SMART START

A square has an area of 144 in^2. What is the length, to the nearest inch, of the diagonal?

- 15 inches
- 16 inches
- 17 inches
- 18 inches

Brianna starts from home and walks 6 km west. She then takes a left and walks 8 km south.

Using the *shortest* distance between her home and current location, how far is Brianna from her home in the scenario described?

- $\sqrt{2}$km
- $2\sqrt{7}$km
- 10 km
- 14 km

Example 1 Use the Distributive Property

Use the Distributive Property to factor each polynomial.

a. $27y^2 + 18y$

Find the GCF of each term.

$27y^2 = 3 \cdot 3 \cdot 3 \cdot y \cdot y$ — Factor each term.

$18y = 2 \cdot 3 \cdot 3 \cdot y$ — Circle common factors.

GCF $= 3 \cdot 3 \cdot y$ or $9y$

Write each term as the product of the GCF and the remaining factors. Use the Distributive Property to *factor out* the GCF.

$27y^2 + 18y = 9y(3y) + 9y(2)$ — Rewrite each term using the GCF.

$= 9y(3y + 2)$ — Distributive Property

b. $-4a^2b - 8ab^2 + 2ab$

$-4a^2b = -1 \cdot 2 \cdot 2 \cdot a \cdot a \cdot b$ — Factor each term.

$-8ab^2 = -1 \cdot 2 \cdot 2 \cdot 2 \cdot a \cdot b \cdot b$ — Circle common factors.

$2ab = 2 \cdot a \cdot b$

GCF $= 2 \cdot a \cdot b$ or $2ab$

$-4a^2b - 8ab^2 + 2ab = 2ab(-2a) - 2ab(4b) + 2ab(1)$ — Rewrite each term using the GCF.

$= 2ab(-2a - 4b + 1)$ — Distributive Property

Guided Practice

1A. $15w - 3v$ **$3(5w - v)$**

1B. $7u^2t^2 + 21ut^2 - ut$ **$ut(7ut + 21t - 1)$**

Ind St Clair/Blend Images/Getty Images

Example 2 Factor by Grouping

Factor $4qr + 8r + 3q + 6$.

$4qr + 8r + 3q + 6$	Original expression
$= (4qr + 8r) + (3q + 6)$	Group terms with common factors.
$= 4r(q + 2) + 3(q + 2)$	Factor the GCF from each group.

Notice that $(q + 2)$ is common in both groups, so it becomes the GCF.

$= (4r + 3)(q + 2)$	Distributive Property

Guided Practice

Factor each polynomial.

2A. $rn + 5n - r - 5$ **$(r + 5)(n - 1)$**

2B. $3np + 15p - 4n - 20$ **$(n + 5)(3p - 4)$**

Example 3 Factor by Grouping with Additive Inverses

Factor $2mk - 12m + 42 - 7k$.

$2mk - 12m + 42 - 7k$

$= (2mk - 12m) + (42 - 7k)$	Group terms with common factors.
$= 2m(k - 6) + 7(6 - k)$	Factor the GCF from each group.
$= 2m(k - 6) + 7[(-1)(k - 6)]$	$6 - k = -1(k - 6)$
$= 2m(k - 6) - 7(k - 6)$	Associative Property
$= (2m - 7)(k - 6)$	Distributive Property

Guided Practice

Factor each polynomial.

3A. $c - 2cd + 8d - 4$

3B. $3p - 2p^2 - 18p + 27$

Example 4 Solve Equations

Solve each equation. Check your solutions.

a. $(2d + 6)(3d - 15) = 0$

$(2d + 6)(3d - 15) = 0$	Original equation
$2d + 6 = 0$ or $3d - 15 = 0$	Zero Product Property
$2d = -6$ $\quad$ $3d = 15$	Solve each equation.
$d = -3$ $\quad$ $d = 5$	Divide.

The roots are -3 and 5.

CHECK Substitute -3 and 5 for d in the original equation.

$$(2d + 6)(3d - 15) = 0 \qquad (2d + 6)(3d - 15) = 0$$
$$[2(-3) + 6][3(-3) - 15] \stackrel{?}{=} 0 \qquad [2(5) + 6][3(5) - 15] \stackrel{?}{=} 0$$
$$(-6 + 6)(-9 - 15) \stackrel{?}{=} 0 \qquad (10 + 6)(15 - 15) \stackrel{?}{=} 0$$
$$(0)(-24) \stackrel{?}{=} 0 \qquad 16(0) \stackrel{?}{=} 0$$
$$0 = 0 \checkmark \qquad 0 = 0 \checkmark$$

b. $c^2 = 3c$

$c^2 = 3c$	Original equation
$c^2 - 3c = 0$	Subtract $3c$ from each side to get 0 on one side of the equation.
$c(c - 3) = 0$	Factor by using the GCF to get the form $ab = 0$.
$c = 0$ or $c - 3 = 0$	Zero Product Property
$c = 3$	Solve each equation.
The roots are 0 and 3.	Check by substituting 0 and 3 for c.

Guided Practice

4A. $3n(n + 2) = 0$ **4B.** $8b^2 - 40b = 0$ **4C.** $x^2 = -10x$

8-6 page 503

KeyConcept **Factoring $x^2 + bx + c$**

Words	To factor trinomials in the form $x^2 + bx + c$, find two integers, m and p, with a sum of b and a product of c. Then write $x^2 + bx + c$ as $(x + m)(x + p)$.
Symbols	$x^2 + bx + c = (x + m)(x + p)$ when $m + p = b$ and $mp = c$.
Example	$x^2 + 6x + 8 = (x + 2)(x + 4)$, because $2 + 4 = 6$ and $2 \cdot 4 = 8$.

Example 1 b and c are Positive

Factor $x^2 + 9x + 20$.

In this trinomial, $b = 9$ and $c = 20$. Since c is positive and b is positive, you need to find two positive factors with a sum of 9 and a product of 20. Make an organized list of the factors of 20, and look for the pair of factors with a sum of 9.

Factors of 20	Sum of Factors
1, 20	21
2, 10	12
4, 5	9

The correct factors are 4 and 5.

$x^2 + 9x + 20 = (x + m)(x + p)$ Write the pattern.

$= (x + 4)(x + 5)$ $m = 4$ and $p = 5$

CHECK You can check this result by multiplying the two factors. The product should be equal to the original expression.

$(x + 4)(x + 5) = x^2 + 5x + 4x + 20$ FOIL Method

$= x^2 + 9x + 20$ ✓ Simplify.

Guided Practice

Factor each polynomial.

1A. $d^2 + 11d + 24$

1B. $9 + 10t + t^2$

Example 2 *b* is Negative and *c* is Positive

Factor $x^2 - 8x + 12$. Confirm your answer using a graphing calculator.

In this trinomial, $b = -8$ and $c = 12$. Since c is positive and b is negative, you need to find two negative factors with a sum of -8 and a product of 12.

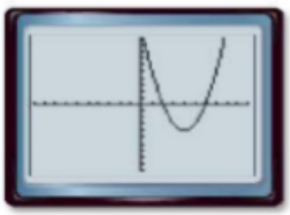
Factors of 12	Sum of Factors
$-1, -12$	-13
$-2, -6$	-8
$-3, -4$	-7

The correct factors are -2 and -6.

$x^2 - 8x + 12 = (x + m)(x + p)$ Write the pattern.

$= (x - 2)(x - 6)$ $m = -2$ and $p = -6$

CHECK Graph $y = x^2 - 8x + 12$ and $y = (x - 2)(x - 6)$ on the same screen. Since only one graph appears, the two graphs must coincide. Therefore, the trinomial has been factored correctly. ✓



[−10, 10] scl: 1 by [−10, 10] scl: 1

Guided Practice

Factor each polynomial.

2A. $21 - 22m + m^2$

2B. $w^2 - 11w + 28$

Example 3 c is Negative

Factor each polynomial. Confirm your answers using a graphing calculator.

a. $x^2 + 2x - 15$

In this trinomial, $b = 2$ and $c = -15$. Since c is negative, the factors m and p have opposite signs. So either m or p is negative, but not both. Since b is positive, the factor with the greater absolute value is also positive.

List the factors of -15, where one factor of each pair is negative. Look for the pair of factors with a sum of 2.

Factors of -15	Sum of Factors
$-1, 15$	14
$-3, 5$	2

The correct factors are -3 and 5.

$x^2 + 2x - 15 = (x + m)(x + p)$ Write the pattern.

$= (x - 3)(x + 5)$ $m = -3$ and $p = 5$

CHECK $(x - 3)(x + 5) = x^2 + 5x - 3x - 15$ FOIL Method

$= x^2 + 2x - 15$ ✓ Simplify.

b. $x^2 - 7x - 18$

In this trinomial, $b = -7$ and $c = -18$. Either m or p is negative, but not both. Since b is negative, the factor with the greater absolute value is also negative.

List the factors of -18, where one factor of each pair is negative. Look for the pair of factors with a sum of -7.

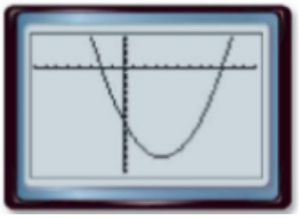
Factors of -18	Sum of Factors
$1, -18$	-17
$2, -9$	-7
$3, -6$	-3

The correct factors are 2 and -9.

$x^2 - 7x - 18 = (x + m)(x + p)$ Write the pattern.

$= (x + 2)(x - 9)$ $m = 2$ and $p = -9$

CHECK Graph $y = x^2 - 7x - 18$ and $y = (x + 2)(x - 9)$ on the same screen.



Guided Practice

3A. $y^2 + 13y - 48$

3B. $r^2 - 2r - 24$

2 Solve Equations by Factoring

A **quadratic equation** can be written in the standard form $ax^2 + bx + c = 0$, where $a \neq 0$. Some equations of the form $x^2 + bx + c = 0$ can be solved by factoring and then using the Zero Product Property.

Example 4 Solve an Equation by Factoring

Solve $x^2 + 6x = 27$. Check your solutions.

$x^2 + 6x = 27$	Original equation
$x^2 + 6x - 27 = 0$	Subtract 27 from each side.
$(x - 3)(x + 9) = 0$	Factor.
$x - 3 = 0$ or $x + 9 = 0$	Zero Product Property
$x = 3$ $\quad$ $x = -9$	Solve each equation.

The roots are 3 and -9.

CHECK Substitute 3 and -9 for x in the original equation.

$$x^2 + 6x = 27$$
$$(3)^2 + 6(3) \stackrel{?}{=} 27$$
$$9 + 18 \stackrel{?}{=} 27$$
$$27 = 27 \checkmark$$

$$x^2 + 6x = 27$$
$$(-9)^2 + 6(-9) \stackrel{?}{=} 27$$
$$81 - 54 \stackrel{?}{=} 27$$
$$27 = 27 \checkmark$$

Guided Practice

Solve each equation. Check your solutions.

4A. $z^2 - 3z = 70$

4B. $x^2 + 3x - 18 = 0$