8-1 Adding and Subtracting Polynomials

Write each polynomial in standard form.

11. $x + 2 + 3x^2$

12. $1 - x^4$

13. $2 + 3x + x^2$

14. $3x^5 - 2 + 6x - 2x^2 + x^3$

Find each sum or difference.

15. $(x^3 + 2) + (-3x^3 - 5)$

16. $a^2 + 5a - 3 - (2a^2 - 4a + 3)$

17. $(4x - 3x^2 + 5) + (2x^2 - 5x + 1)$

18. **PICTURE FRAMES** Jean is framing a painting that is a rectangle. What is the perimeter of the frame?

$5x + 3$

$2x^2 - 3x + 1$

Example 1

Write $3 - x^2 + 4x$ in standard form.

Step 1 Find the degree of each term.

3:	degree 0
$-x^2$:	degree 2
$4x$:	degree 1

Step 2 Write the terms in descending order of degree.

$3 - x^2 + 4x = -x^2 + 4x + 3$

Example 2

Find $(8r^2 + 3r) - (10r^2 - 5)$.

$(8r^2 + 3r) - (10r^2 - 5)$

$= (8r^2 + 3r) + (-10r^2 + 5)$ Use the additive inverse.

$= (8r^2 - 10r^2) + 3r + 5$ Group like terms.

$= -2r^2 + 3r + 5$ Add like terms.

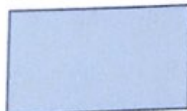## 8-2 Multiplying a Polynomial by a Monomial

Solve each equation.

19. $x^2(x + 2) = x(x^2 + 2x + 1)$

20. $2x(x + 3) = 2(x^2 + 3)$

21. $2(4w + w^2) - 6 = 2w(w - 4) + 10$

22. **GEOMETRY** Find the area of the rectangle.

$3x$

$x^2 + x - 7$

Example 3

Solve $m(2m - 5) + m = 2m(m - 6) + 16$.

$m(2m - 5) + m = 2m(m - 6) + 16$

$2m^2 - 5m + m = 2m^2 - 12m + 16$

$2m^2 - 4m = 2m^2 - 12m + 16$

$-4m = -12m + 16$

$8m = 16$

$m = 2$

8-3 Multiplying Polynomials

Find each product.

23. $(x - 3)(x + 7)$

24. $(3a - 2)(6a + 5)$

25. $(3r - 7t)(2r + 5t)$

26. $(2x + 5)(5x + 2)$

27. **PARKING LOT** The parking lot shown is to be paved. What is the area to be paved?

$2x + 3$

$5x - 4$

Example 4

Find $(6x - 5)(x + 4)$.

$(6x - 5)(x + 4)$

     F   O   I   L

$= (6x)(x) + (6x)(4) + (-5)(x) + (-5)(4)$

$= 6x^2 + 24x - 5x - 20$ Multiply.

$= 6x^2 + 19x - 20$ Combine like terms.

8-4 Special Products

Find each product.

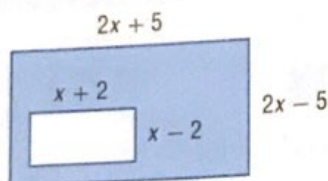28. $(x + 5)(x - 5)$

29. $(3x - 2)^2$

30. $(5x + 4)^2$

31. $(2x - 3)(2x + 3)$

32. $(2r + 5t)^2$

33. $(3m - 2)(3m + 2)$

34. **GEOMETRY** Write an expression to represent the area of the shaded region.

2x + 5

x + 2

2x − 5

x − 2

Example 5

Find $(x - 7)^2$.

$$(a - b)^2 = a^2 - 2ab + b^2$$

Square of a Difference

$$(x - 7)^2 = x^2 - 2(x)(7) + (-7)^2$$

$a = x$ and $b = 7$

$$= x^2 - 14x + 49$$

Simplify.

Example 6

Find $(5a - 4)(5a + 4)$.

$$(a + b)(a - b) = a^2 - b^2$$

Product of a Sum and Difference

$$(5a - 4)(5a + 4) = (5a)^2 - (4)^2$$

$a = 5a$ and $b = 4$

$$= 25a^2 - 16$$

Simplify.

8-5 Using the Distributive Property

Use the Distributive Property to factor each polynomial.

35. $12x + 24y$

36. $14x^2y - 21xy + 35xy^2$

37. $8xy - 16x^3y + 10y$

38. $a^2 - 4ac + ab - 4bc$

39. $2x^2 - 3xz - 2xy + 3yz$

40. $24am - 9an + 40bm - 15bn$
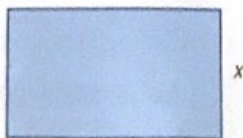
Solve each equation. Check your solutions.

41. $x(3x - 6) = 0$

42. $6x^2 = 12x$

43. $x^2 = 3x$

44. $3x^2 = 5x$

45. **GEOMETRY** The area of the rectangle shown is $x^3 - 2x^2 + 5x$ square units. What is the length?

x

Example 7

Factor $12y^2 + 9y + 8y + 6$.

$$12y^2 + 9y + 8y + 6$$

$$= (12y^2 + 9y) + (8y + 6)$$

Group terms with common factors.

$$= 3y(4y + 3) + 2(4y + 3)$$

Factor the GCF from each group.

$$= (4y + 3)(3y + 2)$$

Distributive Property

Example 8

Solve $x^2 - 6x = 0$. Check your solutions.

Write the equation so that it is of the form $ab = 0$.

$$x^2 - 6x = 0$$

Original equation

$$x(x - 6) = 0$$

Factor by using the GCF.

$$x = 0 \quad \text{or} \quad x - 6 = 0$$

Zero Product Property

$$x = 6$$

Solve.

The roots are 0 and 6. Check by substituting 0 and 6 for x in the original equation.

8-6 Solving $x^2 + bx + c = 0$

Factor each trinomial. Confirm your answers using a graphing calculator.

46. $x^2 - 8x + 15$ **47.** $x^2 + 9x + 20$

48. $x^2 - 5x - 6$ **49.** $x^2 + 3x - 18$

Solve each equation. Check your solutions.

50. $x^2 + 5x - 50 = 0$

51. $x^2 - 6x + 8 = 0$

52. $x^2 + 12x + 32 = 0$

53. $x^2 - 2x - 48 = 0$

54. $x^2 + 11x + 10 = 0$

55. ART An artist is working on a painting that is 3 inches longer than it is wide. The area of the painting is 154 square inches. What is the length of the painting?

Example 9

Factor $x^2 + 10x + 21$

$b = 10$ and $c = 21$, so $m + p$ is positive and mp is positive. Therefore, m and p must both be positive. List the positive factors of 21, and look for the pair of factors with a sum of 10.

Factors of 21	Sum of 10
1, 21	22
3, 7	10

The correct factors are 3 and 7.

$x^2 + 10x + 21 = (x + m)(x + p)$ Write the pattern.

$\qquad\qquad\qquad = (x + 3)(x + 7)$ $m = 3$ and $p = 7$


8-7 Solving $ax^2 + bx + c = 0$

Factor each trinomial, if possible. If the trinomial cannot be factored, write *prime*.

56. $12x^2 + 22x - 14$

57. $2y^2 - 9y + 3$

58. $3x^2 - 6x - 45$

59. $2a^2 + 13a - 24$

Solve each equation. Confirm your answers using a graphing calculator.

60. $40x^2 + 2x = 24$

61. $2x^2 - 3x - 20 = 0$

62. $-16t^2 + 36t - 8 = 0$

63. $6x^2 - 7x - 5 = 0$

64. GEOMETRY The area of the rectangle shown is $6x^2 + 11x - 7$ square units. What is the width of the rectangle?
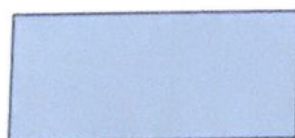
2x − 1

Example 10

Factor $12a^2 + 17a + 6$

$a = 12$, $b = 17$, and $c = 6$. Since b is positive, $m + p$ is positive. Since c is positive, mp is positive. So, m and p are both positive. List the factors of 12(6) or 72, where both factors are positive.

Factors of 72	Sum of 17
1, 72	73
2, 36	38
3, 24	27
4, 18	22
6, 12	18
8, 9	17

The correct factors are 8 and 9.

$12a^2 + 17a + 6 = 12a^2 + ma + pa + 6$

$\qquad\qquad\qquad = 12a^2 + 8a + 9a + 6$

$\qquad\qquad\qquad = (12a^2 + 8a) + (9a + 6)$

$\qquad\qquad\qquad = 4a(3a + 2) + 3(3a + 2)$

$\qquad\qquad\qquad = (3a + 2)(4a + 3)$

So, $12a^2 + 17a + 6 = (3a + 2)(4a + 3)$.

8-8 Differences of Squares

Factor each polynomial.

65. $y^2 - 81$

66. $64 - 25x^2$

67. $16a^2 - 21b^2$

68. $3x^2 - 3$

Solve each equation by factoring. Confirm your answers using a graphing calculator.

69. $a^2 - 25 = 0$

70. $9x^2 - 25 = 0$

71. $81 - y^2 = 0$

72. $x^2 - 5 = 20$

73. EROSION A boulder falls down a mountain into water 64 feet below. The distance d that the boulder falls in t seconds is given by the equation $d = 16t^2$. How long does it take the boulder to hit the water?

Example 11

Solve $x^2 - 4 = 12$ by factoring.

$x^2 - 4 = 12$	Original equation
$x^2 - 16 = 0$	Subtract 12 from each side.
$x^2 - (4)^2 = 0$	$16 = 4^2$
$(x + 4)(x - 4) = 0$	Factor the difference of squares.
$x + 4 = 0$ or $x - 4 = 0$	Zero Product Property
$x = -4$ $x = 4$	Solve each equation.

The solutions are -4 and 4.

8-9 Perfect Squares

Factor each polynomial, if possible. If the polynomial cannot be factored write *prime*.

74. $x^2 + 12x + 36$

75. $x^2 + 5x + 25$

76. $9y^2 - 12y + 4$

77. $4 - 28a + 49a^2$

78. $x^4 - 1$

79. $x^4 - 16x^2$

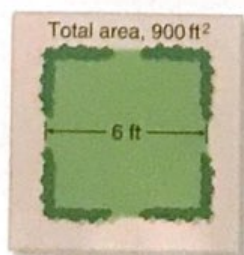Solve each equation. Confirm your answers using a graphing calculator.

80. $(x - 5)^2 = 121$

81. $4c^2 + 4c + 1 = 9$

82. $4y^2 = 64$

83. $16d^2 + 40d + 25 = 9$

84. LANDSCAPING A sidewalk of equal width is being built around a square yard. What is the width of the sidewalk?

Total area, 900 ft²

— 6 ft —

Example 12

Solve $(x - 9)^2 = 144$.

$(x - 9)^2 = 144$	Original equation
$x - 9 = \pm\sqrt{144}$	Square Root Property
$x - 9 = \pm 12$	$144 = 12 \cdot 12$
$x = 9 \pm 12$	Add 9 to each side.
$x = 9 + 12$ or $x = 9 - 12$	Zero Product Property
$x = 21$ $x = -3$	Solve.

CHECK

$(x - 9)^2 = 144$
$(21 - 9)^2 \stackrel{?}{=} 144$
$(12)^2 \stackrel{?}{=} 144$
$144 = 144$ ✔

$(x - 9)^2 = 144$
$(-3 - 9)^2 \stackrel{?}{=} 144$
$(-12)^2 \stackrel{?}{=} 144$
$144 = 144$ ✔