Math 2 U2.6 Solve Quadratics by Completing the Square Name: ____________

Some quadratic equations in the form of $ax^2+bx+c=0$ can be solved easily by factoring. For example, the equation $x^2+6x-16=0$ can be factored easily to $(x+8)(x-2)=0$ to give solutions of $x=-8 \text{ and } x=2$

When a quadratic equation cannot be factored using integers, you have two options. You can use the quadratic formula of you can use a method called **completing the square**. When a = 1, completing the square is the way to go (when a >1, use the quadratic formula).

Example 1: Solve $x^2+8x-10=0$ by completing the square.

Since it cannot be factored using integers, Write the equation in the form $ax^2+bx=-c$	$x^2+8x-10=0$ $x^2+8x=10$
Find $\frac{1}{2}$ of b and add the square of that number $(\frac{b}{2})^2$ to both sides of the equation	Think $b=8$ $\frac{1}{2}b=4$ and $4^2=16$ $x^2+8x=10$ $x^2+8x+16=10+16$
The left side is now a perfect square trinomial (PST), so factor it.	$(x+4)(x+4)=26$ $(x+4)^2=26$
Find the square root of each side.	$(x+4)^2=26$ $x+4=\pm\sqrt{26}$
Solve for x	$x=-4\pm\sqrt{26}$

...ve each quadratic by completing the square.

1) $a^2 + 2a - 3 = 0$

$a^2 + 2a = 3$
$a^2 + 2a + 1 = 4$
$(a+1)^2 = 4$
$a+1 = \pm 4$
$a = 1; -3$

2) $a^2 - 2a - 8 = 0$

$a^2 - 2a = 8$
$a^2 - 2a + 1 = 9$
$(a-1)^2 = 9$
$a - 1 = \pm 3$
$a = 4; -2$

3) $p^2 + 16p - 22 = 0$

$p^2 + 16p = 22$
$p^2 + 16p + 64 = 86$
$(p+8)^2 = 86$
$p + 8 = \sqrt{86}$
$p = -8 \pm \sqrt{86}$

4) $k^2 + 8k + 12 = 0$

$K^2 + 8K = -12$
$K^2 + 8K + 16 = 4$
$(K+4)^2 = 4$
$K + 4 = \pm 2$
$K = -2, -6$

5) $r^2 + 2r - 33 = 0$

$r^2 + 2r = 33$
$r^2 + 2r + 1 = 34$
$(r+1)^2 = 34$
$r + 1 = \pm\sqrt{34}$
$r = -1 \pm \sqrt{34}$

6) $a^2 - 2a - 48 = 0$

$a^2 - 2a = 48$
$a^2 - 2a + 1 = 49$
$(a-1)^2 = 49$
$a - 1 = \pm 7$
$8, -6$

7) $m^2 - 12m + 26 = 0$

$m^2 - 12m = -26$
$m^2 - 12m + 36 = \cancel{34}\ 10$
$(m-6)^2 = 10$
$m - 6 = \pm\sqrt{10}$
$m = 6 \pm \sqrt{10}$

8) $x^2 + 12x + 20 = 0$

$x^2 + 12x = -20$
$x^2 + 12x + 36 = 16$
$(x+6)^2 = 16$
$x + 6 = \pm 4$
$-2; -10$

9) $k^2 - 8k - 48 = 0$

$K^2 - 8K = 48$
$K^2 - 8K + 16 = 64$
$(K-4)^2 = 64$
$K - 4 = \pm 8$
$K = 12 \text{ or } -4$

10) $p^2 + 2p - 63 = 0$

$p^2 + 2p = 63$
$p^2 + 2p + 1 = 64$
$(p+1)^2 = 64$
$p + 1 = \pm 8$
$p = 7; -9$

11) $m^2 + 2m - 48 = -6$

$m^2 + 2m = 42$
$m^2 + 2m + 1 = 43$
$(m+1)^2 = 43$
$m + 1 = \pm\sqrt{43}$
$m = -1 \pm \sqrt{43}$

12) $p^2 - 8p + 21 = 6$

$p^2 - 8p = -15$
$p^2 - 8p + 16 = 1$
$(p-4)^2 = 1$
$p - 4 = \pm\sqrt{1}$
$p - 4 = \pm 1$
$5; -3$

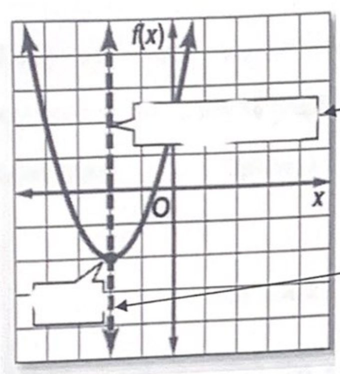
Name ____________

Graphing Quadratics Review Worksheet

Fill in each blank using the word bank.

vertex	minimum	axis of symmetry	x-intercepts
parabola	maximum	zeros/roots	~~$ax^2 + bx + c$~~

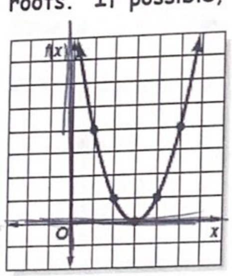
1. Standard form of a quadratic function is y = ax^2+bx+c
2. The shape of a quadratic equation is called a parabola



3. axis of symmetry
4. vertex

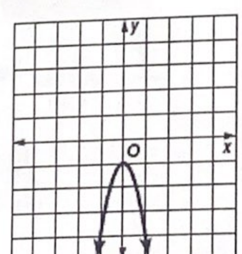
5. When the vertex is the highest point on the graph, we call that a maximum.
6. When the vertex is the lowest point on the graph, we call that a Minimum.
7. Our solutions are the x-intercepts.
8. Solutions to quadratic equations are called zeros/roots.

Determine whether the quadratic functions have two real roots, one real root, or no real roots. If possible, list the zeros of the function.



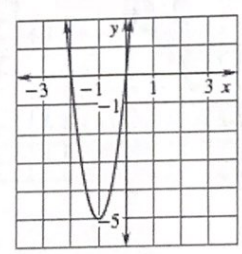
9. Number of roots: 1

Zero(s): 3



10. Number of roots: 0

Zero(s): none



11. Number of roots: 2

Zero(s): 0, -2

12. Given the graph, identify the following.

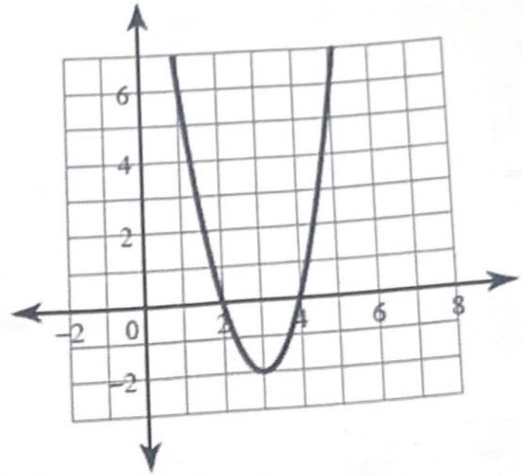
Axis of symmetry: x = 3

Vertex: (3, -2)

How many zeros: 2 which are: 2, 4

Domain: all real #s

Range: $y \geq -2$



Graph the following quadratic functions by using critical values and/or factoring.

You need three points to graph and don't necessarily need all the information listed.

Remember: **Option 1:** **If it factors, find the zeros.**

The middle of the two factors is the axis of symmetry.

Option 2: **If it doesn't factor, find the axis of symmetry with** $x = \frac{-b}{2a}$

Plug the x**-value into the original equation to find the** y**-value of the vertex. The** y**-intercept is at (0,** c**)**

13. $y = x^2 - 2x - 3$ factor or critical values?

x	-1	0	1	2	3
y	0	-3	-4	-3	0

Identify the zeros/roots: -1 and 3

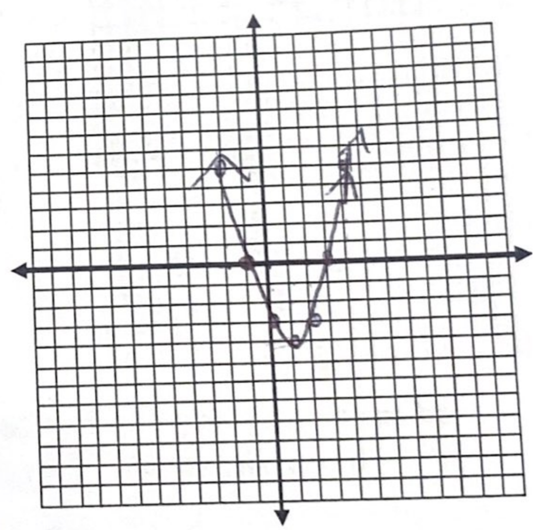
Does it have a minimum or maximum? minimum

Axis of symmetry: x = 1 Vertex: (1, -4

y-intercept: (0, -3)

Domain: all real #s Range: $y \geq -4$

Graph at least 5 points



14. $y = -x^2 - 4x + 5$ factor or critical values?

x	-5	-4	-3	-2	-1	0	1
y	0	5	8	9	8	5	0

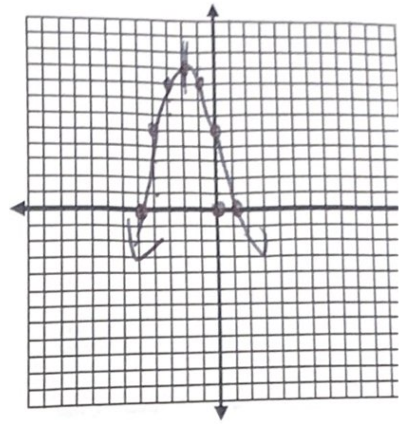
Identify the zeros/roots: -5 and 1

Does it have a minimum or maximum? Minimum y=9

Axis of symmetry: $x=-2$ Vertex: (-2, 9)

y-intercept: (0, 5) Graph at least 5 points

Domain: all real #s Range: $y \le 9$



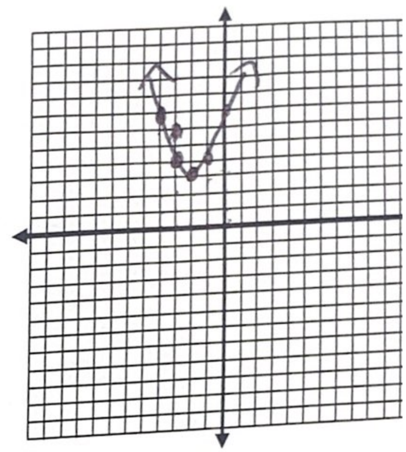
15. $y = x^2 + 4x + 7$ factor or critical values?

x	-4	-3	-2	-1	0
y	7	4	3	4	7

Axis of symmetry: $x=-2$ Vertex: (-2, 3)

Max or Min? min

y-intercept: (0, 7) Graph at least 3 points



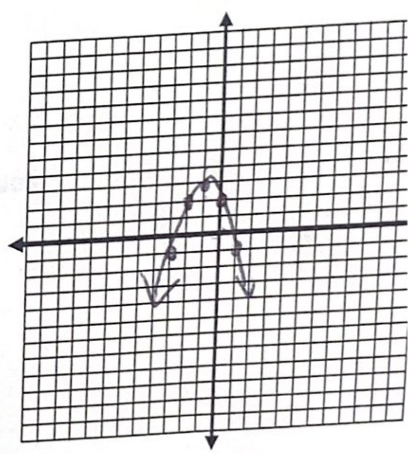
16. $y = -x^2 - 2x + 2$ factor or critical values?

x	-3	-2	-1	0	1
y	-1	2	3	2	-1

Axis of symmetry: $x=-1$ Vertex: (-1, 2)

Max or Min? max

y-intercept: (0, 2) Graph at least 5 points



17. A bottlenose dolphin jumps out of the water. The path the dolphin travels can be modeled by $h = -0.2d^2 + 2d$, where h represents the height of the dolphin and d represents horizontal distance.

$\frac{-b}{2a}$ $\frac{-2}{2(-.2)} = 5$

a. What is the maximum height the dolphin reaches? y=5 5ft

b. How far did the dolphin jump? 10ft (zero of the function)