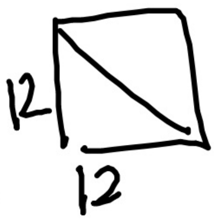
SMART START

1

A square has an area of 144 in^2. What is the length, to the nearest inch, of the diagonal?

- 15 inches
- 16 inches
- 17 inches
- 18 inches



$a^2+b^2=c^2$

$12^2+12^2=c^2$

$144+144=c^2$

$\sqrt{288}=\sqrt{c^2}$

17

2

Brianna starts from home and walks 6 km west. She then takes a left and walks 8 km south.

Using the *shortest* distance between her home and current location, how far is Brianna from her home in the scenario described?

- $\sqrt{2}$km
- $2\sqrt{7}$km
- 10 km
- 14 km

$6^2+8^2=c^2$

$36+64=c^2$

$\sqrt{100}=\sqrt{c^2}$

$10=c$

Example 1 Use the Distributive Property

Use the Distributive Property to factor each polynomial.

a. $27y^2 + 18y$

Find the GCF of each term.

$27y^2 = 3 \cdot 3 \cdot 3 \cdot y \cdot y$ — Factor each term.

$18y = 2 \cdot 3 \cdot 3 \cdot y$ — Circle common factors.

$\text{GCF} = 3 \cdot 3 \cdot y$ or $9y$

Write each term as the product of the GCF and the remaining factors. Use the Distributive Property to *factor out* the GCF.

$27y^2 + 18y = 9y(3y) + 9y(2)$ — Rewrite each term using the GCF.

$= 9y(3y + 2)$ — Distributive Property

$27y^2 + 18y$

$9y(3y+2)$

b. $-4a^2b - 8ab^2 + 2ab$

$-4a^2b = -1 \cdot 2 \cdot 2 \cdot a \cdot a \cdot b$ — Factor each term.

$-8ab^2 = -1 \cdot 2 \cdot 2 \cdot 2 \cdot a \cdot b \cdot b$ — Circle common factors.

$2ab = 2 \cdot a \cdot b$

$\text{GCF} = 2 \cdot a \cdot b$ or $2ab$

$-4a^2b - 8ab^2 + 2ab = 2ab(-2a) - 2ab(4b) + 2ab(1)$ — Rewrite each term using the GCF.

$= 2ab(-2a - 4b + 1)$ — Distributive Property

GuidedPractice

1A. $15w - 3v$ $3(5w - v)$

1B. $7u^2t^2 + 21ut^2 - ut$ $ut(7ut + 21t - 1)$

Example 2 Factor by Grouping

Factor $4qr + 8r + 3q + 6$.

$4qr + 8r + 3q + 6$	Original expression
$= (4qr + 8r) + (3q + 6)$	Group terms with common factors.
$= 4r(q + 2) + 3(q + 2)$	Factor the GCF from each group.

Notice that $(q + 2)$ is common in both groups, so it becomes the GCF.

$= (4r + 3)(q + 2)$	Distributive Property

GuidedPractice

Factor each polynomial.

2A. $rn + 5n - r - 5$ $(r + 5)(n - 1)$ **2B.** $3np + 15p - 4n - 20$ $(n + 5)(3p - 4)$

$(4qr + 8r) + (3q + 6)$

$4r(q + 2) + 3(q + 2)$

$(q + 2)(4r + 3)$

Example 3 Factor by Grouping with Additive Inverses

Factor $2mk - 12m + 42 - 7k$.

$$2mk - 12m + 42 - 7k$$

$= (2mk - 12m) + (42 - 7k)$	Group terms with common factors.
$= 2m(k - 6) + 7(6 - k)$	Factor the GCF from each group.
$= 2m(k - 6) + 7[(-1)(k - 6)]$	$6 - k = -1(k - 6)$
$= 2m(k - 6) - 7(k - 6)$	Associative Property
$= (2m - 7)(k - 6)$	Distributive Property

GuidedPractice

Factor each polynomial.

3A. $c - 2cd + 8d - 4$

3B. $3p - 2p^2 - 18p + 27$

$(c-2cd)+(8d-4)$

$c(1-2d)+4(2d-1)$

$c(1-2d)+-4(-2d+1)$

$c(1-2d)+-4(1-2d)$

$(c+-4)(1-2d)$

$(2mk-12m)+(42-7k)$

$2m(k-6)+7(6-k)$

$2m(k-6)+-7(-6+k)$

$2m(k-6)+-7(k-6)$

$(2m-7)(k-6)$

Example 4 Solve Equations

Solve each equation. Check your solutions.

a. $(2d + 6)(3d - 15) = 0$

$(2d + 6)(3d - 15) = 0$		Original equation
$2d + 6 = 0$ or	$3d - 15 = 0$	Zero Product Property
$2d = -6$	$3d = 15$	Solve each equation.
$d = -3$	$d = 5$	Divide.

The roots are -3 and 5.

CHECK Substitute -3 and 5 for d in the original equation.

$(2d + 6)(3d - 15) = 0$	$(2d + 6)(3d - 15) = 0$
$[2(-3) + 6][3(-3) - 15] \stackrel{?}{=} 0$	$[2(5) + 6][3(5) - 15] \stackrel{?}{=} 0$
$(-6 + 6)(-9 - 15) \stackrel{?}{=} 0$	$(10 + 6)(15 - 15) \stackrel{?}{=} 0$
$(0)(-24) \stackrel{?}{=} 0$	$16(0) \stackrel{?}{=} 0$
$0 = 0$ ✓	$0 = 0$ ✓

$2d + 6 = 0$
$-6 \quad -6$
$\frac{2d}{2} = \frac{-6}{2}$
$d = -3$

$3d - 15 = 0$
$+15 \quad +15$
$\frac{3d}{3} = \frac{15}{3}$
$d = 5$

b. $c^2 = 3c$

$c^2 = 3c$	Original equation
$c^2 - 3c = 0$	Subtract $3c$ from each side to get 0 on one side of the equation.
$c(c - 3) = 0$	Factor by using the GCF to get the form $ab = 0$.
$c = 0$ or $c - 3 = 0$	Zero Product Property
$c = 3$	Solve each equation.
The roots are 0 and 3.	Check by substituting 0 and 3 for c.

$c^2 = 3c$
$c^2 - 3c = 0$
$c(c - 3) = 0$
$c = 0 \qquad c - 3 = 0$
$c = 3$

GuidedPractice

4A. $3n(n + 2) = 0$ **4B.** $8b^2 - 40b = 0$ **4C.** $x^2 = -10x$

A $3n(n+2)=0$

$3n=0 \quad n+2=0$

$n=0 \quad n=-2$

B $8b^2-40b=0$

$8b(b-5)=0$

$8b=0 \quad b-5=0$

$b=0 \quad b=5$

C $x^2=-10x$

$x^2+10x=0$

$x(x+10)=0$

$x=0 \quad x+10=0$

$x=0 \quad x=-10$

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b x

KeyConcept **Factoring $x^2 + bx + c$**

Words	To factor trinomials in the form $x^2 + bx + c$, find two integers, m and p, with a sum of b and a product of c. Then write $x^2 + bx + c$ as $(x + m)(x + p)$.
Symbols	$x^2 + bx + c = (x + m)(x + p)$ when $m + p = b$ and $mp = c$.
Example	$x^2 + 6x + 8 = (x + 2)(x + 4)$, because $2 + 4 = 6$ and $2 \cdot 4 = 8$.

Example 1 b and c are Positive

Factor $x^2 + 9x + 20$.

In this trinomial, $b = 9$ and $c = 20$. Since c is positive and b is positive, you need to find two positive factors with a sum of 9 and a product of 20. Make an organized list of the factors of 20, and look for the pair of factors with a sum of 9.

Factors of 20	Sum of Factors
1, 20	21
2, 10	12
4, 5	9

The correct factors are 4 and 5.

$$x^2 + 9x + 20 = (x + m)(x + p)$$ Write the pattern.

$$= (x + 4)(x + 5)$$ $m = 4$ and $p = 5$

CHECK You can check this result by multiplying the two factors. The product should be equal to the original expression.

$$(x + 4)(x + 5) = x^2 + 5x + 4x + 20$$ FOIL Method

$$= x^2 + 9x + 20 \checkmark$$ Simplify.

Guided Practice

Factor each polynomial.

1A. $d^2 + 11d + 24$

1B. $9 + 10t + t^2$

$x^2+9x+20$

~~1~~	~~20~~
~~2~~	~~10~~
4	5

$(x+4)(x+5)$

$x^2+5x+4x+20$

$x^2+9x+20$

(A) $d^2+11x+24$

1	24
~~2~~	~~12~~
3	8
4	6

$(x+3)(x+8)$

(B) $9+10t+t^2$

$t^2+10t+9$

1	9
3	3

$(t+1)(t+9)$

Example 2 b is Negative and c is Positive

Factor $x^2 - 8x + 12$. Confirm your answer using a graphing calculator.

In this trinomial, $b = -8$ and $c = 12$. Since c is positive and b is negative, you need to find two negative factors with a sum of -8 and a product of 12.

Factors of 12	Sum of Factors
$-1, -12$	-13
$-2, -6$	-8
$-3, -4$	-7

The correct factors are -2 and -6.

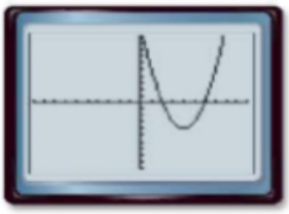
-1	12
-2	6
-3	4

$(x-2)(x-6)$

$x^2 - 8x + 12 = (x + m)(x + p)$ Write the pattern.

$= (x - 2)(x - 6)$ $m = -2$ and $p = -6$

CHECK Graph $y = x^2 - 8x + 12$ and $y = (x - 2)(x - 6)$ on the same screen. Since only one graph appears, the two graphs must coincide. Therefore, the trinomial has been factored correctly. ✓



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Guided Practice

Factor each polynomial.

2A. $21 - 22m + m^2$

2B. $w^2 - 11w + 28$

Example 3 c is Negative

Factor each polynomial. Confirm your answers using a graphing calculator.

a. $x^2 + 2x - 15$

In this trinomial, $b = 2$ and $c = -15$. Since c is negative, the factors m and p have opposite signs. So either m or p is negative, but not both. Since b is positive, the factor with the greater absolute value is also positive.

List the factors of -15, where one factor of each pair is negative. Look for the pair of factors with a sum of 2.

Factors of -15	Sum of Factors
$-1, 15$	14
$-3, 5$	2

The correct factors are -3 and 5.

$x^2 + 2x - 15 = (x + m)(x + p)$ Write the pattern.

$= (x - 3)(x + 5)$ $m = -3$ and $p = 5$

CHECK $(x - 3)(x + 5) = x^2 + 5x - 3x - 15$ FOIL Method

$= x^2 + 2x - 15$ ✓ Simplify.

-1	15
-3	5

$(x-3)(x+5)$

b. $\mathbf{x^2 - 7x - 18}$

In this trinomial, $b = -7$ and $c = -18$. Either m or p is negative, but not both. Since b is negative, the factor with the greater absolute value is also negative.

List the factors of -18, where one factor of each pair is negative. Look for the pair of factors with a sum of -7.

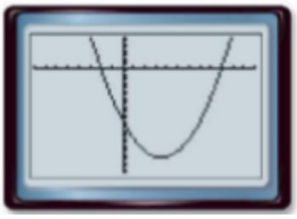
Factors of -18	Sum of Factors
$1, -18$	-17
$2, -9$	-7
$3, -6$	-3

The correct factors are 2 and -9.

$x^2 - 7x - 18 = (x + m)(x + p)$ — Write the pattern.

$= (x + 2)(x - 9)$ — $m = 2$ and $p = -9$

CHECK Graph $y = x^2 - 7x - 18$ and $y = (x + 2)(x - 9)$ on the same screen.



GuidedPractice

3A. $y^2 + 13y - 48$

3B. $r^2 - 2r - 24$

~~1~~	~~48~~
~~2~~	~~24~~
3	16
4	12
6	8

$(y-3)(y+16)$

~~1~~	~~24~~
~~2~~	~~12~~
~~3~~	~~8~~
4	-6

$(r+4)(r-6)$

2 Solve Equations by Factoring

A **quadratic equation** can be written in the standard form $ax^2 + bx + c = 0$, where $a \neq 0$. Some equations of the form $x^2 + bx + c = 0$ can be solved by factoring and then using the Zero Product Property.

Example 4 Solve an Equation by Factoring

Solve $x^2 + 6x = 27$. Check your solutions.

$x^2 + 6x = 27$	Original equation
$x^2 + 6x - 27 = 0$	Subtract 27 from each side.
$(x - 3)(x + 9) = 0$	Factor.
$x - 3 = 0$ or $x + 9 = 0$	Zero Product Property
$x = 3$ $\quad$ $x = -9$	Solve each equation.

The roots are 3 and −9.

CHECK Substitute 3 and −9 for x in the original equation.

$$x^2 + 6x = 27 \qquad x^2 + 6x = 27$$
$$(3)^2 + 6(3) \stackrel{?}{=} 27 \qquad (-9)^2 + 6(-9) \stackrel{?}{=} 27$$
$$9 + 18 \stackrel{?}{=} 27 \qquad 81 - 54 \stackrel{?}{=} 27$$
$$27 = 27 \checkmark \qquad 27 = 27 \checkmark$$

$x^2+6x=27$

$x^2+6x-27=0$

−1	27
−3	9

$(x-3)(x+9)=0$

$x-3=0 \quad x+9=0$

$x=3 \quad x=-9$

Guided Practice

Solve each equation. Check your solutions.

4A. $z^2 - 3z = 70$

4B. $x^2 + 3x - 18 = 0$