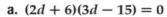
# KeyConcept Zero Product Property

Words If the product of two factors is 0, then at least one of the factors must be 0.

Symbols For any real numbers a and b, if ab = 0, then a = 0, b = 0, or both a and b equal zero.

## **Example 4** Solve Equations





$$(2d+6)(3d-15) = 0$$

Original equation

$$2d + 6 = 0$$
 or  $3d - 15 = 0$ 

or 
$$3a - 15 = 0$$

**Zero Product Property** 

$$2d = -6$$

$$3d = 15$$

Solve each equation.

$$d = -3$$

$$d = 5$$

Divide.

The roots are -3 and 5.

**CHECK** Substitute -3 and 5 for d in the original equation.

$$(2d + 6)(3d - 15) = 0$$

$$(2d + 6)(3d - 15) = 0$$
  $(2d + 6)(3d - 15) = 0$ 

$$[2(-3) + 6][3(-3) - 15] \stackrel{?}{=} 0$$
  $[2(5) + 6][3(5) - 15] \stackrel{?}{=} 0$ 

$$[2(5) + 6][3(5) - 15] \stackrel{?}{=} 0$$

$$(-6+6)(-9-15)\stackrel{?}{=}0$$
  $(10+6)(15-15)\stackrel{?}{=}0$ 

$$(10 \pm 6)(15 - 15) \stackrel{?}{=} 0$$

$$(0)(-24) \stackrel{?}{=} 0$$

$$16(0) \stackrel{?}{=} 0$$

$$0 = 0$$

$$0 = 0$$



**b.** 
$$c^2 = 3c$$

$$c^2 = 3c$$

Original equation

$$c^2 - 3c = 0$$

Subtract 3c from each side to get 0 on one side of the equation.

$$c(c-3)=0$$

Factor by using the GCF to get the form ab = 0.

$$c = 0$$
 or  $c - 3 = 0$  Zero Product Property

$$c = 3$$

c = 3 Solve each equation.

The roots are 0 and 3.

Check by substituting 0 and 3 for c.

#### GuidedPractice

**4A.** 
$$3n(n+2) = 0$$
 **4B.**  $8b^2 - 40b = 0$  **4C.**  $x^2 = -10x$ 

**4B.** 
$$8b^2 - 40b = 0$$

**4C.** 
$$x^2 = -10x$$